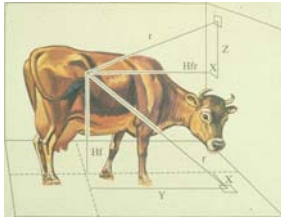


## Local vs. Global Illumination & Radiosity



An early application of radiative heat transfer in stables.

## Last Time?

CSCI-6962 Advanced Computer Graphics Cutler

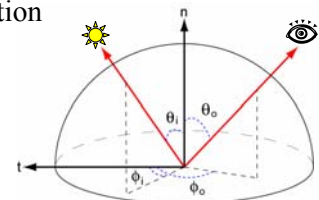
## Today

- **Local Illumination**
  - BRDF
  - Ideal Diffuse Reflectance
  - Ideal Specular Reflectance
  - The Phong Model
- Why is Global Illumination Important?
- The Rendering Equation
- Radiosity Equation/Matrix
- Calculating the Form Factors
- Advanced Radiosity

CSCI-6962 Advanced Computer Graphics Cutler

## BRDF

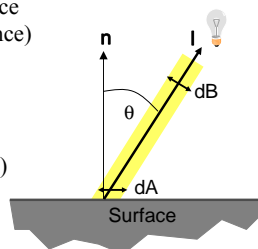
- Ratio of light coming from one direction that gets reflected in another direction
- Bidirectional Reflectance Distribution Function
  - 4D
  - $R(\theta_i, \phi_i; \theta_o, \phi_o)$



CSCI-6962 Advanced Computer Graphics Cutler

## Incoming radiance

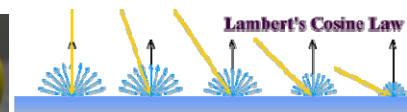
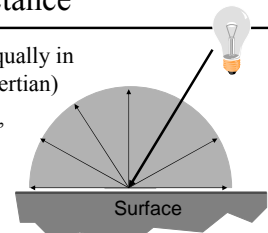
- The amount of light received by a surface depends on incoming angle
  - Bigger at normal incidence (Winter/Summer difference)
- By how much?
  - $dB = dA \cos \theta$
  - Same as:  $\mathbf{l} \cdot \mathbf{n}$  (dot product with normal)



CSCI-6962 Advanced Computer Graphics Cutler

## Ideal Diffuse Reflectance

- Assume surface reflects equally in all directions (a.k.a. Lambertian)
- An ideal diffuse surface is, at the microscopic level, a very rough surface
- Examples: chalk, clay, some paints



CSCI-6962 Advanced Computer Graphics Cutler

### Ideal Specular Reflectance

- Assume surface reflects only in mirror direction
  - View dependent
- Microscopic surface elements are oriented in the same direction as the surface
- Examples: mirrors, highly polished metals

CSCI-6962 Advanced Computer Graphics Cutler

### Non-Ideal Reflectors

- Real materials tend to deviate significantly from ideal mirror reflectors
- Highlight is blurry
- They are not ideal diffuse surfaces either ...

CSCI-6962 Advanced Computer Graphics Cutler

### Non-Ideal Reflectors

- Most light reflects in the ideal reflected direction
- Microscopic surface variations will reflect light just slightly offset
- How much light is reflected?

CSCI-6962 Advanced Computer Graphics Cutler

### The Phong Model

- How much light is reflected “specularly”?
  - Depends on the angle between the ideal reflection direction and the viewer direction  $\alpha$ .

$$L_o = k_s (\cos \alpha)^q \frac{L_i}{r^2}$$

$k_s$ : specular reflection coefficient  
 $q$ : specular reflection exponent

Effect of the  $q$  exponent

### The Phong Model

- Sum of three components:
  - diffuse reflection + specular reflection + “ambient”.

