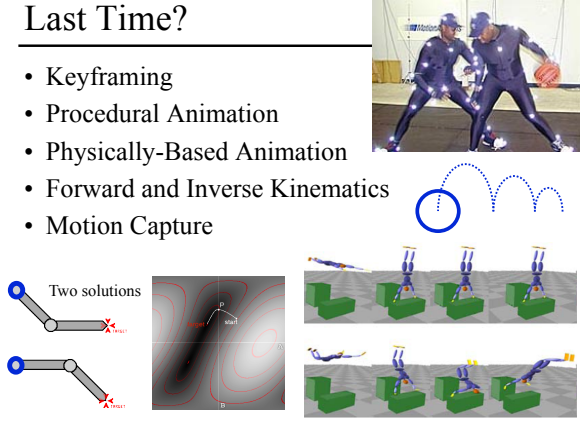


Ray Tracing

Last Time?

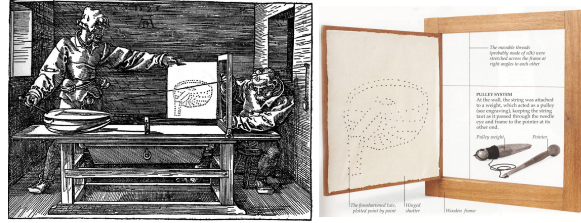
- Keyframing
- Procedural Animation
- Physically-Based Animation
- Forward and Inverse Kinematics
- Motion Capture



- ## Today
- Ray Casting
 - Ray-Plane Intersection
 - Ray-Sphere Intersection
 - Point in Polygon
 - Ray Tracing
 - Recursive Ray Tracing
 - Distribution Ray Tracing

Durer's Ray Casting Machine

- Albrecht Durer, 16th century



Ray Casting

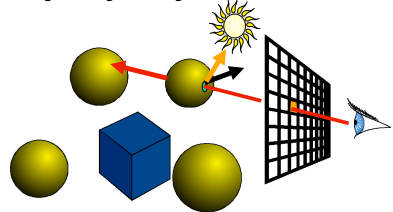
For every pixel
Construct a ray from the eye

For every object in the scene
Find **intersection** with the ray

Keep if closest

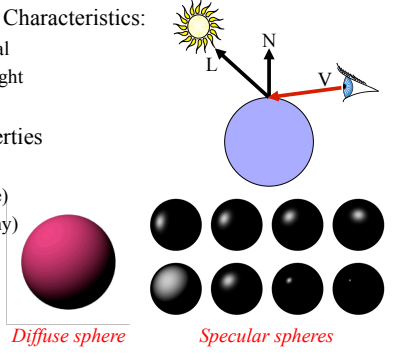
Shade depending on light and **normal** vector

Finding the intersection and normal is the central part of ray casting



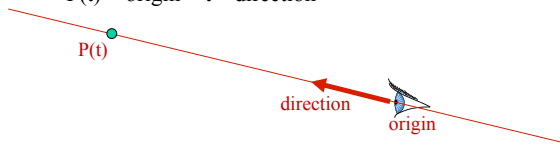
A Note on Local Shading

- Surface/Scene Characteristics:
 - surface normal
 - direction to light
 - viewpoint
- Material Properties
 - color/texture
 - diffuse (matte)
 - specular (shiny)
 - ...
- More later!



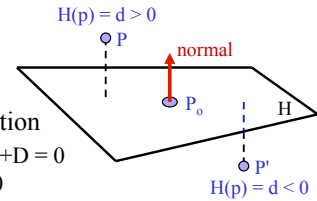
Ray Representation?

- Two vectors:
 - Origin
 - Direction (normalized is better)
- Parametric line (*explicit* representation)
 - $P(t) = \text{origin} + t * \text{direction}$



3D Plane Representation?

- Plane defined by
 - $P_o = (x,y,z)$
 - $n = (A,B,C)$
- *Implicit* plane equation
 - $H(P) = Ax+By+Cz+D = 0$
 - $= n \cdot P + D = 0$
- Point-Plane distance?
 - If n is normalized, distance to plane, $d = H(P)$
 - d is the *signed distance*!

