Voxels & Collisions

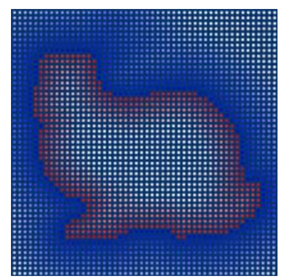
Last Time?

Today

- Implicit Surfaces
- Voxels
- Collisions
- Implicit Integration

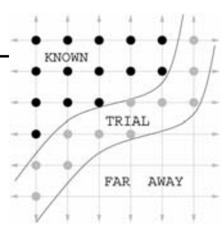
Implicit Surfaces

- For a sphere:
  $H(x,y,z) = x^2 + y^2 + z^2 - r^2$
- If $H(x,y,z) = 0$, on surface
- If $H(x,y,z) > 0$, outside surface
- If $H(x,y,z) < 0$, inside surface

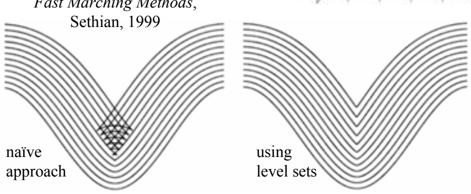


Level Sets

- Efficient method for computing signed distance field



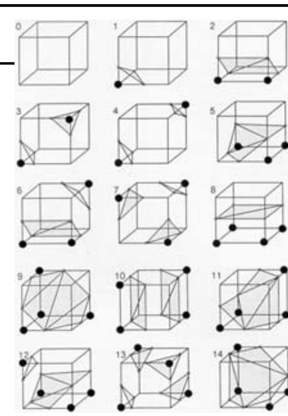
Level Set Methods and Fast Marching Methods, Sethian, 1999



Marching Cubes

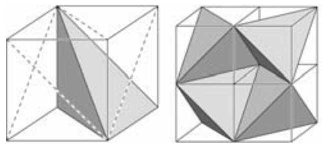
- Polygonization: extract triangle mesh from signed distance field

"Marching Cubes: A High Resolution 3D Surface Construction Algorithm", Lorensen and Cline, SIGGRAPH '87.



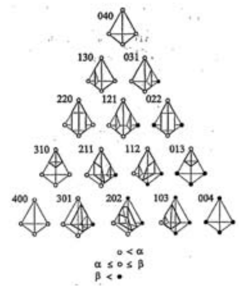
“Marching Tetrahedra”

Jules Bloomenthal
“An implicit surface polygonizer”
Graphics Gems IV



“When the Blobs Go Marching Two by Two”, *Jeff Lander,* Gamasutra

“Marching Tetrahedra”



“Interval volume tetrahedrization”
Visualization '97
Nielson & Sung

Similarly, we can create volumetric models:

Questions?

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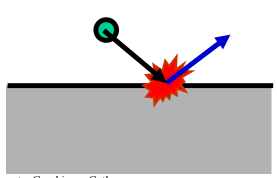
- Collisions
- Implicit Integration

Collisions

- Detection
- Response
- Overshooting problem
  (when we enter the solid)



Detecting Collisions

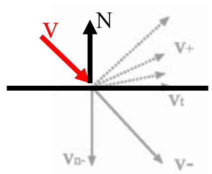
- Easy with implicit equations of surfaces
- H(x,y,z)=0 at surface
- H(x,y,z)<0 inside surface
- So just compute H and you know that you’re inside if it’s negative

- More complex with other surface definitions

Collision Response

- tangential velocity v_t unchanged
- normal velocity v_n reflects:

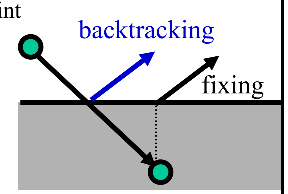
$$v = v_t + v_n$$
$$v \leftarrow v_t - \varepsilon v_n$$



- coefficient of restitution (1 for elastic, 0 for plastic)
- change of velocity = $-(1+\epsilon)v$
- change of momentum *Impulse* = $-m(1+\epsilon)v$

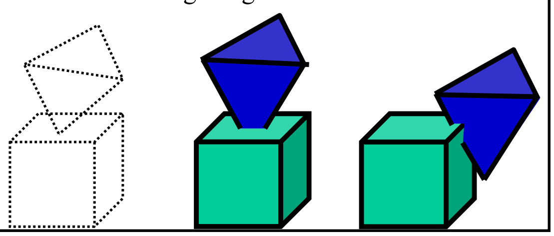
Collisions - Overshooting

- Usually, we detect collision when it's too late: we're already inside
- Solutions: back up
  - Compute intersection point
  - Compute response there
  - Advance for remaining fractional time step
- Other solution: Quick and dirty fixup
  - Just project back to object closest point



Collision Detection for Solids

- How to detect collision between 2 polyhedra?
- Need an inside/outside test
- Test if a vertex is inside the other polyhedron
- But treat also edge-edge intersection

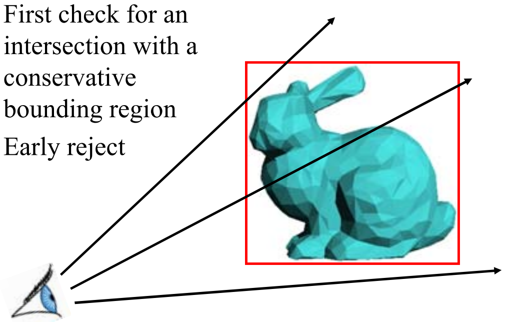


Cost of Detection?

