Programming Language Syntax

Read: Scott, Chapter 2.1

Announcements

- No class Tuesday next week
- HW1 will be out today
- Office hours schedule coming any day now
  - Check Submitty and course page: https://www.cs.rpi.edu/~milanova/csci4430/

Timeline

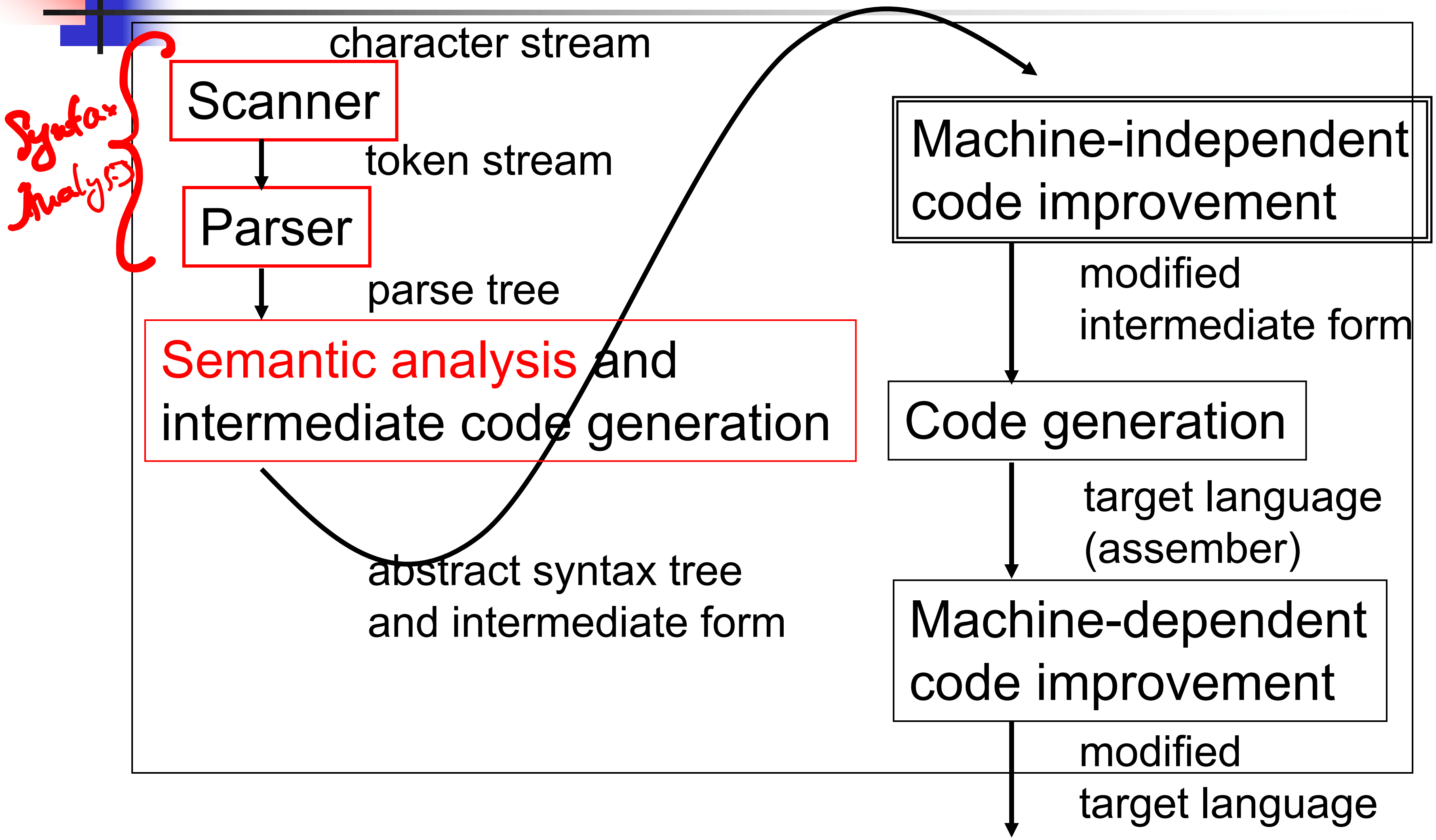
- Mid 1950’s: FORTRAN
- 1969: Hoare logic
- 1970’s: Verification
- Late 1970’s: Enthusiasm cools
- 1979: “Can programming be liberated…” by Backus
- 1980’s and on: Research on functional programming and type theory

