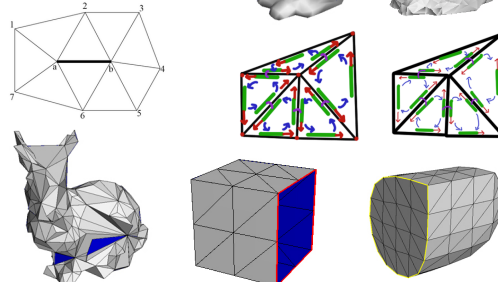


Subdivision Surfaces

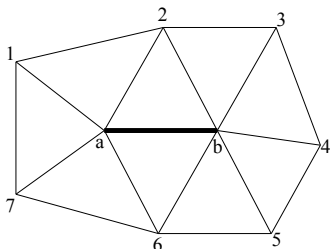
Homework 1:

- Questions/Comments?



Questions on Homework?

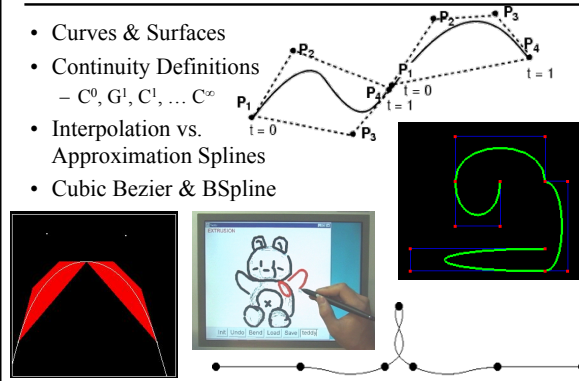
- What's an illegal edge collapse?



- To be legal, the ring of vertex neighbors *must be unique* (have no duplicates)!

Last Time?

- Curves & Surfaces
- Continuity Definitions
 - $C^0, G^1, C^1, \dots, C^\infty$
- Interpolation vs. Approximation Splines
- Cubic Bezier & BSpline



Today

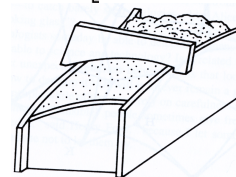
- Spline Surfaces / Patches
 - Tensor Product
 - Bezier Patches
 - Trimming Curves
- Subdivision Surface "Zoo"
- Misc. Mesh/Surface Vocabulary
- "Subdivision Surfaces in Character Animation"
- "Piecewise Smooth Surface Reconstruction"
- Interpolating Subdivision

Tensor Product

- Of two vectors:

$$[a_1 \ a_2 \ a_3] \otimes [b_1 \ b_2 \ b_3 \ b_4] = \begin{bmatrix} a_1b_1 & a_2b_1 & a_3b_1 \\ a_1b_2 & a_2b_2 & a_3b_2 \\ a_1b_3 & a_2b_3 & a_3b_3 \\ a_1b_4 & a_2b_4 & a_3b_4 \end{bmatrix}$$

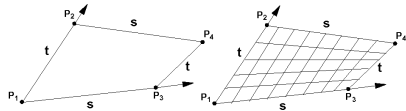
- Similarly, we can define a surface as the tensor product of two curves....



Farin, Curves and Surfaces for Computer Aided Geometric Design

Bilinear Patch

Bi-lerp a (typically non-planar) quadrilateral

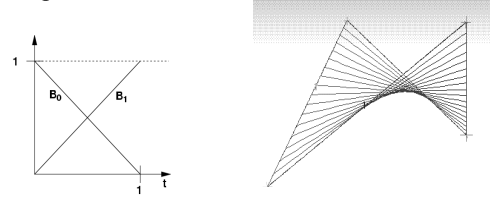


Notation: $L(P_1, P_2, \alpha) \equiv (1 - \alpha)P_1 + \alpha P_2$

$$Q(s, t) = L(L(P_1, P_2, t), L(P_3, P_4, t), s)$$

Bilinear Patch

- Smooth version of quadrilateral with non-planar vertices...



- But will this help us model smooth surfaces?
- Do we have control of the derivative at the edges?

