Functional Programming with Scheme

Keep reading: Scott, Chapter 11.1-11.3, 11.5-11.6, Scott, 3.6

Lecture Outline

- Scheme
  - Exercises with map, foldl and foldr

  - Binding with let, let*, and letrec
  - Scoping in Scheme
  - Closures

  - Scoping, revisited

Exercises

(define (rev2 lis)
(foldr (lambda (x y) (append y (list x))) lis '()))

(cons x '())

current elem

partial result

(foldr op lis id))

(e_1 … e_{n-1} e_n) id

(e_1 … e_{n-1}) res_1

…

(e_1) res_{n-1}

res_n

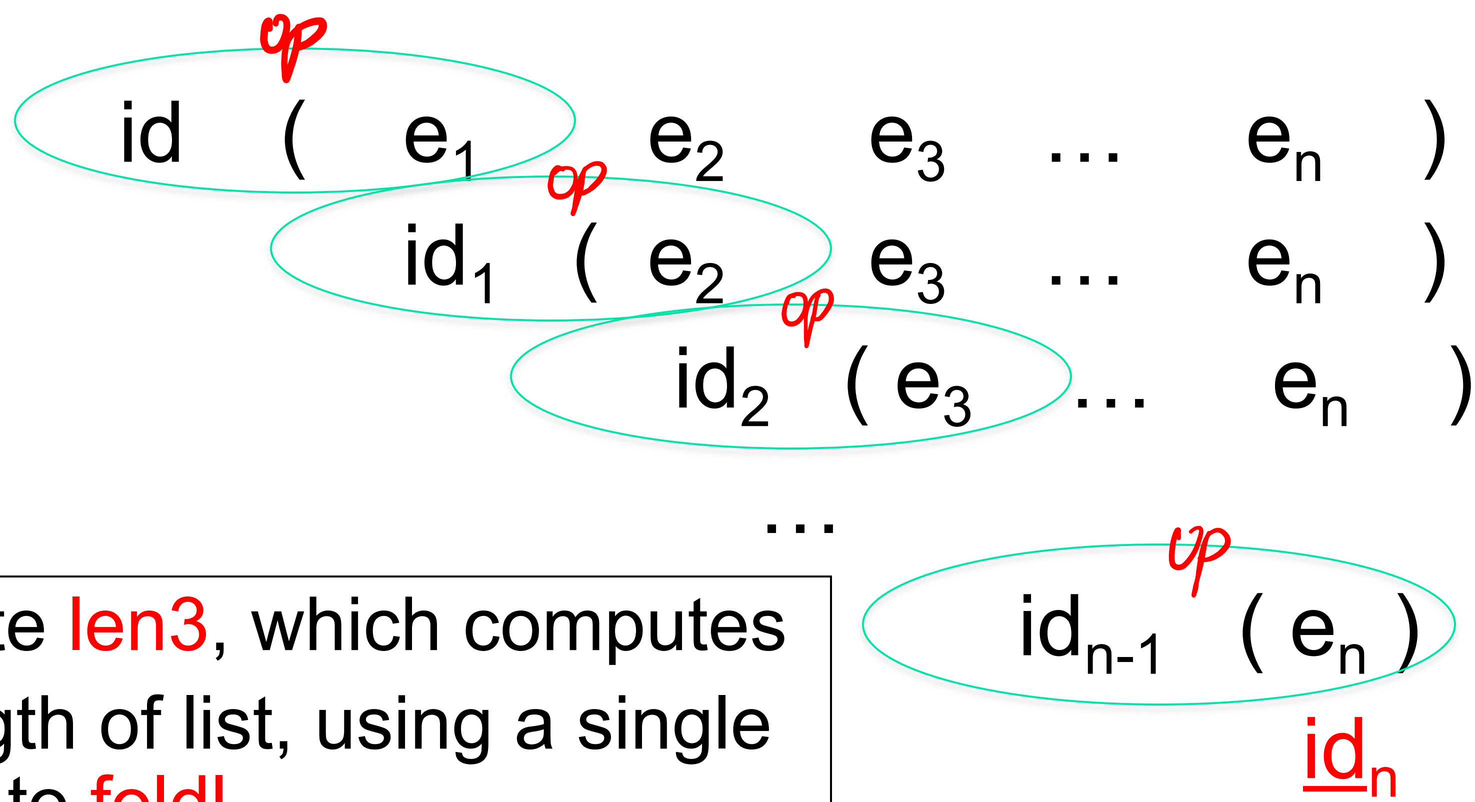
Write rev2, which reverses a list, using a single call to foldr
(define (rev2 lis) (foldr …))