- Mid 2000’s: Z3 and resurgence of verification

Lecture Outline

- Formal languages
- Regular expressions
- Context-free grammars
  - Derivation
  - Parse
  - Parse trees
  - Ambiguity
- Expression grammars

Last Class: Compiler



Syntax and Semantics

- Syntax is the form or structure of expressions, statements, and program units of a given language
  - Syntax of a Java **while** statement:
    - `while (` *boolean_expr* `)` *statement*

- Semantics is the meaning of expressions, statements and program units of a given language
  - Semantics of `while (` *boolean_expr* `)` *statement*
    - Execute *statement* repeatedly (0 or more times) as long as *boolean_expr* evaluates to `true`

LALR

Formal Languages

- Theoretical foundations – Automata theory
- A language is a set of strings (also called sentences) over a finite alphabet
- A generator is a set of rules that generate the strings in the language
- A recognizer reads input strings and determines whether they belong to the language
- Languages are characterized by the complexity of generation/recognition rules
  - E.g., regular languages
  - E.g., context-free languages

Question

- What are the classes of formal languages?

- The Chomsky hierarchy:
  - Regular languages
  - Context-free languages
  - Context-sensitive languages
  - Recursively enumerable languages

Formal Languages

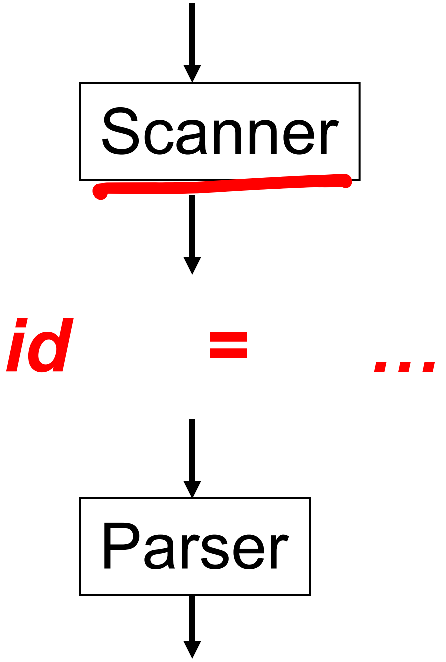
- Generators and recognizers become more complex as languages become more complex
  - Regular languages
    - Describe PL tokens (e.g., keywords, identifiers, numeric literals)
    - Generated by Regular Expressions
    - Recognized by a Finite Automaton (scanner)
  - Context-free languages
    - Describe more complex PL constructs (e.g., expressions and statements)
    - Generated by a Context-free Grammar
    - Recognized by a Push-down Automaton (parser)
  - Even more complex constructs

Formal Languages

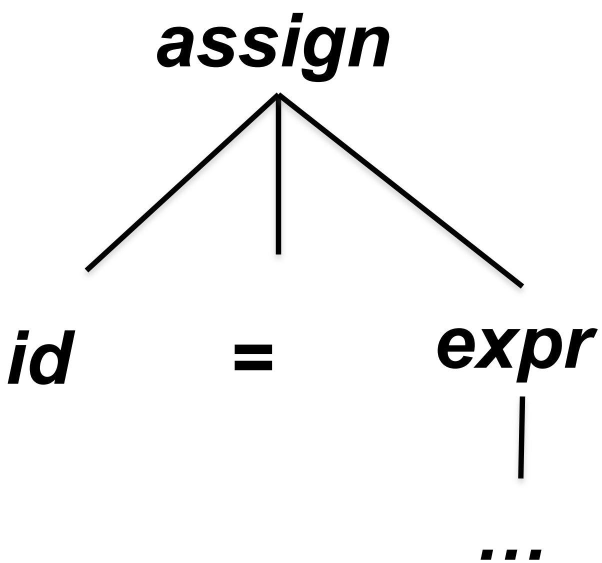
- Main application of formal languages: enable proof of relative difficulty of computational problems
- Our focus: formal languages provide the formalism for describing PL constructs
  - A compelling application of formal languages!
  - Building a scanner
  - Building a parser   LL, LR, LALR
  - Central issue: build efficient, linear-time parsers