Ruled Surfaces in Art & Architecture

<http://www.bergenwood.no/wp-content/media/images/frozenmusic.jpg>

Chiras Iulia
Astri Isabella
Matiss Shteinerts



Antoni Gaudi
Children's School
Barcelona

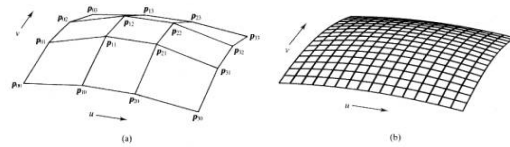
<http://www.lonelyplanetimages.com/images/399954>

Bicubic Bezier Patch

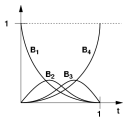
Notation: $CB(P_1, P_2, P_3, P_4, \alpha)$ is Bézier curve with control points P_i evaluated at α

Define "Tensor-product" Bézier surface

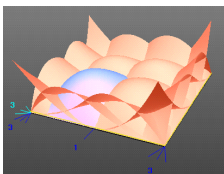
$$Q(s, t) = CB(\begin{matrix} CB(P_{00}, P_{01}, P_{02}, P_{03}, t), \\ CB(P_{10}, P_{11}, P_{12}, P_{13}, t), \\ CB(P_{20}, P_{21}, P_{22}, P_{23}, t), \\ CB(P_{30}, P_{31}, P_{32}, P_{33}, t), \\ s \end{matrix})$$



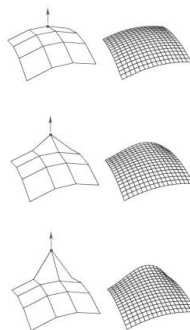
Editing Bicubic Bezier Patches



Curve Basis Functions

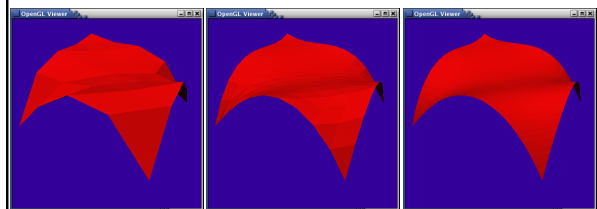


Surface Basis Functions



Bicubic Bezier Patch Tessellation

- Given 16 control points and a tessellation resolution, we can create a triangle mesh



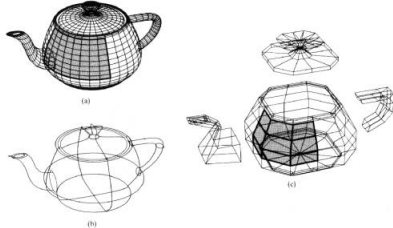
resolution:
5x5 vertices

resolution:
11x11 vertices

resolution:
41x41 vertices

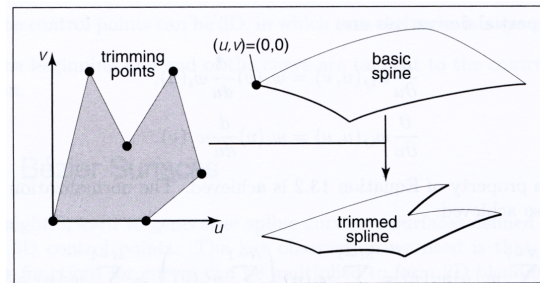
Modeling with Bicubic Bezier Patches

- Original Teapot specified with Bezier Patches



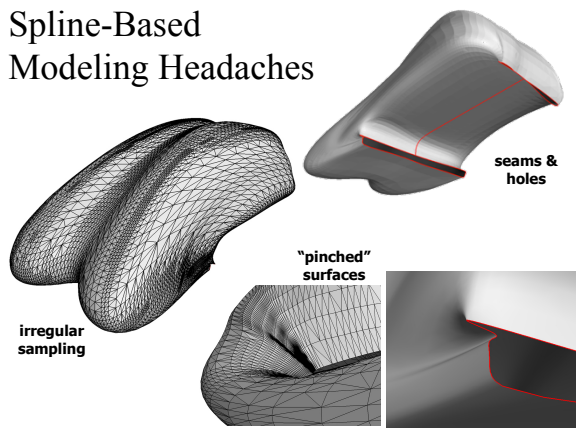
- But it's not "watertight": it has intersecting surfaces at spout & handle, no bottom, a hole at the spout tip, a gap between lid & base

Trimming Curves for Patches



Shirley, Fundamentals of Computer Graphics

Spline-Based Modeling Headaches



Questions?

