Quantitative Texton Sequences for Legible Bivariate Maps

Colin Ware

Abstract—Representing bivariate scalar maps is a common but difficult visualization problem. One solution has been to use two dimensional color schemes, but the results are often hard to interpret and inaccurately read. An alternative is to use a color sequence for one variable and a texture sequence for another. This has been used, for example, in geology, but much less studied than the two dimensional color scheme, although theory suggests that it should lead to easier perceptual separation of information relating to the two variables. To make a texture sequence more clearly readable the concept of the quantitative texton sequence (QTonS) is introduced. A QTonS is defined a sequence of small graphical elements, called textons, where each texton represents a different numerical value and sets of textons can be densely displayed to produce visually differentiable textures. An experiment was carried out to compare two bivariate color coding schemes with two schemes using QTonS for one bivariate map component and a color sequence for the other. Two different key designs were investigated (a key being a sequence of colors or textures used in obtaining quantitative values from a map). The first design used two separate keys, one for each dimension, in order to measure how accurately subjects could *independently* estimate the underlying scalar variables. The second key design was two dimensional and intended to measure the *overall integral accuracy* that could be obtained. The results show that the accuracy is substantially higher for the QTonS/color sequence schemes. A hypothesis that texture/color sequence combinations are better for independent judgments of mapped quantities was supported. A second experiment probed the limits of spatial resolution for QTonSs.

Index Terms—Bivariate maps, texture, texton, legibility, quantitative texton sequence, QTonS.

1 INTRODUCTION

A common problem in data visualization is how to help people understand the relationships between variables that are continuous over a plane—in other words, scalar fields. For example, we might wish to understand how patterns of atmospheric pressure and temperature interrelate on a weather map, or we might wish to know how the distribution of a marine organism co-varies with a nutrient.

One solution is to display the variables with side-by-side maps, but this can involve much looking back and forth to identify correspondences. If the two variables can be combined in a single image then the resulting bivariate map can potentially make the comparison easier, assuming that the two underlying variables can still be independently perceived.

Scalar field maps can support a number of tasks. They allow numerical values to be estimated at any point over the plane. For example, on a weather map the actual values of the temperature or atmospheric pressure are often of fundamental interest and a key is usually provided to translate from colors or textures on the map to numerical values. Scalar maps can also allow the user to see patterns in the data such as ridges, local maxima, minima or saddles. In the present work the primary focus is on reading numeric values accurately; although, we will also pay attention to the need to convey pattern information. Specifically, we explore the use of a texture sequence for one map component and a color sequence for the other with the goal of producing a map where both components are highly legible and accurately readable.

1.1 Prior Work on Bivariate Map Displays

To begin the discussion of bivariate map displays it is useful to consider the desirable properties of a good solution. Here is a list of what are perhaps the most important.

Is each map displayed in a way that is perceptually monotonic? Montonicity, in this case, means that as the underlying data increases the display attribute should also increase along some perceptually increasing dimension [2,24]. For example, luminance is perceptually monotonic but the spectrum sequence of hues is not [26].

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Can quantities be read accurately? The ability to read values from a key is critical for some applications and unimportant for others where only the patterns embedded in the data may be more critical.

Are the display dimensions perceptually independent? For example this means that variation in one of the two mapped quantities should not hinder our ability to read the other [3,19,22,24]

Do the display dimensions allow integral judgments to be made? Often information from the two data maps must be integrated to reach some conclusion [24].

Depending on the visual reasoning task, different properties may have different weights. For example, if we wish to visually classify vegetation based on two measurements then integrality is important because the judgment is fundamentally holistic; e.g does an area represent grassland or forest? But if we wish to compare and contrast two variables (e.g. temperature and pressure) then separability is important.

Color vision is fundamentally three dimensional and up to three data dimensions can be coded using color. A solution to the two map problem, that has been extensively studied is the bivariate color coding scheme [20, 21, 24]. Wainer and Francolini [27], however, found that such schemes can be difficult to interpret. Garner's theory of integral and separable perceptual dimensions [8] suggests that color dimensions are not very independent perceptually; changes in one color dimension are likely to interfere with accurate perception of changes in the other. Rheingans [22] suggested that using hue and value will be a way of mitigating this and obtaining some degree of separability. Bergman, et al. [1] went further to suggest that high spatial frequency information should be mapped to lightness and low spatial frequency mapped to hue. This, however, entails going beyond using a simple transformation such as Smith's [23] HSV color space where changes in hue can cause large changes in perceived luminance. HSV is a simple geometric transformation to RGB monitor values and has only a very rough relationship to psychophysically determined hue saturation and value [21].

A recently developed alternative to using a two dimensional color coding scheme is use interwoven small patches color (e.g. red and green) with each color patch varying in saturation to display a different scalar value [9,16]. Saturation scales only can be expected to yield a few clearly readable steps for continuous maps [26] although Hagh-Shenas et al. achieved quite low errors for display

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with 13 uniform regions representing the Midwestern states of the USA [9].