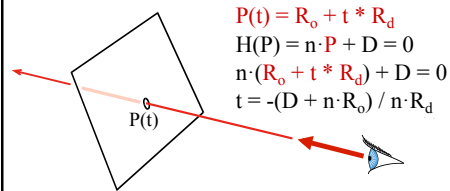


Explicit vs. Implicit?

- Ray equation is *explicit* $P(t) = R_o + t * R_d$
 - Parametric
 - Generates points
 - Harder to verify that a point is on the ray
- Plane equation is *implicit* $H(P) = n \cdot P + D = 0$
 - Solution of an equation
 - Does not generate points
 - Verifies that a point is on the plane

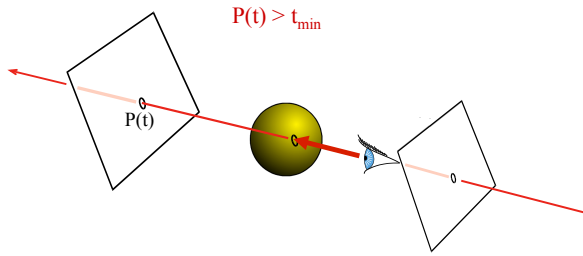
Ray-Plane Intersection

- Intersection means both are satisfied
- So, insert explicit equation of ray into implicit equation of plane & solve for t



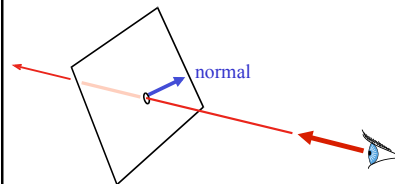
Additional Housekeeping

- Verify that intersection is closer than previous
 - $P(t) < t_{\text{current}}$
- Verify that it is not out of range (behind eye)
 - $P(t) > t_{\text{min}}$



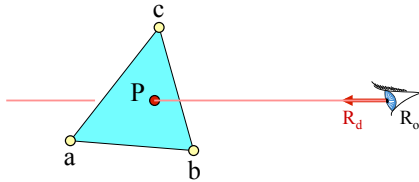
Normal

- Needed for shading
 - diffuse: dot product between light and normal
- Normal of a plane is constant!



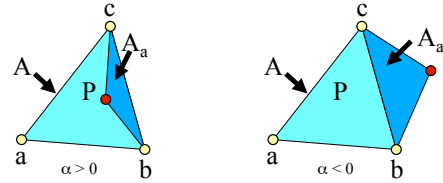
Ray-Triangle Intersection

- Use barycentric coordinates:
 - $P(\alpha, \beta, \gamma) = \alpha a + \beta b + \gamma c$
with $\alpha + \beta + \gamma = 1$
 - If $0 < \alpha < 1$ & $0 < \beta < 1$ & $0 < \gamma < 1$
then the point is inside the triangle!



How Do We Compute α, β, γ ?

- Ratio of opposite sub-triangle area to total area
 - $\alpha = A_d/A$ $\beta = A_b/A$ $\gamma = A_c/A$
- Use signed areas for points outside the triangle



*But how do I know if the point is outside the triangle?
That's what I was trying to determine!*

Using Cramer's Rule...

- Used to solve for one variable at a time in system of equations

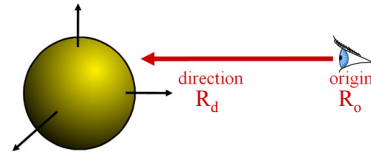
$$\beta = \frac{\begin{vmatrix} a_x - R_{ox} & a_x - c_x & R_{dx} \\ a_y - R_{oy} & a_y - c_y & R_{dy} \\ a_z - R_{oz} & a_z - c_z & R_{dz} \end{vmatrix}}{|A|} \quad \gamma = \frac{\begin{vmatrix} a_x - b_x & a_x - R_{ox} & R_{dx} \\ a_y - b_y & a_y - R_{oy} & R_{dy} \\ a_z - b_z & a_z - R_{oz} & R_{dz} \end{vmatrix}}{|A|}$$

$$t = \frac{\begin{vmatrix} a_x - b_x & a_x - c_x & a_x - R_{ox} \\ a_y - b_y & a_y - c_y & a_y - R_{oy} \\ a_z - b_z & a_z - c_z & a_z - R_{oz} \end{vmatrix}}{|A|}$$

| | denotes the determinant
Can be copied mechanically into code

Sphere Representation?