- Bezier Patches?

or

- Triangle Mesh?

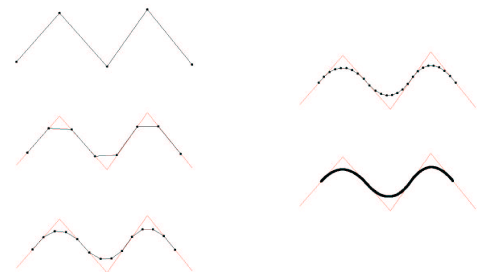


Henrik Wann Jensen

Today

- Spline Surfaces / Patches
- **Subdivision Surface "Zoo"**
 - Doo Sabin (anything!)
 - Loop (triangles only)
 - Catmull Clark (turns everything into quads)
 - ... many others!
- Misc. Mesh/Surface Vocabulary
- "Subdivision Surfaces in Character Animation"
- "Piecewise Smooth Surface Reconstruction"
- Interpolating Subdivision

Chaikin's Algorithm



Doo-Sabin Subdivision

Idea: introduce a new vertex for each face
At the midpoint of old vertex, face centroid

Doo-Sabin Subdivision

<http://www.ke.ics.saitama-u.ac.jp/xuz/pic/doo-sabin.gif>

Loop Subdivision

Shirley, Fundamentals of Computer Graphics

Loop Subdivision

Subdivision Rules. The masks for the Loop scheme are shown in Figure 4.3. For boundaries and edges tagged as *crease* edges, special rules are used. These rules produce a cubic spline curve along the boundary/crease. The curve only depends on control points on the boundary/crease.

Figure 4.3: Loop subdivision: in the picture above, β can be chosen to be either $\frac{1}{8}(5/8 - (\frac{1}{4} + \frac{1}{4} \cos \frac{2\pi}{n}))^2$ (original choice of Loop [16]), or, for $n > 3$, $\beta = \frac{1}{8n}$ as proposed by Warren [33]. For $n = 3$, $\beta = 3/16$ can be used.

SIGGRAPH 2000 course notes
Subdivision for Modeling and Animation (page 70)

Catmull Clark Subdivision

$$v^{j+1} = \frac{v^j + v_j^{j+1} + f_j^{j+1}}{4} \quad (1)$$

where subscripts are taken modulo the valence of the central vertex v^0 . (The valence of a vertex is the number of edges incident to it.) Finally, a vertex point v^j is computed as

$$v^{j+1} = \frac{n-2}{n} v^j + \frac{1}{n^2} \sum_{j=1}^n v_j^{j+1} + \frac{1}{n^2} \sum_{j=1}^n f_j^{j+1} \quad (2)$$

Vertices of valence 4 are called ordinary; others are called extraordinary.

Figure 4: The situation around a vertex v^0 of valence n .

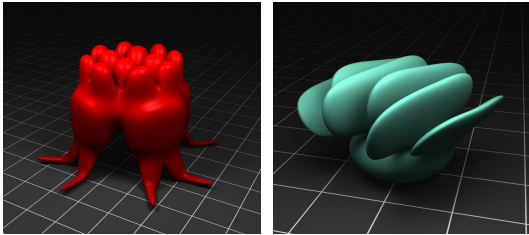
"Subdivision Surfaces in Character Animation",
DeRose, Kass & Truong, SIGGRAPH 1998

Adding creases to Loop Subdivision

- Vertex & edge masks
- Limit masks
 - Position
 - Tangent

(1) smooth edge (2) regular crease edge (3) non-regular crease edge

Questions?