Exercises

(define (len3 lis)
(foldl (lambda (x y) (+ x 1)) lis 0)
)

partial result → x; next element → y

(foldl op lis id)



Write len3, which computes length of list, using a single call to foldl

(define (len3 lis) (foldl …))

Exercises

(1 (2 (3))) → (1 2 3)

```
(define (foldl op lis id)
   (if (null? lis) id
      (foldl op (cdr lis) (op id (car lis)))) )
```

apply

- Write flatten3 using map and foldl/foldr

```
(define (flatten3 lis)
     (cond ((null? lis) lis)
           ((atom? lis) (list lis))
           (else (foldl append (map flatten3 lis) '()))
     )
)
```

- Write flatten4 this time using foldl but not map.

Ref. implementation:

```
(define (flatten lis)
  (cond ((null? lis) '())
        ((atom? lis) (cons lis '()))
        (else (append (flatten (car lis))
                      (flatten (cdr lis))))))
```

```
(define (flatten4 lis)
  (cond ((null? lis) '())
        ((atom? lis) (list lis))
        (else (foldl (lambda (x y)
                        (append x (flatten4 y)))
                     lis
                     '()
)))
```

Exercises

- Write a function that counts the appearances of symbols a, b and c in a list of flat lists
  - (count-sym ‘((a b) (c a) (a b d)) yields

    ((a 3) (b 2) (c 1))
  - Natural idea: use map and fold
- map and fold (or map and reduce), are the foundation of Google’s MapReduce model
  - Canonical MapReduce example [Dean and Ghemawat OSDI’04] is WordCount

Tail Recursion, A Bit More

- A tail expression is an expression that occurs in tail context. Defined inductively as follows:
  - The body of function is a tail expression
  - If (if e1 e2 e3) is a tail expression, then e2 and e3 are tail expressions
- Examples

```
(define (foldl op lis id)
  (if (null? lis) id
      (foldl op (cdr lis) (op id (car lis)))))
```