Garner's theory of separable/integral dimensions [8] suggests that better perceptual separation between two scalar values may occur if one is mapped to color and the other is mapped to variation in some aspect of visual texture. This is because texture and color are more separable than any two color dimensions. Healey and Enns [11] showed that simple target shapes could be rapidly detected in a texture-color combination display. They found that glyph variability did not measurably interfere with the detection of targets shown using glyph color. Conversely, random colors did interfere somewhat with detection of targets shown using a glyph-based texture. The glyphs field they used for their empirical studies, however, was only a 20x20 grid capable of representing very little detail. Most bivariate mapping problems require a much greater density than this. When using texture to display quantitative information, density is necessarily sacrificed since legible glyphs must necessarily take up some space to be independently resolvable [25]. Nevertheless, distinguishable textures can be quite fine and a secondary goal in this research was to design and evaluate the use of high density, small texton textures.

2 Quantitative Texton Sequences

The solution we propose is the quantitative texton sequence (QTonS) to display data for one of two maps, with a conventional color sequence for the other. The purpose of the QTonS is to create an ordered sequence of textures with an unambiguously perceived value associated with a member of the sequence.

A texton is here defined as a small texture element that when presented in a dense field forms a texture. The term originates with Julesz [13] who gave it a specific theoretical meaning, but since that time it has come to have the more generic sense in which it is used here. A number of researchers have investigated the use of textons in displaying both univariate and multivariate fields [2, 10, 11, 12, 25]. Pickett and Grinstein [18] developed little stick figure-like glyps where each arm represented a variable and presented them in dense fields. Ware and Knight proposed, based on perceptual theory, that the primary dimensions of texture are size, orientation, and amplitude [25]. Bertin [2] recommended using the texture grain – defined as the number of marks per unit area. Healy and Enns [11] studied the effectiveness of using texture density, regularity and height to display independent variables. However, in each of these prior studies the mapping between data and the texture appearance was to a continuously varied property of the texture, such as density, size or orientation of the textons. A drawback of all of such schemes is that any continuous single variation (such as texton size) is likely to be susceptible to simultaneous contrast effects [25] causing errors in the same way that simultaneous lightness or color contrast causes errors in color-coded maps [4, 6, 26].

For completeness we must also note that non-texton based textures, such as stripes of different widths and orientations, have also been used to display quantitative information [2].

We define a QTonS to be a *series of texton elements such that textons in the series are each visually distinct and collectively represent an increasing or decreasing sequence of numerical values.* If the design is successful in creating visually distinct QTonS contrast induced errors should be eliminated. In addition to this basic definition there are a number of other properties that are desirable for QTonS.

Monotonicity. As with color sequences QTonS elements should be *perceptually* ordered on some dimension such as grain size, density, or overall lightness.

Legibility. Each quantitative texton should be capable of being used to make texture that is clearly distinct from the other textures created by the other textons. Achieving maximum legibility must take into account the background color. In particular, there must be reasonable luminance contrast between the textons and the background.

Small size. In order to reveal the greatest possible information in the underlying data field QTonS elements should be small so that they can be packed into dense textures.

Minimal interference. Often QTonS textures will be used overlaying other color-coded information to provide a bivariate map display. Because of this it is important that the QTonS elements minimally obscure and distort the background information display. The most straightforward way of achieving this is to ensure that the texton elements do not obscure more than 50% of the background. Having highly saturated colors for the background color coded variable and black or white for the QTonS textures is likely to help with this.

The two ten-step QTonSs shown in Figure 1 were designed through an informal iterative process that involved repeated adjustments to create textons that are clearly ordered and where each has a roughly equal perceptual distance from its neighbors. A design breakthrough came with the realization that the sequence could be split in the middle, using black and white sub-sequences as shown. Fig. 1(a) shows a sequence where if the elements are properly spaced in a regular grid the lines join to form a series of cross hatching schemes. Fig 1 (b) is designed so that each symbol in the sequence is distinctive in shape as well as being ordered in the lightness darkness dimension.

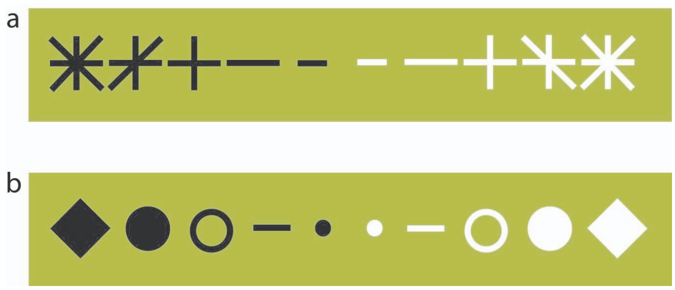


Fig.1. Two examples of QTonS sequences. Both are intended to be more closely spaced when used in a grid to form a texture. These are shown as textures in Figure 2.