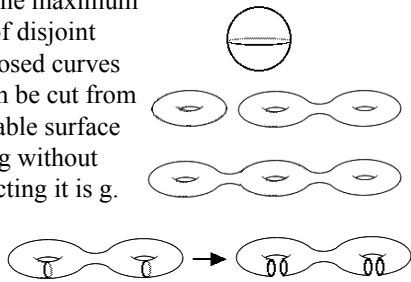
Justin Legakis

Today

- Spline Surfaces / Patches
- Subdivision Surface “Zoo”
- **Misc. Mesh/Surface Vocabulary**
- “Subdivision Surfaces in Character Animation”
- “Piecewise Smooth Surface Reconstruction”
- Interpolating Subdivision

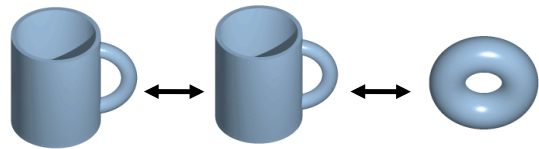
Misc. Mesh/Surface Vocabulary

- *Genus*: The maximum number of disjoint simple closed curves which can be cut from an orientable surface of genus g without disconnecting it is g .



Misc. Mesh/Surface Vocabulary

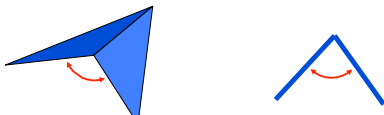
- *Homeomorphic/Topological equivalence*: a continuous stretching and bending of the object into a new shape



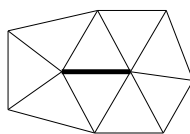
http://en.wikipedia.org/wiki/Image:Mug_and_Torus_morph.gif

Misc. Mesh/Surface Vocabulary

- *Dihedral Angle*:
 - the angle between the planes of two triangular faces
 - “looking down the edge” between two faces, the angle between the faces.

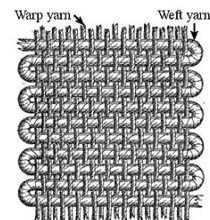


- *Valence (a.k.a. degree)*: the number of edges incident to the vertex.



Misc. Mesh/Surface Vocabulary

- *Warp & weft*: Yarns used in weaving. Because the weft does not have to be stretched in the way that the warp is, it can generally be less strong.



<http://en.wikipedia.org/wiki/Weft>

Today

- Spline Surfaces / Patches
- Subdivision Surface “Zoo”
- Seams In Subdivision
- Misc. Mesh/Surface Vocabulary
- “Subdivision Surfaces in Character Animation”
- “Piecewise Smooth Surface Reconstruction”
- Interpolating Subdivision

Reading for Today

- DeRose, Kass, & Truong, "Subdivision Surfaces in Character Animation", SIGGRAPH 1998



Figure 5: Geri's hand as a piecewise smooth Catmull-Clark surface. Infinitely sharp creases are used between the skin and the finger nails.

Subdivision Surfaces in Character Animation

- Catmull Clark Subdivision Rules
- Semi-sharp vs. Infinitely-sharp creases
- Mass-Spring Cloth (*next week*)
- Hierarchical Mesh for Collision
- Texturing Subdivision Surfaces

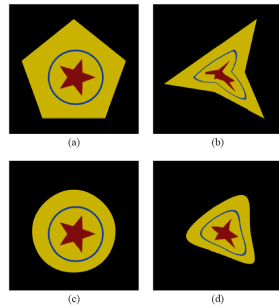
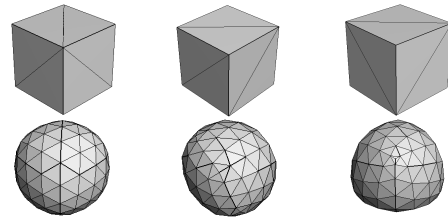


Figure 11: (a) A texture mapped regular pentagon comprised of 5 triangles; (b) the pentagonal model with its vertices moved; (c) A subdivision surface whose control mesh is the same 5 triangles in (a), and where boundary edges are marked as creases; (d) the subdivision surface with its vertices positioned as in (b).