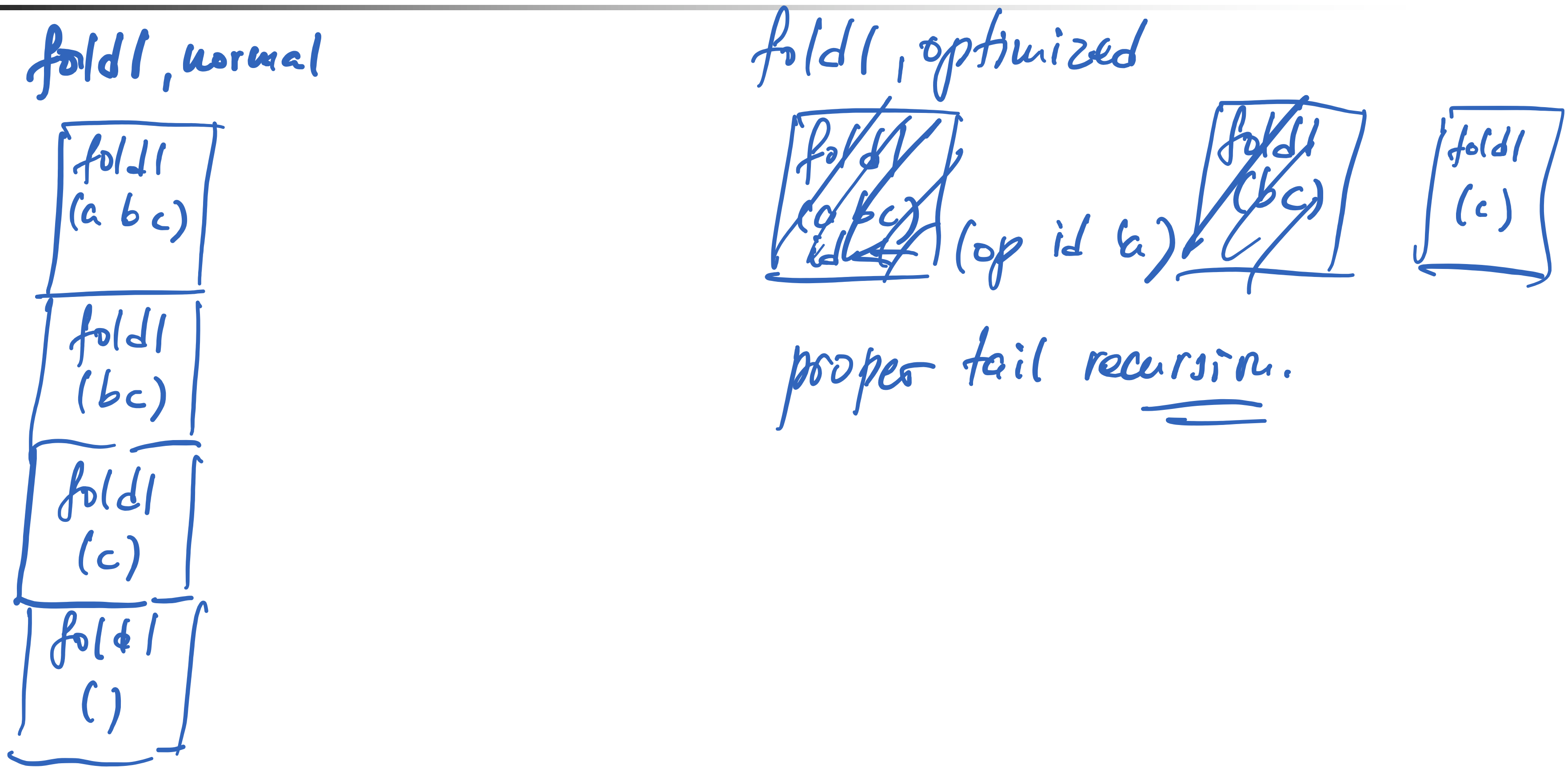
Tail Recursion, A Bit More

- A tail call is a tail expression that is a function call. E.g.,

```
(define (foldl op lis id)
   (if (null? lis) id
      (foldl op (cdr lis) (op id (car lis))) )
```

- A tail recursive function is a function whose "leaf" tail expressions are either returns or tail calls to itself (still informal)
- Tail calls give rise to efficient implementation of Continuation Passing Style (CPS)

Tail Recursion, A Bit More



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Let Expressions

let, let*, letrec

Let-expr ::= (let (Binding-list) S-expr1)

Let*-expr ::= (let* (Binding-list) S-expr1)

Binding-list ::= (Var S-expr) { (Var S-expr) }

- let and let* expressions define a binding between each Var and the S-expr value, which holds during execution of S-expr1
- let evaluates the S-exprs in Binding-list in current environment "in parallel"
- let* evaluates the S-exprs from left to right

- Associate values with variables for the local computation

Questions

(let ((x 2)) (* x x)) **yields** 4

Binding list — S-expr1

let ((x 2) (y 1)) (+ x y))

(let ((x 2)) (let ((y 1)) (+ x y))) **yields** what? 3

Sexpr (* 2 x) evaluated in empty environment

(let ((x 10) (y (* 2 x))) (* x y)) **yields** what?

ERROR

S-expr (* 2 x) evaluated in ((x 10)) environment

(let* ((x 10) (y (* 2 x))) (* x y)) **yields** what?