The two ten-step QTonSs shown in Figure 1 were designed through an informal iterative process that involved repeated adjustments to create textons that are clearly ordered and where each has a roughly equal perceptual distance from its neighbors. A design breakthrough came with the realization that the sequence could be split in the middle, using black and white sub-sequences as shown. Fig. 1(a) shows a sequence where if the elements are properly spaced in a regular grid the lines join to form a series of cross hatching schemes. Fig 1 (b) is designed so that each symbol in the sequence is distinctive in shape as well as being ordered in the lightness darkness dimension.

Both sequences use black and white textons combined with changes in the area covered so that the overall effect is a continuum from dark to light. They are analogous to "divergent" or double-ended color sequences [3]. Such sequences are useful to show values above and below a mean, as in the case for atmospheric pressure. The sequences can be seen in the form of textures in Figure 2 (Schemes QTS_1 and QTS_2)

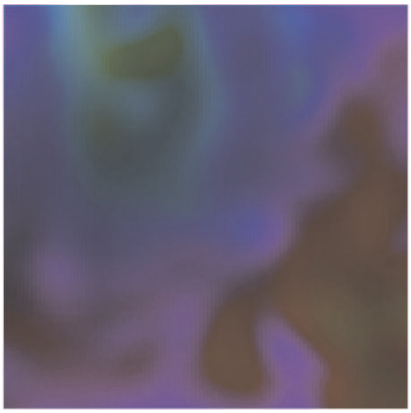
3 Study Goals and Hypotheses

To evaluate the effectiveness of QTonS/color sequence combinations of bivariate maps we carried out two experiments. In the first, the two different QTonSs were compared to two bivariate color mapping schemes and also to a conventional texton sequence where textons consisting of circular dot elements of variable size were used. In the second, the limits of QTonS density was explored.

To investigate the integrality/separability of the different schemes we used two different kinds of keys (see Figures 3 and 4). One was a two dimensional matrix, that represented all combinations of the two display dimensions. This allowed for direct matches to the appearance of any part of a map. The other was a pair of keys — one key for each display dimension. Because a particular map combination was unlikely to be present on either key it required that separable judgments of the two variables be made.

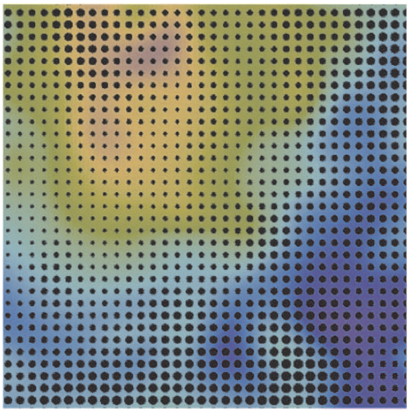
Scheme 1: Green Red (GR)

One data dimension is mapped to green and the other to red. We chose this scheme because it is effective in signaling when the two maps are uncorrelated, because there will be lots of red and green color; or correlated, because the colors will range from black to yellow. It also has the advantage that it uses the color space effectively; red and green colors have the greatest luminance range for both LCD and CRT monitors.



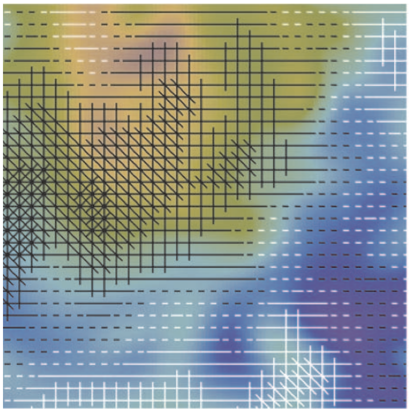
Scheme 2: Hue Lightness (HL)

One data dimension is mapped to a hue sequence and the other to lightness. The hue sequence used is blue – green – red – brown. An attempt was made to equate the hues in terms of heterochromatic brightness for a given value [29]. The value scale was started at 10% because hues become indistinguishable close to black.



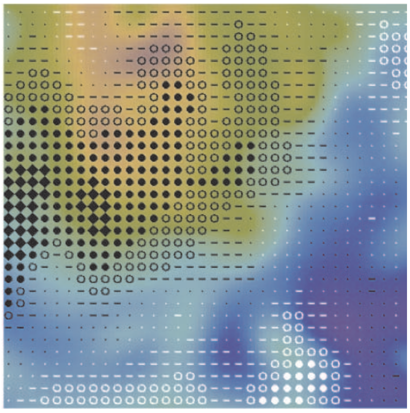
Scheme 3: Spectrum Dot Glyph (DOT)

One data dimension is mapped to a spectrum color sequence and the other to a series of dots of increasing size. This sequence is intended to be representative of the kinds of textures that are commonly used in practice.



Scheme 4: Spectrum QTonS 1 (QTS_1)

One data dimension is mapped to a spectrum color sequence and the other to the 10 step QTonS sequence shown in Figure 1a. This 10 step sequence is designed to clearly show positive and negative values by making those above zero white and those below zero black. The area of each glyph increased monotonically above and below zero. In addition the lines are designed to fuse when used in a dense grid to provide the hatch textures as shown.