- Implicit sphere equation
 - Assume centered at origin (easy to translate)
 - $H(P) = P \cdot P - r^2 = 0$



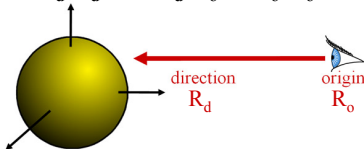
Ray-Sphere Intersection

- Insert explicit equation of ray into implicit equation of sphere & solve for t

$$P(t) = R_o + t \cdot R_d \quad H(P) = P \cdot P - r^2 = 0$$

$$(R_o + tR_d) \cdot (R_o + tR_d) - r^2 = 0$$

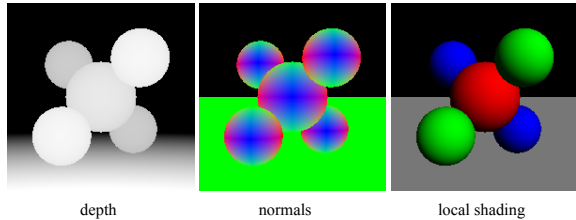
$$R_d \cdot R_d t^2 + 2R_d \cdot R_o t + R_o \cdot R_o - r^2 = 0$$



Ray-Sphere Intersection

- Quadratic: $at^2 + bt + c = 0$
 - $a = 1$ (remember, $\|R_d\| = 1$)
 - $b = 2R_d \cdot R_o$
 - $c = R_o \cdot R_o - r^2$
- with discriminant $d = \sqrt{b^2 - 4ac}$
- and solutions $t_{\pm} = \frac{-b \pm d}{2a}$
- What does it mean if there are no solutions, 1 solution, or 2 solutions?

Questions?



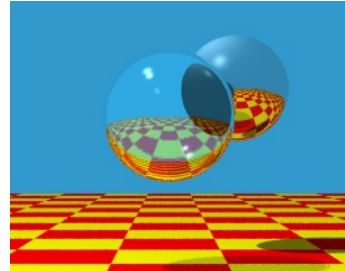
depth

normals

local shading

Reading for Today

- "An improved illumination model for shaded display"
Turner Whitted, 1980.

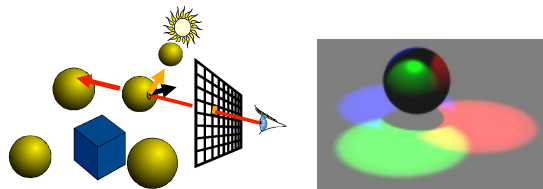


Today

- Ray Casting
- Ray Tracing
 - Shadows
 - Reflection
 - Refraction
- Recursive Ray Tracing
- Distribution Ray Tracing

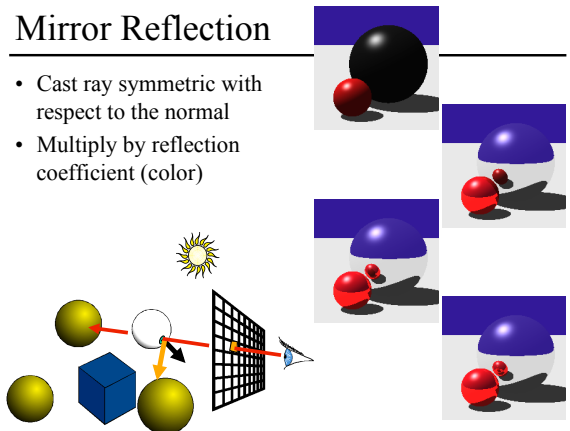
How Can We Add Shadows?