Let Expressions

Letrec-expr ::= (letrec (Binding-list) S-expr1)

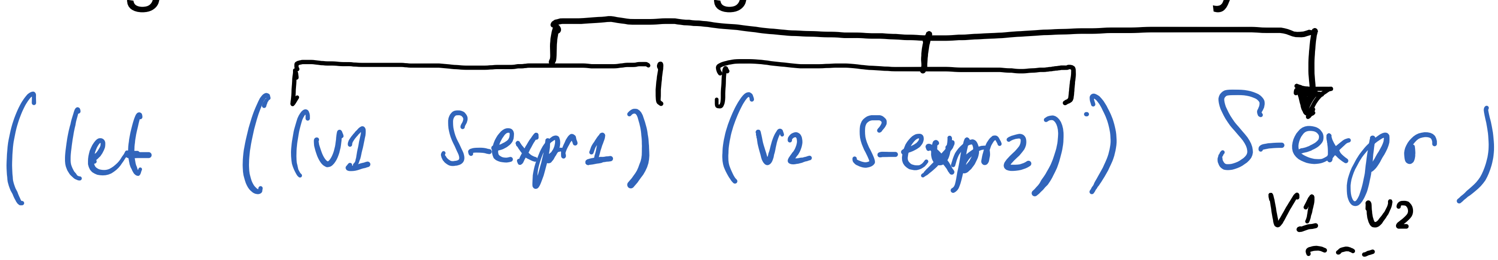
Binding-list ::= (Var **S-expr**) { (Var **S-expr**) }

- letrec Vars are bound to fresh locations holding undefined values; **S-exprs** are evaluated "in parallel" in augmented environment
- letrec allows for definition of mutually recursive functions

```
(letrec (( even? (lambda (n) (if (zero? n) #t (odd? (- n 1)))) )
         ( odd?   (lambda (n) (if (zero? n) #f (even? (- n 1)))) )
         )
    (even? 88)
)
```

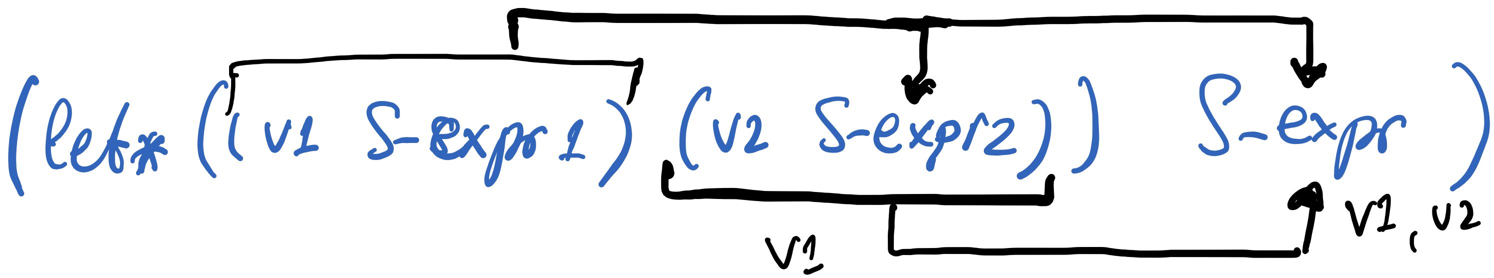
Regions (Scopes) in Scheme

- let, let* and letrec give rise to block structure
- They have the same syntax but define different regions (scopes)
- let
  - Region where binding is active: body of let



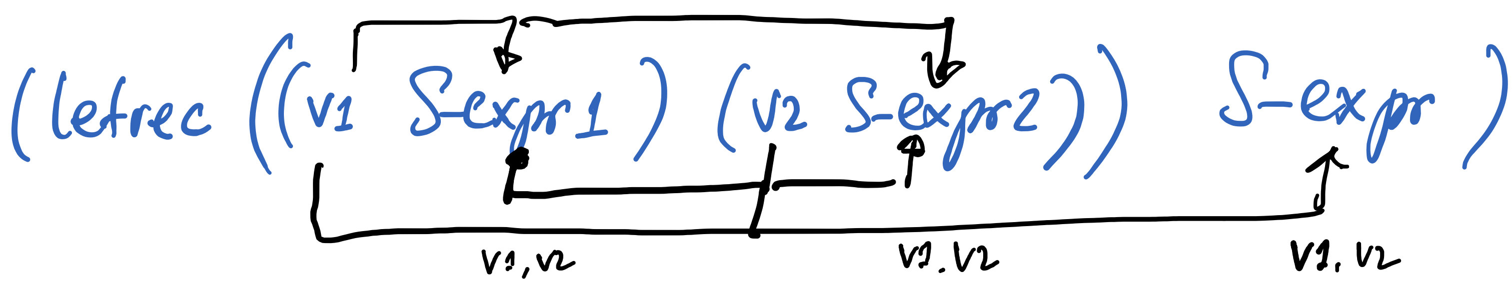
Regions (Scopes) in Scheme

- let, let* and letrec give rise to block structure
- They have the same syntax but define different regions (scopes)
- let*
  - Region: all bindings to the right plus body of let*