Scheme 5: Spectrum QTonS 2 (QTS_2)

One data dimension is mapped to a spectrum color sequence and the other to the 10 step QTonS sequence shown in Figure 1b. This 10 step sequence is designed to clearly show positive and negative values by making those above zero white and those below zero black. The area of each glyph increased monotonically above and below zero. The texture glyphs are designed to be very distinct from their neighbors in the sequence.

Fig. 2. Five different bivariate mapping schemes were evaluated. These sample images show a fraction (one eighth) of the area that was displayed in the experiment.

Figure 2 provides details regarding the specific schemes that were evaluated.

The following hypotheses were formulated regarding expected errors.

- The two spectrum/QTonS schemes will produce lower errors than the two bivariate color schemes and lower errors than the spectrum/dot scheme.
- For the two integral schemes (bivariate color: GR, HL) the direct matching keys should result in greater accuracy.
- For the separable color/texture schemes (DOT, QTS_1, QTS_2) there should be no difference in accuracy depending on the type of key.
- Luminance will result in the greatest error in the hue-value (HL) scheme.

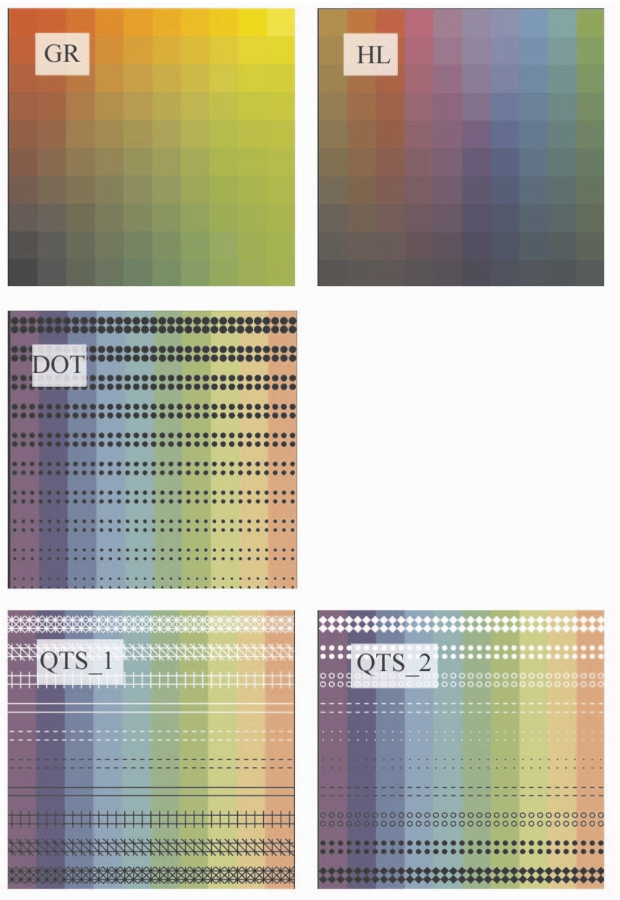


Fig. 3. The set of 2D keys used in the direct matching conitions.The labels were not part of the original display.

4 Experiment One

Five different bivariate schemes were evaluated with two different kinds of keys (a 5x2 design). Each of the bivariate map representation schemes was carefully designed to be a good example of its type.

4.1 Bivariate Map Generation and Viewing Conditions

The experiment used pairs of artificially generated, smoothly varying scalar fields. These were constructed by summing a set of randomly positioned and oriented two-dimensional gabor functions into a 600x600 array. The use of gabors enabled us to control the dominant spatial frequency components. The generation procedure used two different gabor sizes. To give large scale variation 40 gabors were added with mean wavelength of 6 degrees of visual angle (0.167 cycles/deg). Also added were 120 gabors with mean wavelength of 1.5 degrees and proportionately lower amplitude. After generation the two artificial fields were scaled to have a minimum value of 0.0 and a maximum value of 1.0. The examples shown in Figure 2 represent about one eighth of the total area of the synthetic field displayed to subjects.

A high resolution IBM T221 LCD monitor was employed for the study. This has a screen resolution of 80 pixels per cm. The display was presented in a 15 cm x 15 cm (1200 pixel x 1200 pixel) viewport. The screen was viewed from 57 cm yielding a 15 degree visual angle for the display. The grid of textons was 100x100 within square display. This yielded 6.6 textons per degree of visual angle for an overall density of 44 textons/deg^2.

The mean display luminance was approximately 50 cd/m^2 and the display was viewed in a dimly lit room.