Catmull-Clark in Pixar Production

- Based on quadrilaterals
 - Like NURBS, specifically cubic bsplines
 - Implicit adjacency in subdivided microgeometry
 - Better than triangles for symmetric objects

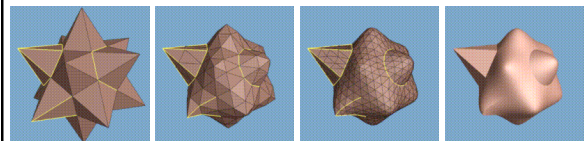


Today

- Spline Surfaces / Patches
- Subdivision Surface “Zoo”
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- “Piecewise Smooth Surface Reconstruction”
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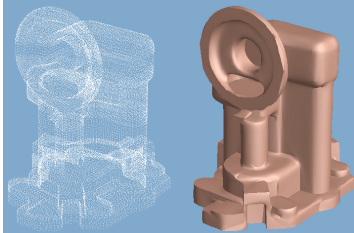
Reading for Today

- Hoppe et al., “Piecewise Smooth Surface Reconstruction” SIGGRAPH 1994



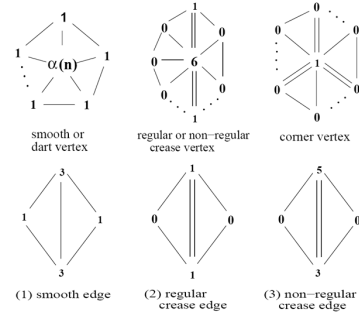
Piecewise Smooth Surface Reconstruction

- From input: scanned mesh points
 - Estimate topological type (genus)
 - Mesh optimization (a.k.a. simplification)
 - Smooth surface optimization



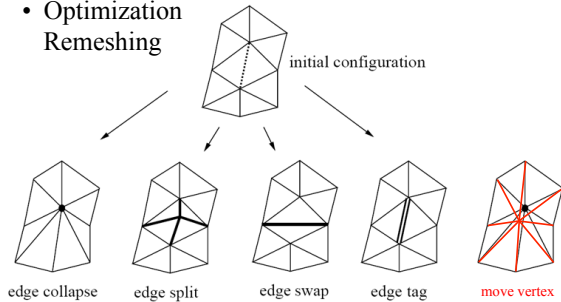
Adding creases to Loop Subdivision

- Vertex & edge masks
- Limit masks
 - Position
 - Tangent



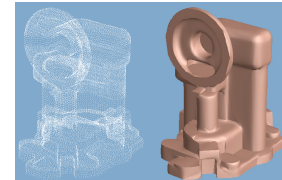
Piecewise Smooth Surface Reconstruction

- Optimization Remeshing



Piecewise Smooth Surface Reconstruction

- Crease subdivision masks *decouple* behavior of surface on either side of crease
- Crease rules cannot model a cone
- Optimization can be done locally
 - subdivision control points have only local influence
- Results
 - Noise?
 - Applicability?
 - Limitations?
 - Running Time

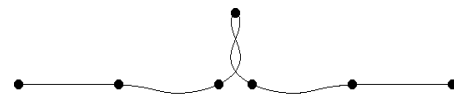


Today

- Spline Surfaces / Patches
- Subdivision Surface “Zoo”
- Seams In Subdivision
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- “Subdivision Surfaces in Character Animation”
- “Piecewise Smooth Surface Reconstruction”
- **Interpolating Subdivision**

Interpolation vs. Approximation Curves

- Interpolation Curve – over constrained → lots of (undesirable?) oscillations



- Approximation Curve – more reasonable?