A Single Pass

position = initial + rate * 60;



- Scanner emits next token
- Parser consumes the token and continues building the parse tree (typically bottom up)



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Regular Expressions

- Simplest structure
- Formalism to describe the simplest programming language constructs, the tokens
  - each symbol (e.g., “+”, “-”) is a token
  - an identifier (e.g., position, rate, initial) is a token
  - a numeric constant (e.g., 59) is a token
  - etc.
- Recognized by a finite automaton

Regular Expressions

- A Regular Expression is one of the following:
  - A character, e.g., **a**
  - The empty string, denoted by **ε**
  - Two regular expressions next to each other, $R_1 R_2$ $L_{R1} \times L_{R2}$
    - Meaning: $R_1 R_2$ generates the language of strings that are made up of any string generated by R_1, followed by any string generated by R_2
  - Two regular expressions separated by |, $R_1 \mid R_2$
    - Meaning: $R_1 \mid R_2$ generates the language that is the union of the strings generated by R_1 with the strings generated by R_2

Question

- What is the language defined by reg. exp. (a | b) (a a | b b) ?

  { a, b } { aa , bb }

  { aaa , abb, baa, bbb }

- We saw concatenation and alternation. What operation is still missing?

  *

Regular Expressions

- A Regular Expression is one of the following:
  - A character, e.g., **a**
  - The empty string, denoted by ε
  - $R_1\ R_2$
  - $R_1 \mid R_2$
  - Regular expression followed by a Kleene star, R*
    - Meaning: the concatenation of zero or more strings generated by R
    - E.g., **a*** generates {ε, **a**, **aa**, **aaa**, … }
    - E.g., **(a|b)*** generates all strings of **a**'s and **b**'s

abab

Regular Expressions

- Precedence
  - Kleene * has highest precedence
  - Followed by concatenation
  - Followed by alternation |

  - E.g., `a b | c` is `(a b) | c` not `a (b | c)`
    - Generates `{ab,c}` not `{ab,ac}`
  - E.g., `a b*` generates `{a,ab,abb,…}` not `{ε, ab, abab, ababab,…}`

Question

- What is the language defined by regular expression `(0 | 1)* 1` ?

- What about `0* (1 0* 1 0*)*` ?

Regular Expressions in Programming Languages

- Describe tokens
- Let
  - *letter* → **a|b|c| … |z**
  - *digit* → **1|2|3|4|5|6|7|8|9|0**

- Which token is this?

1. *letter (letter | digit)** ? identifier
2. *digit digit ** ? non-negative int. constant
3. *digit *. digit digit ** ?

Regular Expressions in Programming Languages

- Which token is this:

number → *integer* | *real*

real → *integer exponent* | *decimal* (*exponent* | **ε**)

decimal → *digit** (**.** *digit* | *digit* **.**) *digit**

exponent → (**e** | **E**) (**+** | **-** | **ε**) *integer*

integer → *digit digit**

digit → **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **0**

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Context-Free Grammars

- Unfortunately, regular languages cannot specify all constructs in programming
- E.g., can we write a regular expression that specifies valid arithmetic expressions?
  - `id * ( id + id * ( number - id ) )`
    - Among other things, we need to ensure that parentheses are matched!
    - Answer is no. We need context-free languages and context-free grammars!

Grammar

- A grammar is a formalism to describe the strings of a (formal) language
- A grammar consists of a set of terminals, set of nonterminals, a set of productions, and a start symbol
  - Terminals are the characters in the alphabet
  - Nonterminals represent language constructs
  - Productions are rules for forming syntactically correct constructs
  - Start symbol tells where to start applying the rules

Notation

Specification of identifier:

Regular expression: *letter* (*letter* | *digit*)*

BNF: *<digit> ::= 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0*
<letter> ::= a | b | c | … | x | y | z
<id> ::= <letter> | <id> <letter> | <id> <digit>

Textbook and slides: (also BNF)

digit → `1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0`
letter → `a | b | c | d | … | z`
id → *letter* | *id letter* | *id digit*