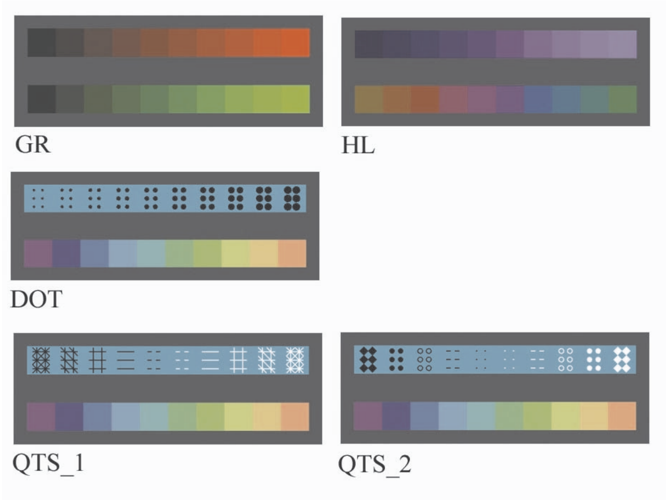


Fig. 4. The set of 1D keys used in the separable judgments condition

4.2 Task

There was a training phase for the participants where the task was first demonstrated. Following this they carried out two practice trials using each of the 5 schemes. This was done prior to both the matching key and separable key trial blocks (see below).

On each trial the subject was presented with the cursor in a different randomly determined position always more than 2 degrees inside the boundaries of the display (illustrated in Figure 5). With the matching key their task was to click on the cell most closely matching the target value. With the separate keys they clicked twice to select the two components. There was no time limit, and subjects were urged to work carefully. They typically took about 8 seconds per trial on average.

4.3 Trials

Each scheme was tested in groups of 12 trials using a particular scheme the 5 different schemes randomly ordered (60 trials in a block). They completed four such blocks, two with the matching key (I) and two with the separate keys (S) with either an ISIS or an SISI ordering. This yielded a total of 24 trials per condition per subject. Half the participants received the integral condition first and half the participants received the separable condition first. The whole experiment took about 45 minutes including breaks between trial blocks.

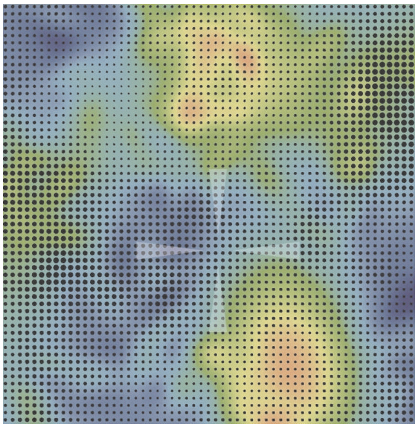


Fig. 5. The cursor specifying a target point.

4.4 Participants

The 14 participants were undergraduate students. They were required to have 20:20 vision or better and were paid for participating.

4.5 Results and Discussion for Experiment One

For each display type we computed the mean error for both components of the bivariate map (*e1* and *e2*) and a combined error.

*Error = sqrt(e1*e1 + e2*e2)*

Because the data are highly skewed log transformed values were used in the analysis; although, simple arithmetic mean values are shown in the figures illustrating the results.

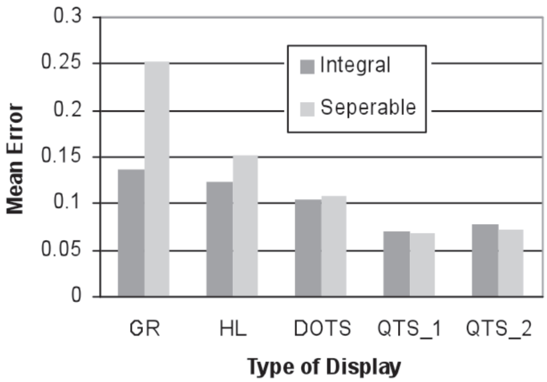


Fig. 6. Summary results showing the mean error for each of the display types. Integral results were obtained with the two-dimensional key whereas separable results were obtained with the independent keys.

Figure 6 summarizes the results for the combined error. A within subjects ANOVA revealed a highly significant main effect for the different mapping schemes [$F(4,52) = 137.4$, $p < 0.001$] with the two QTonS schemes yielding the lowest errors. There was also a highly significant main effect for key type [$F(1,13) = 30.5$, $p < 0.001$]; and a significant interaction [$F(4,52) = 18.5$,$p < 0.001$] between key type and the type of display. Because of the interaction we conducted separate Tukey HSD tests on the integral key and the separable key data. For the integral data this showed all the conditions to be different except for the two bivariate color schemes (RG, HL) and the two QTonS schemes (QTS_1, QTS_2). For the separable data all the conditions were different except for the two QTonS schemes (QTS_1, QTS_2).

To test the hypothesis that the bivariate color sequences would perform better with the matching key than with the separate keys two additional ANOVAs were run. The first was run for the three texture/spectrum schemes with the two kinds of keys. This found no significant difference depending on the key type. The second was run on the integral bivariate color sequences (RG, HL). This revealed a highly significant effect of key [$F(1,13) = 55.5$, $p < 0.001$]. As predicted, errors were significantly smaller with the separate keys.