CSCI-6962 Advanced Computer Graphics Cutler

### Ambient Illumination

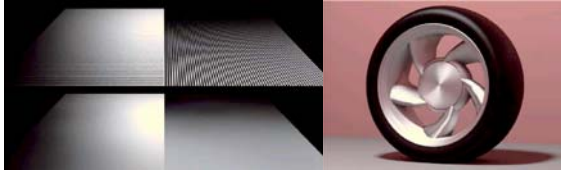
- Avoids the complexity of global illumination.
- Represents the reflection of all indirect illumination.
- This is a total hack!

$$L(\omega_r) = k_a$$

CSCI-6962 Advanced Computer Graphics Cutler

## Anisotropic BRDFs

- Surfaces with strongly oriented microgeometry
- Examples:
  - brushed metals, hair, fur, cloth, velvet



Source: Westin et.al 92

CSCI-6962 Advanced Computer Graphics Cutler

## Questions?

CSCI-6962 Advanced Computer Graphics Cutler

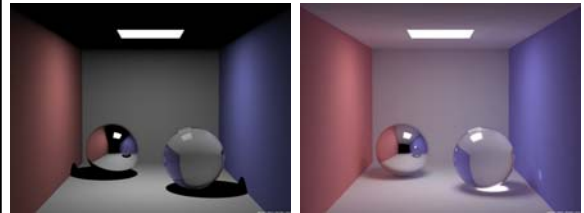
## Today

- Local Illumination
- **Why is Global Illumination Important?**
  - **The Cornell Box**
  - **Radiosity vs. Ray Tracing**
- The Rendering Equation
- Radiosity Equation/Matrix
- Calculating the Form Factors
- Advanced Radiosity

CSCI-6962 Advanced Computer Graphics Cutler

## Why Global Illumination?

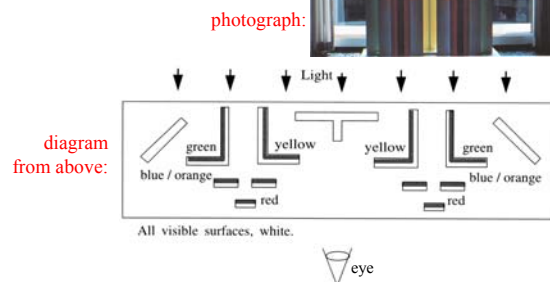
- Simulate all light inter-reflections (indirect lighting)
  - in a room, a lot of the light is indirect: it is reflected by walls.
- How have we dealt with this so far?
  - Ambient term to fake some uniform indirect light



CSCI-6962 Advanced Computer Graphics Cutler

## Why Radiosity?

- Sculpture by John Ferren
- *Diffuse* panels



## Radiosity vs. Ray Tracing



Original sculpture by John Ferren lit by daylight from behind.

Ray traced image. A standard ray tracer cannot simulate the interreflection of light between diffuse surfaces.

Image rendered with radiosity. note color bleeding effects.

CSCI-6962 Advanced Computer Graphics Cutler

## The Cornell Box

direct illumination (0 bounces)      1 bounce      2 bounces

images by Micheal Callahan  
[http://www.cs.utah.edu/~shirley/classes/cs684\\_98/students/callahan/bounce/](http://www.cs.utah.edu/~shirley/classes/cs684_98/students/callahan/bounce/)

## The Cornell Box

photograph

simulation

Goral, Torrance, Greenberg & Battaile  
*Modeling the Interaction of Light Between Diffuse Surfaces*  
 SIGGRAPH '84

## The Cornell Box

- Careful calibration and measurement allows for comparison between physical scene & simulation

photograph      simulation

Light Measurement Laboratory  
 Cornell University, Program for Computer Graphics

## Two approaches for global illumination

- Radiosity
  - View-independent
  - Diffuse only
- Monte-Carlo Ray-tracing
  - Send tons of indirect rays

CSCI-6962 Advanced Computer Graphics    Cutler

## Radiosity vs. Ray Tracing

- Ray tracing is an *image-space* algorithm
  - If the camera is moved, we have to start over
- Radiosity is computed in *object-space*
  - View-independent (just don't move the light)
  - Can pre-compute complex lighting to allow interactive walkthroughs

