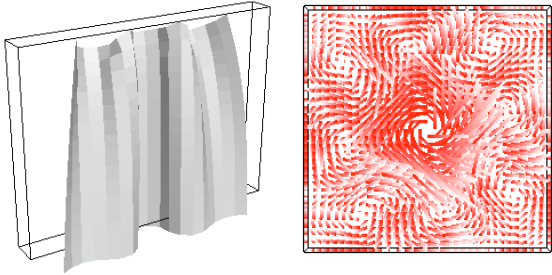


Rigid Body Dynamics, Collision Response, & Deformation

HW2: Cloth & Fluid Simulation



Last Time?

- Navier-Stokes Equations
- Conservation of Momentum & Mass
- Incompressible Flow

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = -\frac{\partial p}{\partial x} + g_x + \nu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2})$$

$$\frac{\partial v}{\partial t} + \frac{\partial vu}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} = -\frac{\partial p}{\partial y} + g_y + \nu(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2})$$

$$\frac{\partial w}{\partial t} + \frac{\partial wu}{\partial x} + \frac{\partial wv}{\partial y} + \frac{\partial w^2}{\partial z} = -\frac{\partial p}{\partial z} + g_z + \nu(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2})$$

Today

- Rigid Body Dynamics
- Collision Response
- Non-Rigid Objects
- Finite Element Method
- Deformation
- Level-of-Detail

Rigid Body Dynamics

- How do we simulate this object's motion over time?
- We could discretize the object into many particles...
 - But a rigid body does *not* deform
 - Only a few *degrees of freedom*
- Instead, we use only one particle at the center of mass
- Compute net force & net torque

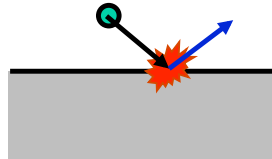
Nice Reference Material: <http://www.pixar.com/companyinfo/research/pbm2001/>

Degree of Freedom (DOF)

- Rotations:
 - 1 DOF: knee
 - 2 DOF: wrist
 - 3 DOF: arm
- Translations count too... → 6 DOF

Collisions

- Detection
- Response
- Overshooting problem (when we enter the solid)



Collision Response

- tangential velocity v_t unchanged
- normal velocity v_n reflects:

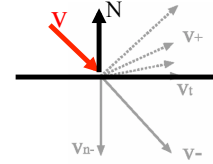
$$v = v_t + v_n$$

$$v \leftarrow v_t - \epsilon v_n$$

- coefficient of restitution
 - 1 for elastic
 - 0 for plastic

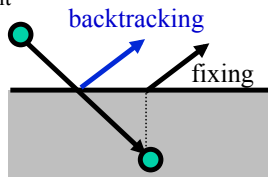
- change of velocity = $-(1+\epsilon)v$

- change of momentum *Impulse* = $-m(1+\epsilon)v$



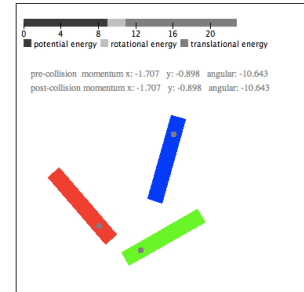
Collisions - Overshooting

- Usually, we detect collision when it's too late: we're already inside
- Solutions: back up
 - Compute intersection point
 - Compute response there
 - Advance for remaining fractional time step
- Other solution: Quick and dirty fixup
 - Just project back to object closest point



Energy & Rigid Body Collisions

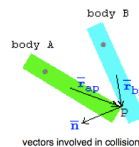
- Total Energy = Kinetic Energy + Potential Energy + Rotational Energy
- Total Energy stays constant if there is no damping and no friction
- Rotational Energy is constant between collisions



<http://www.myphysicslab.com/collision.html>

Collision Between Two Objects

- Suppose a vertex on body A is colliding into an edge of body B at point P. Define the following variables:

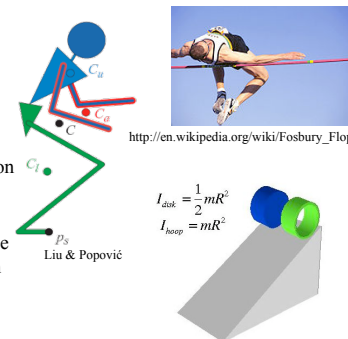


- m_a, m_b = mass of bodies A, B
- \vec{r}_{ap} = distance vector from center of mass of body A to point P
- \vec{r}_{bp} = distance vector from center of mass of body B to point P
- ω_{a1}, ω_{b1} = initial pre-collision angular velocity of bodies A, B
- ω_{a2}, ω_{b2} = final post-collision angular velocity of bodies A, B
- $\vec{v}_{a1}, \vec{v}_{b1}$ = initial pre-collision velocities of center of mass bodies A, B
- $\vec{v}_{a2}, \vec{v}_{b2}$ = final post-collision velocities of center of mass bodies A, B
- \vec{v}_{ap1} = initial pre-collision velocity of impact point on body A
- \vec{v}_{bp1} = initial pre-collision velocity of impact point on body B
- \vec{n} = normal (perpendicular) vector to edge of body B
- ϵ = elasticity (0 = inelastic, 1 = perfectly elastic)

<http://www.myphysicslab.com/collision.html>

Center of Mass & Moment of Inertia

- Center of Mass: mean location of all mass in the system
- Moment of Inertia: a measure of an object's resistance to changes to its rotation
- If a solid cylinder & a hollow tube have the same radius & the same mass, which will reach the bottom of the ramp first?



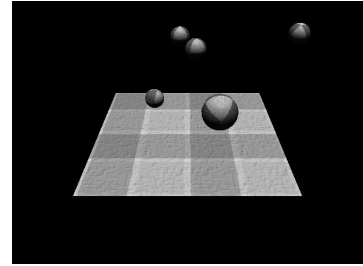
<http://solomon.physics.sc.edu/~tedeschi/demo/demo12.html>
<http://hyperphysics.phy-astr.gsu.edu/hbase/hoocy12.html>

Advanced Collisions

- What about Friction?
- What if the contact between two objects is not a single point?
- What if more than two objects collide simultaneously?

Rigid Body Dynamics

- Physics
 - Velocity
 - Acceleration
 - Angular Momentum
- Collisions
- Friction



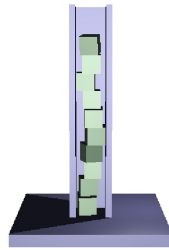
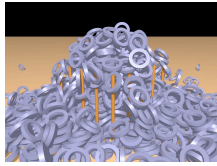
from: Darren Lewis

<http://www-cs-students.stanford.edu/~dalewis/cs448a/rigidbody.html>

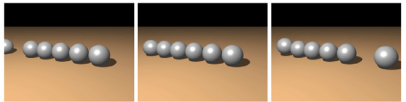
Collisions

Victor J. Milenkovic & Harald Schmidt
Optimization-Based Animation
 SIGGRAPH 2001

- We know how to simulate bouncing really well
- But resting collisions are harder to manage



Guendelman, Bridson & Fedkiw
Nonconvex Rigid Bodies with Stacking
 SIGGRAPH 2003

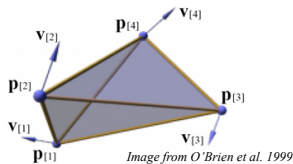


Today

- Rigid Body Dynamics
- Collision Response
- **Non-Rigid Objects**
- **Finite Element Method**
- Deformation
- Level-of-Detail

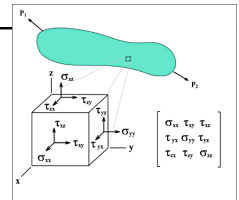
Simulation of Non-Rigid Objects

- We modeled string & cloth using mass-spring systems. Can we do the same?
- Yes...
- But a more physically accurate model uses *volumetric elements*:



Strain & Stress

- Stress
 - the internal distribution of forces within a body that balance and react to the loads applied to it
 - *normal stress & shear stress*
- Strain
 - material deformation caused by stress.
 - measured by the change in length of a line or by the change in angle between two lines



http://en.wikipedia.org/wiki/Image:Stress_tensor.png

$$\epsilon = \frac{\Delta l}{l_0}$$

Finite Element Method

- To solve the continuous problem (deformation of all points of the object)
 - Discretize the problem
 - Express the interrelationship
 - Solve a big linear system
- More principled than Mass-Spring