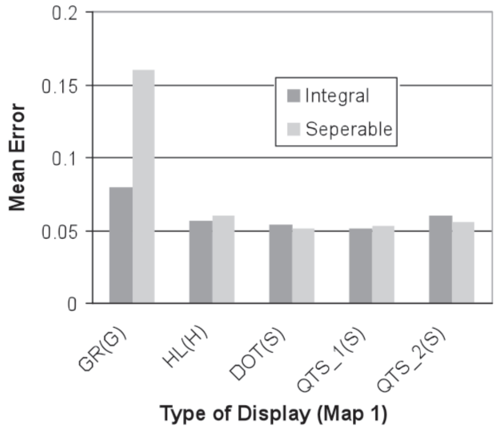


Fig. 7. The error for the first of the two maps being read. (G) Green component; (H) Hue component; (S) Spectrum component.

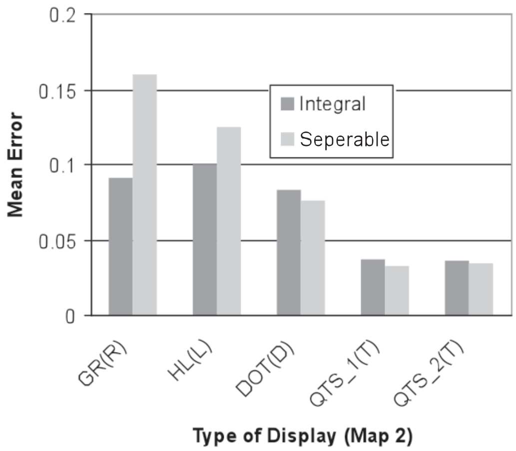


Fig. 8. The error for the second of the two maps being read. (R) Red component; (L) Lightness component; (D) Dots component; (T) Texton component.

Although the combined error results are informative it is also instructive to examine separately the errors obtained from each of the components of the bivariate map display (see Figures 7 and 8 for a summary). Most notably, it can be seen that the QTonS component of the QTonS/spectrum scheme gave the smallest overall error by a wide margin (see QTS_1 and QTS_2 in Figure 8). Optimal performance on the task would yield a mean error of .025 because of the .1 unit width of each key band. The average mean errors we obtained for the QTonS component of the key were .035, about 40% larger. It is worth noting, however, that the median was only .027, remarkably close to the optimal value.

Also, as predicted, the error was markedly less for the hue component of the hue/lightness scheme compared to the lightness component. This is in line with previous findings that errors are largest for grey-scale schemes [26]. Another finding was that for the red/green scheme the green component yielded the smallest error, at least with the matching key. This is presumably because of the greater luminance range for the green component of a monitor compared to the red component. (All of the above findings were statistically significant at .01 level or better.)

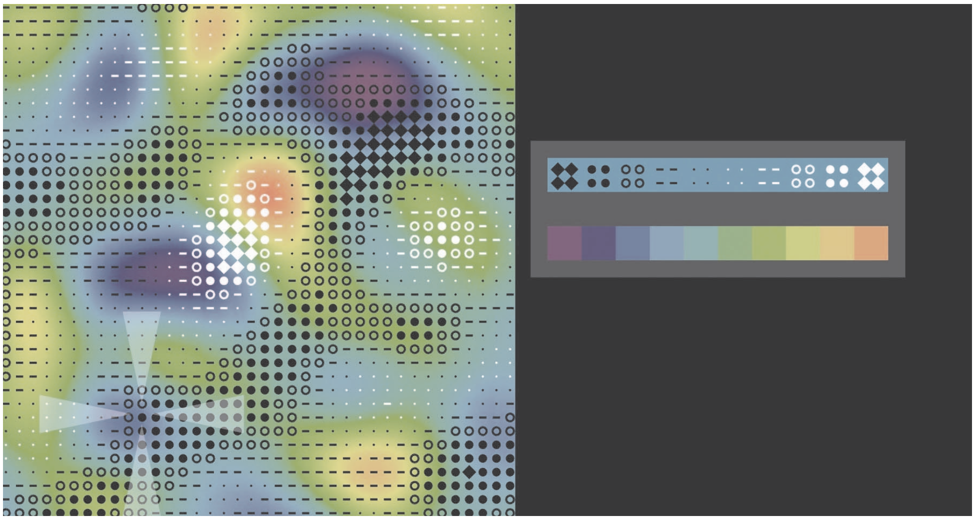


Fig. 9. The display screen used in experiment 2. This is the example with 10 textons per degree. To see this example to scale look at this image from approximately 1.1 meters.

5 EXPERIMENT TWO

The purpose of the second experiment was to probe the limits of QTonS density. To accomplish this, the density was varied from 5 to 40 textons per linear degree of visual angle. Only the two QTonS schemes were investigated.