CSCI-6962 Advanced Computer Graphics    Cutler

## Questions?

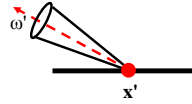
Lightscape    <http://www.lightscape.com>

## Today

- Local Illumination
- Why is Global Illumination Important?
- **The Rendering Equation**
- Radiosity Equation/Matrix
- Calculating the Form Factors
- Advanced Radiosity

CSCI-6962 Advanced Computer Graphics Cutler

## The Rendering Equation

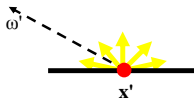


$$L(x', \omega') = E(x', \omega') + \int \rho_x(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

$L(x', \omega')$  is the radiance from a point on a surface in a given direction  $\omega'$

CSCI-6962 Advanced Computer Graphics Cutler

## The Rendering Equation

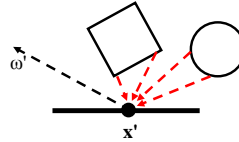


$$L(x', \omega') = E(x', \omega') + \int \rho_x(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

$E(x', \omega')$  is the emitted radiance from a point:  $E$  is non-zero only if  $x'$  is emissive (a light source)

CSCI-6962 Advanced Computer Graphics Cutler

## The Rendering Equation

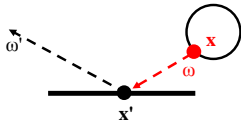


$$L(x', \omega') = E(x', \omega') + \int \rho_x(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

Sum the contribution from all of the other surfaces in the scene

CSCI-6962 Advanced Computer Graphics Cutler

## The Rendering Equation

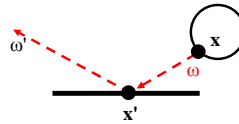


$$L(x', \omega') = E(x', \omega') + \int \rho_x(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

For each  $x$ , compute  $L(x, \omega)$ , the radiance at point  $x$  in the direction  $\omega$  (from  $x$  to  $x'$ )

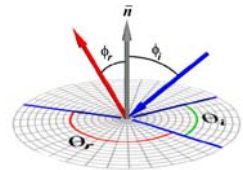
CSCI-6962 Advanced Computer Graphics Cutler

## The Rendering Equation



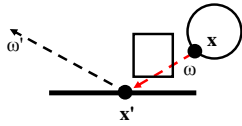
$$L(x', \omega') = E(x', \omega') + \int \rho_x(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

scale the contribution by  $\rho_x(\omega, \omega')$ , the reflectivity (BRDF) of the surface at  $x'$



C

## The Rendering Equation

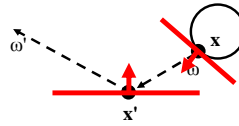


$$L(x', \omega') = E(x', \omega') + \int \rho_x(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

For each  $x$ , compute  $V(x, x')$ , the visibility between  $x$  and  $x'$ :  
 1 when the surfaces are unobstructed along the direction  $\omega$ , 0 otherwise

CSCI-6962 Advanced Computer Graphics Cutler

## The Rendering Equation



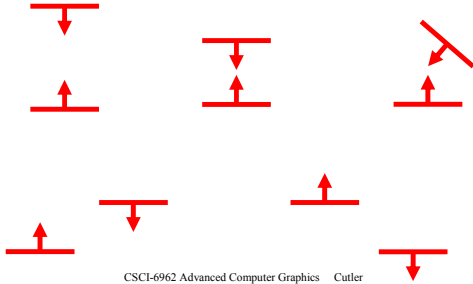
$$L(x', \omega') = E(x', \omega') + \int \rho_x(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

For each  $x$ , compute  $G(x, x')$ , which describes the on the geometric relationship between the two surfaces at  $x$  and  $x'$

CSCI-6962 Advanced Computer Graphics Cutler

## Intuition about $G(x, x')$

- Which arrangement of two surfaces will yield the greatest transfer of light energy? Why?