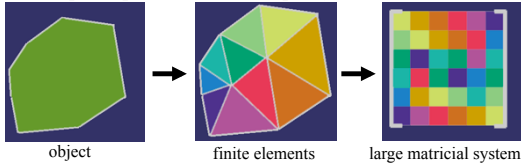


Diagram from Debunne et al. 2001

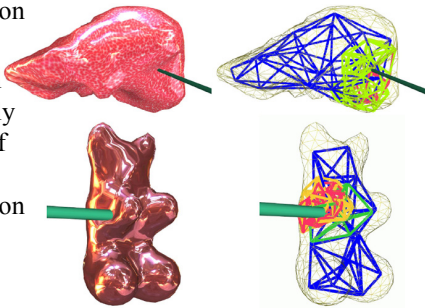
Today

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Level of Detail

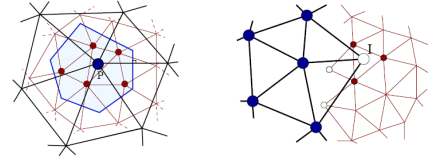
Gilles Debunne, Mathieu Desbrun, Marie-Paule Cani, & Alan H. Barr
Dynamic Real-Time Deformations using Space & Time Adaptive Sampling
 SIGGRAPH 2001

- Interactive shape deformation
- Use high-resolution model only in areas of extreme deformation



Multi-Resolution Deformation

- Use Voronoi diagrams to match parent & child vertices.
- Interpolate values for inactive interface vertices from active parent/child vertices

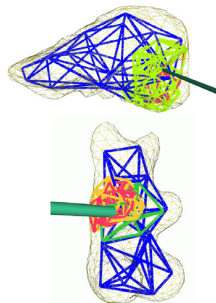


- Need to avoid interference of vibrations between simulations at different resolutions

Debunne et al. "Dynamic Real-Time Deformations using Space & Time Adaptive Sampling", 2001

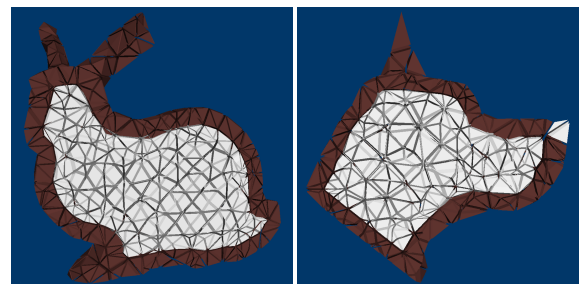
Pre-computation & Simulation

- FEM matrix pre-computed
- Level of detail coupling pre-computed for rest topology
- What to do if connectivity of elements changes?
 - Cloth is cut or torn
 - Surgery simulation



Multiple Materials

Mueller, Dorsey, McMillan, Jagnow, & Cutler
Stable Real-Time Deformations
 Symposium on Computer Animation 2002



Tree Stump



Images from Cutler et al. 2002



Image from Cutler et al. 2002



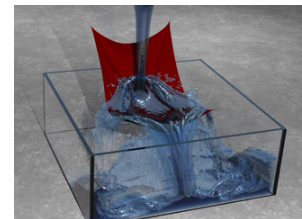
Image from Cutler et al. 2002

Readings for Today:

“Deformable Objects Alive!” Coros, Martin, Thomaszewski, Schumacher, & Sumner, SIGGRAPH 2012



“Coupling Water and Smoke to Thin Deformable and Rigid Shells”, Guendelman, Selle, Losasso, & Fedkiw, SIGGRAPH 2005.



How to read a research paper?

(especially an advanced paper in a new area)

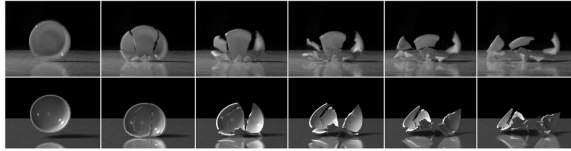
- Multiple readings are often necessary
- Don't necessarily read from front to back
- Lookup important terms
- Target application & claimed contributions
- Experimental procedure
- How well results & examples support the claims
- Scalability of the technique (order notation)
- Limitations of technique, places for future research
- Possibilities for hybrid systems with other work

Components of a well-written research paper?

- Motivation/context/related work
- Contributions of this work
- Clear description of algorithm
 - Sufficiently-detailed to allow work to be reproduced
 - Work is theoretically sound (hacks/arbitrary constants discouraged)
- Results
 - well chosen examples
 - clear tables/illustrations/visualizations
- Conclusions
 - limitations of the method are clearly stated

Reading for Friday:

- James O'Brien & Jessica Hodgins "*Graphical Modeling and Animation of Brittle Fracture*" SIGGRAPH 1999.



- Fracture threshold
- Remeshing
 - need connectivity info!
- Material properties
- Parameter tuning