5.1 Method

The viewing distance was increased to 2.28 meters to allow for very fine textures defined in terms of visual angle. This resulted in an overall display size of 3.75 deg square. At this viewing distance there were 320 pixels per degree of visual angle. Individual textons were generated with the following sizes: 8x8 pixels, 16x16 pixels 32x32 pixels and 64x64 pixels. In terms of visual angle this produced 40, 20, 10 and 5 per linear degree. With the 40 textons/deg textures the individual texture elements were not distinguishable and so the result was a bivariate color mapping scheme. The cursor showing the target areas was also enlarged by a factor of 2 because of the viewing distance. Only the separate keys were used in this experiment and the resulting display is illustrated in Figure 9. Study participants used a wireless mouse to make selections.

The artificial data maps were generated with only the larger size gabors. A sample display is shown in Figure 9.

5.2 Participants

The 10 participants were undergraduate students. They were required to have 20:20 vision or better and were paid for participating.

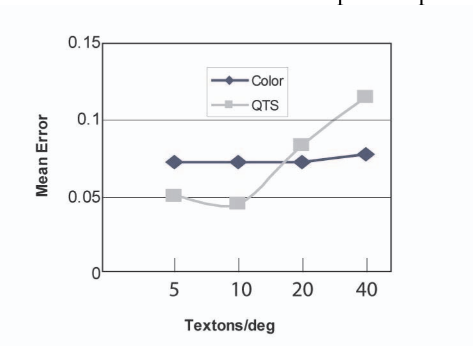


Fig.10. The results from the second study show greatest accuracy with 10 textons/degree

5.3 Results and Discussion for Experiment Two

Figure 10 shows how both the overall QTonS error and the spectrum color error varies with texton density. As expected there was a highly significant main effect of the texton density for the QTonS error component [$F(3,39) = 72.1$, $p < 0.001$]. There was no effect for the color sequence error component. The highest accuracies were obtained with the second largest textons spaced at 10 textons per degree of visual angle. This suggests that the textons used in the first experiment were probably close to the smallest that could be used for optimal accuracy. Note that with textures finer than 10 QTonS per degree the individual elements become much harder to resolve as can be determined by viewing Figure 9 from two meters. Overall, accuracies were somewhat lower than those obtained in the first experiment and this can probably be attributed to the fact that the artificial scalar fields had steeper gradients measured in degrees of visual angle.

6 DISCUSSION

As predicted both of the quantitative texton schemes resulted in substantially lower errors especially compared to the bivariate color mapping schemes. The errors measured for the QTonS component of the display are lower even than those that have been reported for univariate maps (e.g. [26]). Clearly it cannot be claimed that QTonS schemes will always be better than all bivariate color schemes (because we may not have designed the best possible color scheme and it would be possible to construct bad QTonS schemes). Nevertheless, we feel that the alternative schemes that we evaluated are reasonably strong examples of their type and thus the results are suggestive that well designed QTonS schemes can be expected to outperform bivariate color sequences or simple texture/color combinations.

The high accuracy achieved with the 10 level QTonS is especially impressive considering that many examples of bivariate color sequences have used only three or four levels of each variable [4]. The QTonS schemes are clearly successful in overcoming simultaneous contrast induced errors.

The very clear results regarding integral versus separable display dimensions support the relevance of Garner's theory to a task that is very different to the one used by Garner and other psychologists [15]. The tasks described in the psychological literature mostly are some form of speeded classification where subjects rapidly sort stimuli based on one attribute or another. For example, color or shape for separable dimensions, hue or lightness for integral dimensions. With integral stimuli an irrelevant attribute will cause a slowing of the sorting and increased errors [19]. For example, variation in lightness will make it difficult to sort on hue, but variation in shape will not. There is also a theoretical distinction made between perceptual integral vs separable dimension and a response-based or *decisional* integral vs separable dimensions [15]. In the present paper the errors may be as a result of perceptual confusion, or of response difficulties. However, the fact that the separate key solutions produced larger errors with integral schemes seems most likely to have been due to the fact that subjects were not able to abstract dimension on the map and make a match to the key based on that single dimension.

The most obvious disadvantage of the QTonS scheme is that no texture based scheme can be expected to achieve the same spatial resolution as a bivariate color scheme. This problem is made worse by the fact that most computer displays are not capable of displaying textures as fine as the ones used in the present study. Clearly, the method can be used with lower resolution screens, but subjectively the method seems to work best when textons are so fine that they can barely be resolved. At present, high quality printing is likely to be the best way of obtaining the necessary texton density. There is, however, a steady trend towards higher resolution displays. Indeed, if fine textures are found to be useful in information display this gives an additional reason for their development.

The present study has barely begun to scratch the surface of the design space for QTonS. It seems possible that sequences with many more that ten steps can be designed although there is likely to be a tradeoff with the texton density and the rapidity with which the elements can be read. Designing textons so that they are both maximally distinct from their neighbors in the sequence and monotonically increasing on some perceptual dimension, is an interesting challenge.

Besides displaying one variable of a bivariate map the QTonS method can have other uses. Celidnik and Rheingans [5] used patterns overlaid on color sequences to display uncertainty in the data. QTonS could do this with greater accuracy. It is also possible that oriented QTonS elements could be developed to display vector fields. This is a problem we are currently working on.