Find the point to be shaded
 For every light,
 Construct ray from point to light
 For every object
 find intersection of ray with object
 If no objects between point and light
 Add contribution from light



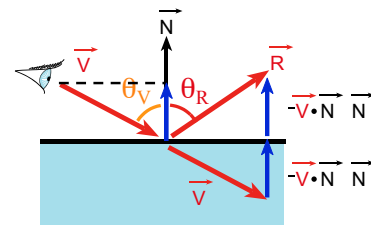
Mirror Reflection

- Cast ray symmetric with respect to the normal
- Multiply by reflection coefficient (color)



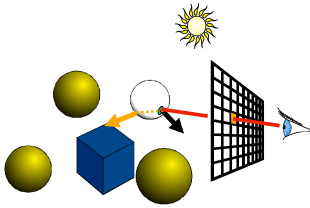
Reflection

- Reflection angle = view angle
- $R = V - 2(V \cdot N)N$

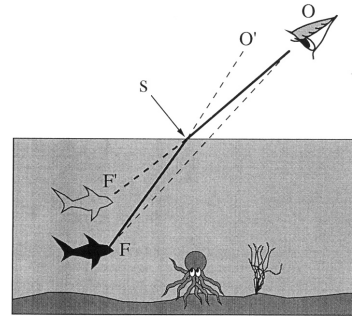


Transparency

- Cast ray in refracted direction
- Multiply by transparency coefficient (color)

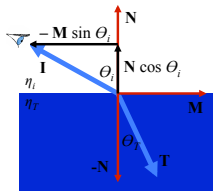


Qualitative Refraction



From "Color and Light in Nature" by Lynch and Livingston

Refraction



Snell-Descartes Law:
 $n_i \sin \theta_i = n_r \sin \theta_r$

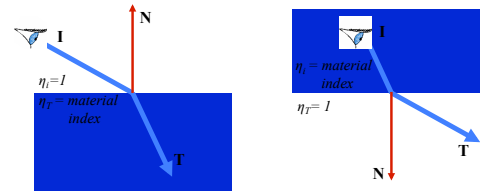
$$\frac{\sin \theta_r}{\sin \theta_i} = \frac{n_i}{n_r} = n_r$$

$$\begin{aligned} \mathbf{I} &= \mathbf{N} \cos \theta_i - \mathbf{M} \sin \theta_i \\ \mathbf{M} &= (\mathbf{N} \cos \theta_i - \mathbf{I}) / \sin \theta_i \\ \mathbf{T} &= -\mathbf{N} \cos \theta_r + \mathbf{M} \sin \theta_r \\ &= -\mathbf{N} \cos \theta_r + (\mathbf{N} \cos \theta_i - \mathbf{I}) \sin \theta_r / \sin \theta_i \\ &= -\mathbf{N} \cos \theta_r + (\mathbf{N} \cos \theta_i - \mathbf{I}) n_r \\ &= [\eta_r \cos \theta_i - \cos \theta_r] \mathbf{N} - \eta_r \mathbf{I} \\ &= [\eta_r \cos \theta_i - \sqrt{1 - \sin^2 \theta_r}] \mathbf{N} - \eta_r \mathbf{I} \\ &= [\eta_r \cos \theta_i - \sqrt{1 - \eta_r^2 \sin^2 \theta_i}] \mathbf{N} - \eta_r \mathbf{I} \\ &= [\eta_r (\mathbf{N} \cdot \mathbf{I}) - \sqrt{1 - \eta_r^2 (1 - (\mathbf{N} \cdot \mathbf{I})^2)}] \mathbf{N} - \eta_r \mathbf{I} \end{aligned}$$

- **Total internal reflection when the square root is imaginary**
- **Don't forget to normalize!**

Refraction & the Sidedness of Objects

- Make sure you know whether you're entering or leaving the transmissive material:



- What about intersecting transparent objects?

Total Internal Reflection



Fig. 3.7A The optical manhole. From under water, the entire celestial hemisphere is compressed into a circle only 97.2° across. The dark boundary defining the edges of the manhole is not sharp due to surface waves. The rays are analogous to the crepuscular type seen in hazy air, Section 1.9. (Photo by D. Grainger)



Fig. 3.7B The optical manhole. Light from the horizon (angle of incidence = 90°) is refracted downward at an angle of 48.6°. This compresses the sky into a circle with a diameter of 97.2° instead of its usual 180°.

From "Color and Light in Nature" by Lynch and Livingston

Questions?

Reading for Today

- "Distributed Ray Tracing", Cook, Porter, & Carpenter, SIGGRAPH 1984.