CSCI-6962 Advanced Computer Graphics Cutler

## Questions?



Museum simulation. Program of Computer Graphics, Cornell University. 50,000 patches. Note indirect lighting from ceiling.

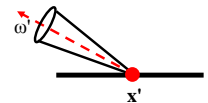
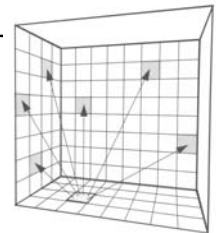
## Today

- Local Illumination
- Why is Global Illumination Important?
- The Rendering Equation
- Radiosity Equation/Matrix
- Calculating the Form Factors
- Advanced Radiosity

CSCI-6962 Advanced Computer Graphics Cutler

## Radiosity Overview

- Surfaces are assumed to be perfectly Lambertian (diffuse)
  - reflect incident light in all directions with equal intensity
- The scene is divided into a set of small areas, or patches.
- The radiosity,  $B_i$ , of patch  $i$  is the total rate of energy leaving a surface. The radiosity over a patch is constant.
- Units for radiosity: Watts / steradian \* meter<sup>2</sup>

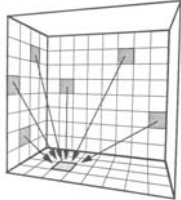


CSCI-6962 Advanced Computer Graphics Cutler

## Radiosity Equation


$$L(x',\omega') = E(x',\omega') + \int \rho_x(\omega,\omega') L(x,\omega) G(x,x') V(x,x') dA$$

↓ Radiosity assumption:  
perfectly diffuse surfaces (not directional)

$$B_{x'} = E_{x'} + \rho_{x'} \int B_x G(x,x') V(x,x')$$


CSCI-6962 Advanced Computer Graphics Cutler

## Continuous Radiosity Equation



$$B_{x'} = E_{x'} + \rho_{x'} \int \underbrace{G(x,x') V(x,x')}_{\text{form factor}} B_x$$

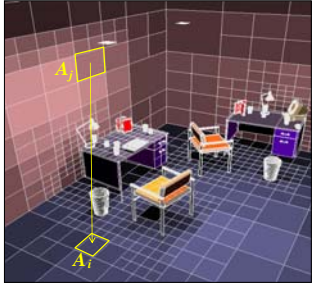
G: geometry term  
V: visibility term

No analytical solution, even for simple configurations

CSCI-6962 Advanced Computer Graphics Cutler

## Discrete Radiosity Equation

Discretize the scene into  $n$  patches, over which the radiosity is constant



$$B_i = E_i + \rho_i \sum_{j=1}^n F_{ij} B_j$$

reflectivity ↓  
form factor ↑

- discrete representation
- iterative solution
- costly geometric/visibility calculations

CSCI-6962 Advanced Computer Graphics Cutler

## The Radiosity Matrix

$$B_i = E_i + \rho_i \sum_{j=1}^n F_{ij} B_j$$

$n$  simultaneous equations with  $n$  unknown  $B_i$  values can be written in matrix form:

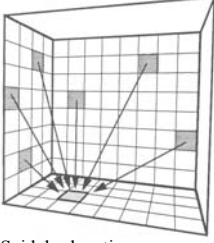
$$\begin{bmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \dots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & & \\ \vdots & & \ddots & \\ -\rho_n F_{n1} & \dots & \dots & 1-\rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

A solution yields a single radiosity value  $B_i$  for each patch in the environment, a view-independent solution.