Regions (Scopes) in Scheme

- let, let* and letrec give rise to block structure
- They have the same syntax but define different regions (scopes)
- letrec
  - Region: entire letrec expression



Restriction: v1, v2 cannot be used as values in S-expr1 or S-expr2.

Let Introduces Nested Scopes

```
(let ((x 10))                                  ;causes x to be bound to 10
    (let ((f (lambda (a) (+ a x)))             ;causes f to be bound to
                                               a lambda expression
        (let ((x 2)) (f 5) ) ))
```

Assuming that Scheme uses static scoping, what would this expression yield?

Question

```
(define (f  z)
  (let* ( (x 5) (f (lambda (z) (* x  z))) )
      (map f z) ) )
```

f is the times-5 function

What does this function do?

Answer: takes a list of numbers, z, and maps it to the times-5 list. E.g., (f ‘(1 2 3)) yields (5 10 15).

Scoping in Scheme: Two Choices

```
(let ((x 10))
   (let ((f (lambda (a) (+ a x))))
      (let ((x 2))
         (* x (f 3) ) ) )
```

a is a "bound" variable

x is a "free" variable; must be found in "outer" scope

With static scoping it evaluates to

(* x ((lambda (a)(+ a x)) 3)) -->

(* 2 ((lambda (a)(+ a 10)) 3)) --> ??? 26

With dynamic scoping it evaluates to

(* x ((lambda (a)(+ a x)) 3)) -->

(* 2 ((lambda (a)(+ a 2)) 3)) --> ??? 10

Scheme Chose Static Scoping

```
(let  ((x 10))
     (let   ((f (lambda (a) (+ a  x))))
          (let ((x  2))
               (*  x  (f  3) ) ) )
```

f is a closure:
The function value: **(lambda (a) (+ a x))**
The environment: **{ x → 10 }** Ref. environment is just the static link.

Scheme chose static scoping:

(* x (lambda (a)(+ a x) 3)) -->

(* 2   ((lambda (a)(+ a 10) 3)) -->

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Closures

- A closure is a function value plus the environment in which it is to be evaluated
  - Function value: e.g., (lambda (x) (+ x y))
  - Environment consists of bindings for variables not local to the function so the closure can eventually be evaluated: e.g., $\{ y \rightarrow 2 \}$
- A closure can be used as a function
  - Applied to arguments
  - Passed as an argument
  - Returned as a value

Closures

- Normally, when let expression exits, its bindings disappear
- Closure bindings (i.e., bindings part of a closure) are special
  - When let exits, bindings become inactive, but they do not disappear
  - When closure is called, bindings become active
  - Closure bindings are "immortal"

```
(let ((x 5))
     (let  (( f  (let ((x 10))  (lambda ()  x ) ) ))
          (list x (f) x (f)) )  )
```

f is { (lambda () x)
x → 10

(5 10 5 10)

Lecture Outline

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  - Scoping, revisited

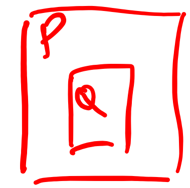
Scoping, revisited (Scott, Ch. 3.6)

- We discussed the two choices for mapping non-local variables to locations
  - Static scoping (early binding)

  and
  - Dynamic scoping (late binding)