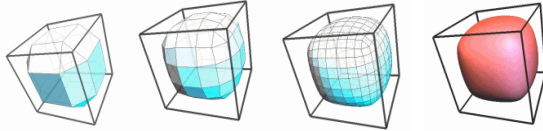


Interpolating Subdivision

- Chaikin:



- Doo-Sabin:



of the centroids of each edge/face

Interpolating Subdivision

- *Interpolation vs. Approximation* of control points
- Handle arbitrary topological type
- Reduce the “extraneous bumps & wiggles”

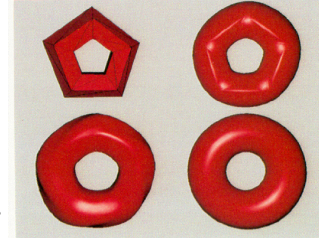
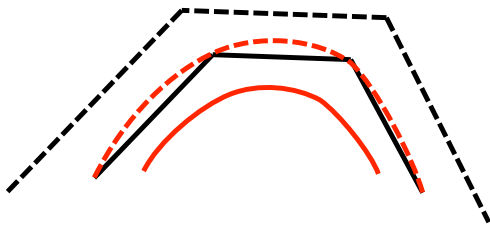


Figure 4: Interpolating a coarsely polygonized torus. Upper left: original mesh. Upper right: Shitman-Séquin interpolation[14]. Lower left: Interpolating Catmull-Clark surface. Lower right: Faired interpolating Catmull-Clark surface.

“Efficient, fair interpolation using Catmull-Clark surfaces”, Halstead, Kass & DeRose, SIGGRAPH 1993

Interpolation of Catmull-Clark Surfaces

- Solve for a new control mesh (generally “bigger”) such that when Catmull-Clark subdivision is applied it interpolates the original control mesh



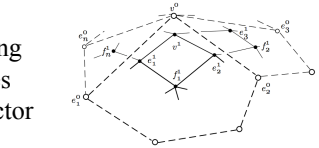
Vertex Position in Limit

- V_n stores the center vertex & surrounding edge & face vertices as a big column vector

$$V_n^{i+1} = S_n V_n^i$$

- When $n = 4$:
($n = \text{valence}$)

$$V_n^\infty := \lim_{i \rightarrow \infty} S_n^i V_n^1$$



$$S_4 = \frac{1}{16} \begin{pmatrix} 9 & 3 & 3 & 3 & 3 & 1 & 1 & 1 & 1 \\ 6 & 6 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 6 & 1 & 6 & 1 & 0 & 1 & 1 & 0 & 0 \\ 6 & 0 & 1 & 6 & 1 & 0 & 1 & 1 & 0 \\ 6 & 1 & 0 & 1 & 6 & 0 & 0 & 1 & 1 \\ 4 & 4 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 4 & 0 & 4 & 4 & 0 & 0 & 4 & 0 & 0 \\ 4 & 0 & 0 & 4 & 4 & 0 & 0 & 4 & 0 \\ 4 & 4 & 0 & 0 & 4 & 0 & 0 & 0 & 4 \end{pmatrix}$$

Solve for New Positions

- Goal: Find the control mesh vertex positions, x (a column vector of 3D points), such that the position of the vertices in the limit match the input vertices, b (also a column vector of points)
- Use Least Squares to solve $Ax = b$ where A is a square matrix with the interpolation rules and connectivity of the mesh
- See paper for extension to match limit normals

Fairing

- Fairing: an additional part or structure added to an aircraft, tractor-trailer, etc. to smooth the outline and thus reduce drag
- Subdivide initial resolution twice so that all constrained vertex positions are independent

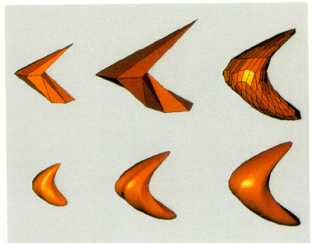
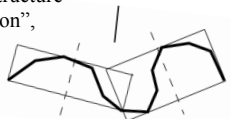


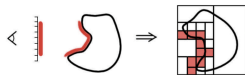
Figure 5: Top row: Original mesh, Interpolating mesh, Faired interpolating mesh. Bottom row: Corresponding Catmull-Clark surfaces. Interpolation introduces wiggles which are removed by fairing.

Reading for Tuesday: *(pick one)*

- "OBB-Tree: A Hierarchical Structure for Rapid Interference Detection", Gottschalk, Lin, Manocha, SIGGRAPH 1996.



- "Painting and Rendering Textures on Unparameterized Models", DeBry, Gibbs, DeLeon, and Robins, SIGGRAPH 2002



Post a comment/question on the LMS discussion by 10am