CSCI-6962 Advanced Computer Graphics Cutler

## Solving the Radiosity Matrix

The radiosity of a single patch  $i$  is updated for each iteration by gathering radiosities from all other patches:


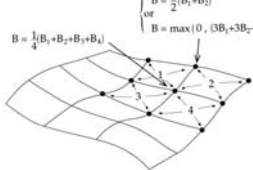
$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_i \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_i \\ \vdots \\ E_n \end{bmatrix} + \begin{bmatrix} \rho_1 F_{1i} & \rho_1 F_{2i} & \dots & \rho_1 F_{ni} \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \rho_i F_{i1} & \rho_i F_{i2} & \dots & \rho_i F_{in} \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_i \\ \vdots \\ B_n \end{bmatrix}$$


This method is fundamentally a Gauss-Seidel relaxation

CSCI-6962 Advanced Computer Graphics Cutler

## Computing Vertex Radiosities

- $B_i$  radiosity values are constant over the extent of a patch.
- How are they mapped to the vertex radiosities (intensities) needed by the renderer?
  - Average the radiosities of patches that contribute to the vertex
  - Vertices on the edge of a surface are assigned values extrapolation

$$B = \frac{1}{2}(B_1 + B_2)$$

$$B = \frac{1}{4}(B_1 + B_2 + B_3 + B_4)$$

$$B = \max(0, (3B_1 + 3B_2 - B_3 - B_4))$$

CSCI-6962 Advanced Computer Graphics Cutler

## Questions?



Factory simulation. Program of Computer Graphics, Cornell University.  
30,000 patches.

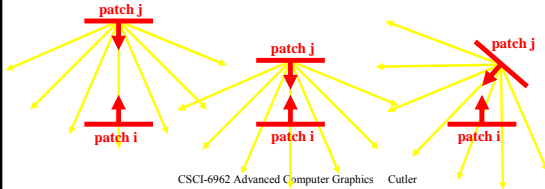
## Today

- Local Illumination
- Why is Global Illumination Important?
- The Rendering Equation
- Radiosity Equation/Matrix
- Calculating the Form Factors
- Advanced Radiosity

CSCI-6962 Advanced Computer Graphics Cutler

## Calculating the Form Factor $F_{ij}$

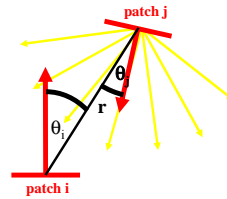
- $F_{ij}$  = fraction of light energy leaving patch j that arrives at patch i
- Takes account of both:
  - geometry (size, orientation & position)
  - visibility (are there any occluders?)



CSCI-6962 Advanced Computer Graphics Cutler

## Calculating the Form Factor $F_{ij}$

- $F_{ij}$  = fraction of light energy leaving patch j that arrives at patch i

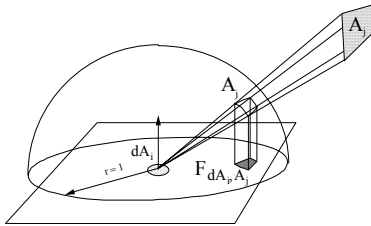


$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi r^2} V_{ij} dA_j dA_i$$

CSCI-6962 Advanced Computer Graphics Cutler

## Form Factor Determination

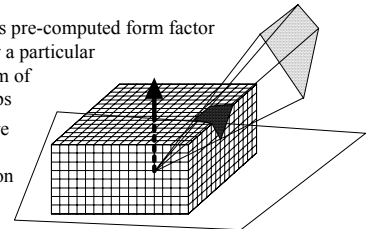
The Nusselt analog: the form factor of a patch is equivalent to the fraction of the unit circle that is formed by taking the projection of the patch onto the hemisphere surface and projecting it down onto the circle.



CSCI-6962 Advanced Computer Graphics Cutler

## Hemicube Algorithm

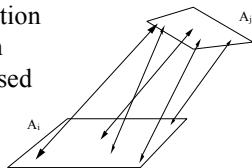
- A hemicube is constructed around the center of each patch
- Faces of the hemicube are divided into "pixels"
- Each patch is projected (rasterized) onto the faces of the hemicube
- Each pixel stores its pre-computed form factor  
The form factor for a particular patch is just the sum of the pixels it overlaps
- Patch occlusions are handled similar to z-buffer rasterization