- Test each edge with each face?
  - $O(N^2)$
- How would you detect collision between two bunnies?
  - $O(N^2)$ is too expensive!
  - Use spatial hierarchy

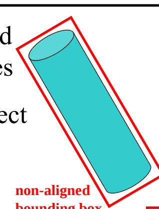
Conservative Bounding Region

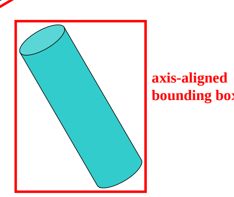
- First check for an intersection with a conservative bounding region
- Early reject



Conservative Bounding Regions

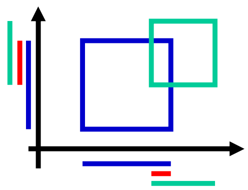
- tight → avoid false positives
- fast to intersect

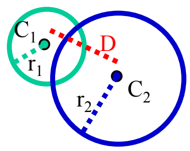




Overlap test

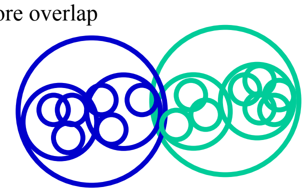
- Overlap between two axis-aligned boxes?
  - Check if the intervals along the 3 dimensions overlap
- Overlap test between two spheres?
  - $D(center_1, center_2) < r_1+r_2$





General Collision Detection

- Put a hierarchy around your objects
- Use the fast overlap test recursively
- Handle exact case at the leaves (when necessary)
- More difficult for self-collision (e.g. cloth)
  - Because there is more overlap



Reduced Deformation

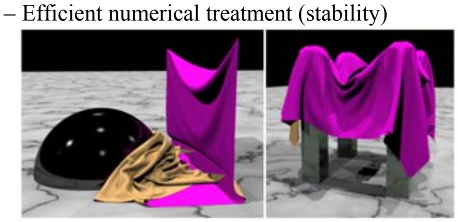
Doug L. James & Dinesh K. Pai
BD-Tree: Output-Sensitive Collision Detection for Reduced Deformable Models
SIGGRAPH 2004

- Collisions are expensive
- Deformation is expensive
- This is a lot of geometry!
- Simplify the simulation model

Cloth Collision

Robert Bridson, Ronald Fedkiw & John Anderson
Robust Treatment of Collisions, Contact and Friction for Cloth Animation
SIGGRAPH 2002

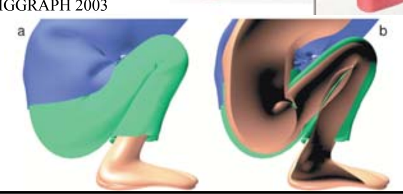
- A cloth has many points of contact
- Stays in contact
- Requires
  - Efficient collision detection
  - Efficient numerical treatment (stability)



Cloth Collision Challenges

- Detecting and resolving prior unavoidable intersections

Baraff, Witkin & Kass,
Untangling Cloth,
SIGGRAPH 2003



Questions?

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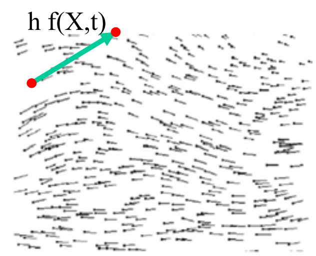
Euler Integration

- Timestep h, move in the direction of f(X, t)



Stiffness & Stability

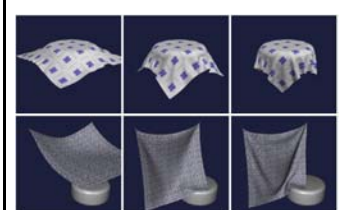
- With an explicit/forward integration scheme:
  $$\mathbf{y}_{k+1} = \mathbf{y}_k + h\,\mathbf{g}(y_k)$$
  we must use a very small timestep to simulate *stable, stiff* cloth.

- Alternatively we can use an implicit/backwards scheme:
  $$\mathbf{y}_{k+1} = \mathbf{y}_k + h\,\mathbf{g}(y_{k+1})$$
  $$\mathbf{y}_k = \mathbf{y}_{k+1} - h\,\mathbf{g}(y_{k+1})$$
  Solving one step is much more expensive (Newton's Method, Conjugate Gradients, …) but overall faster than the thousands of explicit timesteps required for very stiff springs.

Cloth

David Baraff & Andrew Witkin
Large Steps in Cloth Simulation
SIGGRAPH 1998

- Dynamic motion driven by animation



Questions?