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REFERENCES

[1] L. D. Bergman, B. E. Rogowitz and L. A. Treinish. A Rule-based Tool for Assisting Colormap Selections. *Proc of IEEE Visualization,* 118-125, October 1995.
[2] J. Bertin, *Semiology of Graphics*. University of Wisconsin Press. 1983.
[3] C.A. Brewer, Guidelines for selecting colors for diverging schemes on maps. *Cartographic Journal*, 33(2): 79-86, 1996.
[4] C A. Brewer, Prediction of simultaneous contrast between map colors with Hunt's model of color appearance. *Color Research and Application,* 21(3) 221-235, 1996.
[5] A. Cedilnik, and P. Rheingans, Procedural Annotation of Uncertainty Information. Proc *IEEE Visualization,* 2000. Washington, DC. 77-83, 2000.
[6] W. S. Cleveland, and R. McGill, A Color-Caused Optical Illusion on a Statistical Graph. *The American Statistician*, 37:101-105 ,1983.
[7] R. Dunn, A dynamic approach to two variable color mapping. *The American Statistician,* Vol 43(3) 245-252, 1983.
[8] W.R. Garner, *The Processing of Information and Structure*. Erlbaum: Hilldale NJ. 1974.
[9] H. Hagh-Shenas, S. Kim, V. Interrante, C. Healey, Weaving versus blending: a quantitative assessment of the information carrying capacities of two alternative methods for conveying multivariate data with color. *IEEE Transactions on Visualization and Computer Graphics*. 13(6) 1270-1277, 2007.
[10] D.B. Haug, A.M. MacEachren, F.P. Boscoe, D., Brown, M., Marra, C., Polsky, and J. Beedasy, Implementing exploratory spatial data analysis methods for multivariate health statistics. *Proc. GIS/LIS'97* Oct, 1994.
[11] C. Healey, and J. Enns, Large datasets at a glance: Combining textures and colors in scientific visualization. *IEEE Transactions on Visualizations and Computer Graphics*, 5 (2) 145-167, 1999.
[12] C. Healey, and J. Enns, Building perceptual textures to visualize multi-dimensional datasets. *IEEE Transactions on Visualization and Computer Graphics* 5:22, 145-167, 1999.
[13] B. Julesz. Textons, the elements of texture perception, and their interactions. *Nature* 290, 91-97. 1981.
[14] A. Leonowicz, Two-variable choropleth maps as a useful tool for visualization of geographical relationships. *Geographija*, 42, 33-37, 2006.
[15] W.T. Maddox. Perceptual and Decisional Separability. *University of California Press*. 1996.
[16] J.R. Miller. Attribute Blocks: Visualizing multiple continuously defined variables. *IEEE Computer Graphics and Applications*, 73(3) 57-69. 2007.
[17] B. Pham, Spline-based color sequences for univariate, bivariate and trivariate mapping. *Proc. IEEE Visualization*. 202-208, 1990.
[18] R.M. Pickett, and G.G. Grinstein, Iconographic displays for visualizing multidimensional data. *Proc. IEEE Conference on Systems, Man and Cybernetics*. 1, 514-519, 1988.
[19] M.I. Posner, Information reduction in the analysis of sequential tasks. Psychological Review, 71(6) 491-504, 1964.
[20] P. Rheingans, Dynamic color mapping of bivariate qualitative data. *IEEE Vis 97*. 1997.
[21] P. Robertson, and J. O'Callaghan, (1986) The generation of color sequences for univariate and bivariate mapping *IEEE Computer Graphics and Applications*, 6.
[22] P. Rheingans, Task-based color scale design. SPIE. Task-Based Color Scale Design. Proceedings *Applied Image and Pattern Recognition*. SPIE, 35-43. October 1999.
[23] A.R. Smith, (1978). Color Gamut Transform Pairs. *Proc. SIGGRAPH'78. Computer Graphics* 12 (3). 12-19.
[24] E. Trumbo, Theory for Coloring Bivariate Statistical Maps," *The American Statistician,* vol. 35, no. 4, pp. 220-226, 1981.
[25] C. Ware, and Knight, W. Orderable dimensions of visual texture for data display: orientation, size and contrast. *Proc. ACM CHI'92* 203-209, 1992.
[26] C. Ware, Color Sequences for Univariate Maps:" Theory, Experiments and Principles, *IEEE Computer Graphics and Applications,* Sept. 1988, pp. 41-49, 1988.
[27] H. Wainer, and C.M. Fancolini, An empirical inquiry concerning human understanding of two variable map. *The American Statistician,* 34(2) 81-89, 1980.
[28] C.E. Weigle, W. Emigh, G. Iiu, R.M.Taylor II, J.T. Enns, C.G. Healey, oriented sliver textures: A technique for local value estimation of multiple scalar fields. *Proceedings, Graphics Interface*, 2000, 163-170.
[29] G. Wyszecki, W.S. Stiles, *Color Science: Concepts and Methods, Quantitative Data and Formulae*, Wiley, NY. 1982.