- Most languages choose static scoping

Scoping, revisited

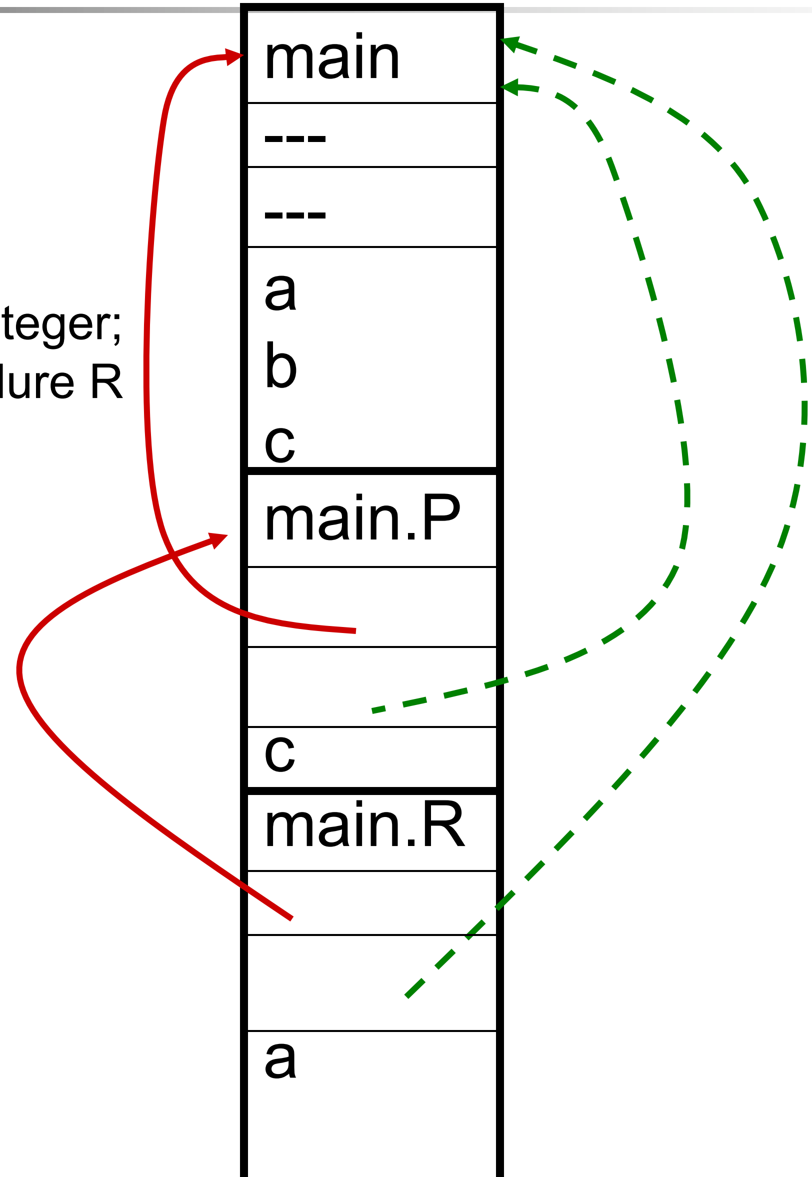


- When we discussed scoping earlier, we assumed that functions were third-class values (i.e., functions cannot be passed as arguments or returned from other functions)

- Functions as third-class values…
  - When functions are third-class values, the function's static reference environment (i.e., closure bindings) is available on the stack. Function cannot outlive its referencing environment!

Functions as Third-Class Values and Static Scoping

```
program
    a, b, c: integer;
    procedure P
        c: integer;
        procedure S
            c, d: integer;
            procedure R
            …
            end R;
            R();
        end S;
        R();
        S();
    end P;
    procedure R
        a: integer;
        = a, b, c;
    end R;
    …; P(); …
end program
```



Static Scoping:
- a bound to R.a,
- b to main.b,
- c to main.c

Scoping, revisited

- Functions as first-class values
  - Static scoping is more involved. Function value may outlive static referencing environment!
  - Therefore, need "immortal" closure bindings
  - In languages that choose static scoping, local variables must have "unlimited extent" (i.e., when stack frame is popped, local variables do not disappear!)