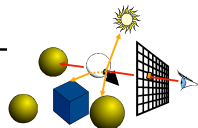
Today

- Ray Casting
- Ray Tracing
- **Recursive Ray Tracing**
- Distribution Ray Tracing

Ray Tracing

```

trace ray
  color = ambient term
  For every light
    cast shadow ray
    color += local shading term
  If mirror
    color += color_refl *
    trace reflected ray
  If transparent
    color += color_trans *
    trace transmitted ray
  
```

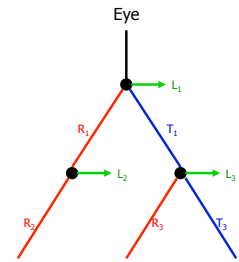
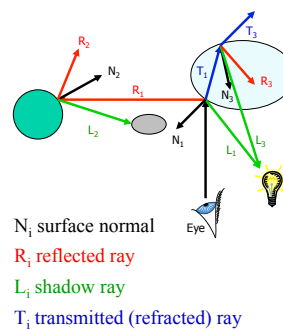


Stopping criteria:

- **Recursion depth**
 - Stop after a number of bounces
- **Ray contribution**
 - Stop if reflected / transmitted contribution becomes too small

- *Does it ever end?*

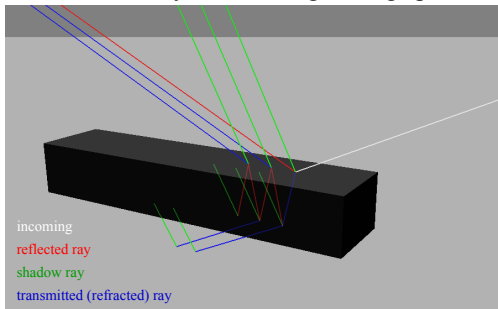
The Ray Tree



Complexity?

Ray Debugging

- Visualize the ray tree for single image pixel



Today

- Ray Casting
- Ray Tracing
- Recursive Ray Tracing
- **Distribution Ray Tracing**
 - Soft shadows
 - Antialiasing (getting rid of jaggies)
 - Glossy reflection
 - Motion blur
 - Depth of field (focus)

Shadows

- one shadow ray per intersection per point light source

point light source

no shadow rays

one shadow ray

Shadows & Light Sources

<http://www.davidfay.com/index.php>

clear bulb

frosted bulb

<http://www.pa.uky.edu/~sciworks/light/preview/bulb2.htm>

Soft Shadows

- multiple shadow rays to sample area light source

area light source

penumbra umbra penumbra

one shadow ray

lots of shadow rays

Antialiasing – Supersampling

- multiple rays per pixel

jaggies

w/ antialiasing

point light

area light

Reflection

- one reflection ray per intersection

perfect mirror

Glossy Reflection

- multiple reflection rays

polished surface

Justin Legakis

Motion Blur

- Sample objects temporally

Rob Cook

Depth of Field

- multiple rays per pixel

Justin Legakis

Ray Tracing Algorithm Analysis

- Ray casting
- Lots of primitives
- Recursive
- Distributed Ray Tracing Effects
 - Soft shadows
 - Anti-aliasing
 - Glossy reflection
 - Motion blur
 - Depth of field

cost \approx height * width *
 num primitives *
 intersection cost *
 size of recursive ray tree *
 num shadow rays *
 num supersamples *
 num glossy rays *
 num temporal samples *
 num focal samples *
 ...
can we reduce this?

these can serve double duty

Raytracing & Epsilon

intersects light @ $t = 25.2$
 intersects light @ $t = 26.9$
 intersects sphere1 @ $t = -0.01$
 intersects sphere2 @ $t = 0.01$
 intersects sphere2 @ $t = 14.3$

Solution: advance the ray start position *epsilon* distance along the ray direction OR ignore all intersections $< \epsilon$ (rather than < 0)

What's a good value for *epsilon*? Depends on hardware precision & scene dimensions

Image from Zachary Lynn

Reading for Friday 3/8:

- Goral, Torrance, Greenberg & Battaile "Modeling the Interaction of Light Between Diffuse Surfaces", SIGGRAPH '84

white paper enclosure
 set of illuminating lights
 camera
 teal cube
 white diffuse surface