CSCI-6962 Advanced Computer Graphics Cutler

## Form Factor from Ray Casting

- Cast  $n$  rays between the two patches
  - $n$  is typically between 4 and 32
  - Compute visibility
  - Integrate the point-to-point form factor
- Permits the computation of the patch-to-patch form factor, as opposed to point-to-patch



CSCI-6962 Advanced Computer Graphics Cutler

## Questions?



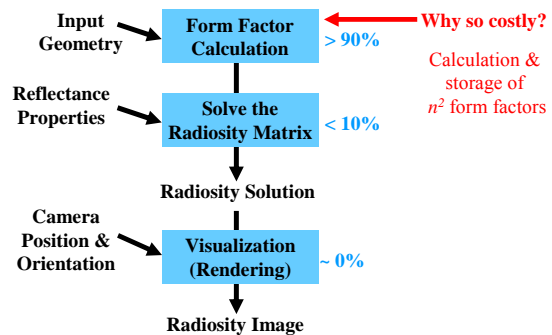
Lightscape <http://www.lightscape.com>

## Today

- Local Illumination
- Why is Global Illumination Important?
- The Rendering Equation
- Radiosity Equation/Matrix
- Calculating the Form Factors
- **Advanced Radiosity**
  - Progressive Radiosity
  - Adaptive Subdivision
  - Discontinuity Meshing
  - Hierarchical Radiosity

CSCI-6962 Advanced Computer Graphics Cutler

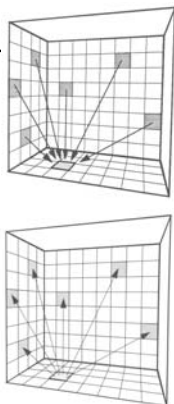
## Stages in a Radiosity Solution



CSCI-6962 Advanced Computer Graphics Cutler

## Progressive Refinement

- Goal: Provide frequent and timely updates to the user during computation
- Key Idea: Update the entire image at every iteration, rather than a single patch
- How? Instead of summing the light received by one patch, distribute the radiance of the patch with the most *undistributed radiance*.



CSCI-6962 Advanced Computer Graphics Cutler

## Reordering the Solution for PR

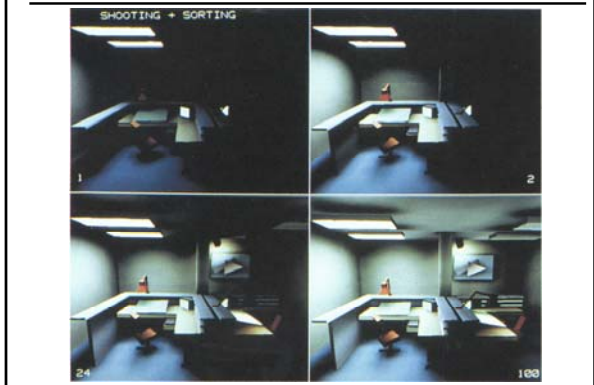
*Shooting:* the radiosity of all patches is updated for each iteration:

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} + \begin{bmatrix} \dots & \rho_1 F_{1i} & \dots \\ \dots & \rho_2 F_{2i} & \dots \\ \vdots & \vdots & \vdots \\ \dots & \rho_n F_{ni} & \dots \end{bmatrix} \begin{bmatrix} \vdots \\ B_i \\ \vdots \end{bmatrix}$$

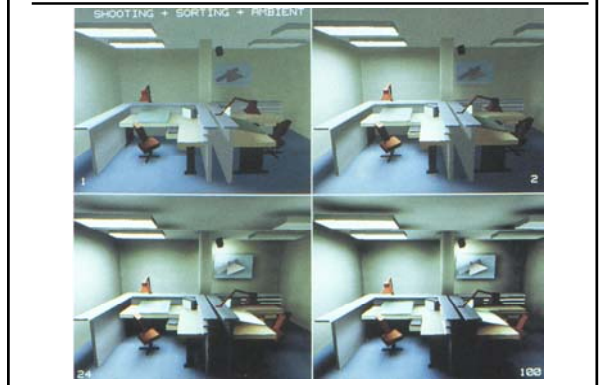
This method is fundamentally a Southwell relaxation