Scoping, revisited

- In functional languages local variables typically have unlimited extent
- In imperative languages local variables typically have limited extent (i.e., when stack frame is popped, local variables disappear)
  - Imperative languages (Fortran, Pascal, C) disallow truly first-class function values
  - More and more languages do allow first-class functions, e.g., Java 8, C++11

More on Dynamic Scoping

- Shallow binding vs. deep binding

- Dynamic scoping with shallow binding
  - Reference environment for function/routine is not created until the function is called
    - I.e., all non-local references are resolved using the most-recent-frame-on-stack rule
  - Shallow binding is usually the default in languages with dynamic scoping
  - All examples of dynamic scoping we saw so far used shallow binding

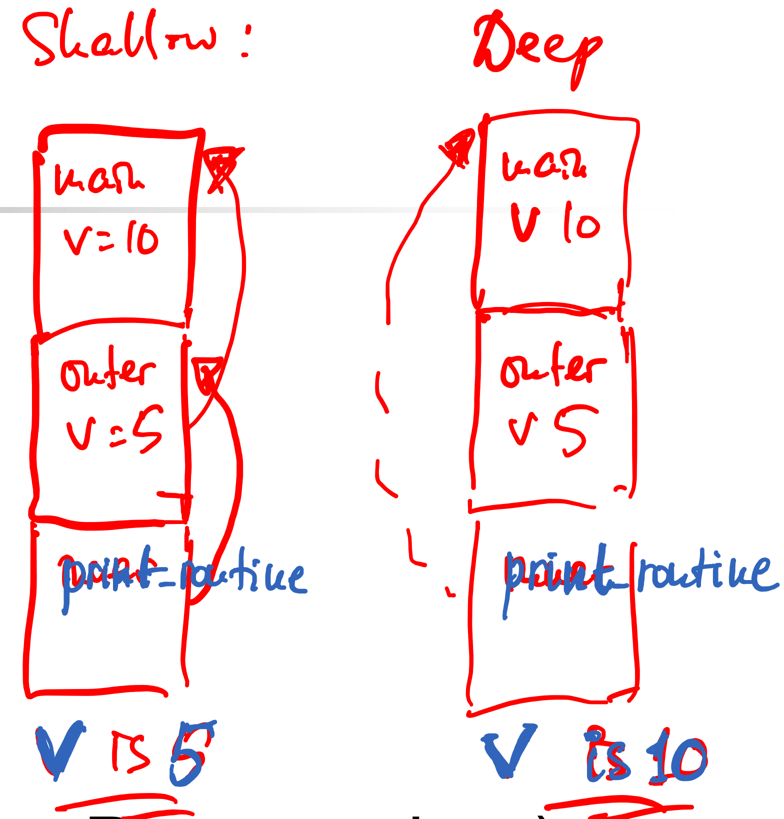
More on Dynamic Scoping

- Dynamic scoping with deep binding
  - When a function/routine is passed as an argument, the code that passes the function/routine has a particular reference environment (the current one!) in mind. It passes this reference environment along with the function value (it passes a closure).

Example

v : integer := 10
people : database

print_routine (p : person)
  if p.age > v
    write_person(p)



other_routine (db : database, P : procedure)
  v : integer := 5
  foreach record r in db
    P(r)

other_routine(people, print_routine) /* call in main */

Exercise

```
(define A
  (lambda ()
    (let* ((x 2)
           (C (lambda (P) (let ((x 4)) (P) )))
           (D (lambda () x))
           (B (lambda () (let ((x 3)) (C D)))))
      (B))))
```

When we call **> (A)** in the interpreter, what gets printed? What would get printed if Scheme used dynamic scoping with shallow binding? Dynamic scoping and deep binding?

Evaluation Order

(define (square x) (* x x))

- Applicative-order (also referred to as eager) evaluation
  - Evaluates arguments before function value

    (square (+ 3 4)) =>

    (square 7) =>

    (* 7 7) =>

    49

Evaluation Order

(define (square x) (* x x))

- Normal-order (also referred to as lazy) evaluation
  - Evaluates function value before arguments

    (square (+ 3 4)) =>

    (* (+ 3 4) (+ 3 4)) =>

    (* 7 (+ 3 4)) =>

    (* 7 7)

    49
- Scheme uses applicative-order evaluation

So Far

- Essential functional programming concepts
  - Reduction semantics
  - Lists and recursion
  - Higher-order functions
    - Map and fold (also known as reduce)
  - Scoping
  - Evaluation order
- Scheme

Coming Up

- Lambda calculus: theoretical foundation of functional programming

- Haskell
  - Algebraic data types and pattern matching
  - Lazy evaluation
  - Type inference
  - Monads

The End