CSCI-6962 Advanced Computer Graphics Cutler

### Progressive Refinement w/out Ambient Term



### Progressive Refinement with Ambient Term



### Questions?



Lightscape <http://www.lightscape.com>

### Today

- Local Illumination
- Why is Global Illumination Important?
- The Rendering Equation
- Radiosity Equation/Matrix
- Calculating the Form Factors
- Advanced Radiosity
  - Progressive Radiosity
  - Adaptive Subdivision
  - Discontinuity Meshing
  - Hierarchical Radiosity

CSCI-6962 Advanced Computer Graphics Cutler

### Increasing the Accuracy of the Solution

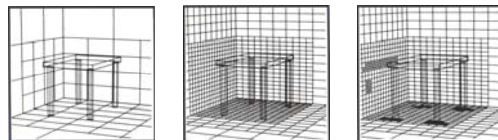
What's wrong with this picture?



- Image quality is a function of patch size
- Compute a solution on a uniform initial mesh, then refine the mesh in areas that exceed some error tolerance:
  - shadow boundaries
  - other areas with a high radiosity gradient

CSCI-6962 Advanced Computer Graphics Cutler

### Adaptive Subdivision of Patches



Coarse patch solution (145 patches)



Improved solution (1021 subpatches)



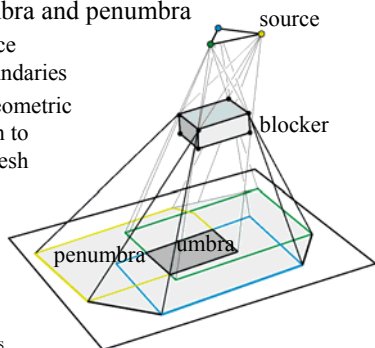
Adaptive subdivision (1306 subpatches)

CSCI-6962 Advanced Computer Graphics Cutler

## Discontinuity Meshing

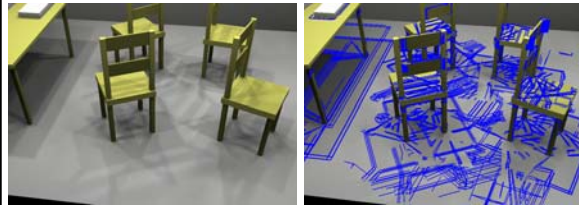
- Limits of umbra and penumbra

- Captures nice shadow boundaries
- Complex geometric computation to construct mesh



CS

## Discontinuity Meshing



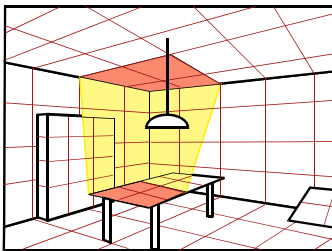
“Fast and Accurate Hierarchical Radiosity Using Global Visibility”  
Durand, Drettakis, & Puech 1999

CSCI-6962 Advanced Computer Graphics Cutler

## Hierarchical Radiosity

- Group elements when the light exchange is not important

- Breaks the quadratic complexity
- Control non trivial, memory cost



CSCI-6962 Advanced Computer Graphics Cutler

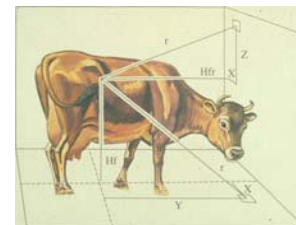
## Practical Problems with Radiosity

- Meshing

- memory
- robustness

- Form factors

- computation



Cow-cow form factor?

- Diffuse limitation

- extension to specular takes too much memory

CSCI-6962 Advanced Computer Graphics Cutler

## Questions?



Lightscape <http://www.lightscape.com>