Nonterminals shown in *italic*

Terminals shown in **typewriter**

Regular Grammars

- Regular grammars generate regular languages
- The rules in regular grammars are of the form:
  - Each left-hand-side (lhs) has exactly one nonterminal
  - Each right-hand-side (rhs) is one of the following
    - A single terminal symbol or
    - A single nonterminal symbol or
    - A nonterminal followed by a terminal

e.g., 1 2^* | 0^+

Kleene +

00^*

$$S \rightarrow A \mid B$$
$$A \rightarrow 1 \mid A\ 2$$
$$B \rightarrow 0 \mid B\ 0$$

Question

- Is this a regular grammar:

  $S \to 0\ A$ $\qquad$ $S \to 0A \mid \varepsilon$

  $A \to S\ 1$

  $S \to \varepsilon$

  - No, this is a context-free grammar
    - It generates 0^n1^n, the canonical example of a context-free language
    - rhs should be nonterminal followed by a terminal, thus, $S \to 0\ A$ is not a valid production

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Context-free Grammars (CFGs)

- Context-free grammars generate context-free languages
  - Most of what we need in programming languages can be specified with CFGs
- Context-free grammars have rules of the form:
  - Each left-hand-side has exactly one nonterminal
  - Each right-hand-side contains an arbitrary sequence of terminals and nonterminals
- A context-free grammar

  e.g. $\mathbf{0}^n\mathbf{1}^n, n \geq 1$ $S \to \mathbf{0}\ S\ \mathbf{1}$

  $S \to \mathbf{0}\ \ \mathbf{1}$

Question

- Examples of non-context-free languages?

  - E.g., $a^n b^m c^n d^m$ $n \geq 1, m \geq 1$
  - E.g., $w\mathbf{c}w$ where w is in `(0|1)*`
  - E.g., $a^n b^n c^n$ $n \geq 1$ (canonical example)

Context-free Grammars

- Can be used to generate strings in the context-free language (derivation)
- Can be used to recognize well-formed strings in the context-free language (parse)

- In Programming Languages and compilers, we are concerned with two special CFGs, called LL and LR grammars

Derivation

Simple context-free grammar for expressions:

$expr \rightarrow \mathbf{id} \mid (\ expr\ ) \mid expr\ op\ expr$

$op \rightarrow + \mid *$

We can generate (derive) expressions:

$$
\begin{aligned}
expr &\Rightarrow expr\ op\ \underline{expr} \\
&\Rightarrow expr\ \underline{op}\ \mathbf{id} \\
&\Rightarrow \underline{expr}\ +\ \mathbf{id} \\
&\Rightarrow expr\ op\ \underline{expr}\ +\ \mathbf{id} \quad \longleftarrow \text{sentential form} \\
&\Rightarrow expr\ \underline{op}\ \mathbf{id}\ +\ \mathbf{id} \\
&\Rightarrow \underline{expr}\ *\ \mathbf{id}\ +\ \mathbf{id} \\
&\Rightarrow \mathbf{id}\ *\ \mathbf{id}\ +\ \mathbf{id} \quad \longleftarrow \text{sentence, string or yield}
\end{aligned}
$$

Derivation

- A derivation is the process that starts from the start symbol, and at each step, replaces a nonterminal with the right-hand side of a production
  - E.g., *expr op* *<u>expr</u>* derives *expr op* **`id`**
    We replaced the right (underlined) *expr* with **`id`**
    due to production $expr \rightarrow$ **`id`**
- An intermediate sentence is called a sentential form
  - E.g., *expr op* **`id`** is a sentential form

Derivation

- The resulting sentence is called yield
  - E.g., `id*id+id` is the yield of our derivation
- What is a left-most derivation?
  - Replaces the left-most nonterminal in the sentential form at each step
- What is a right-most derivation?
  - Replaces the right-most nonterminal in the sentential form at each step
- There are derivations that are neither left- nor right-most

Question

- What kind of derivation is this:

$expr \Rightarrow expr\ op\ \underline{expr}$
$\Rightarrow expr\ \underline{op}$ **`id`**
$\Rightarrow \underline{expr}$ **`+ id`**
$\Rightarrow expr\ op\ \underline{expr}$ **`+ id`**
$\Rightarrow expr\ \underline{op}$ **`id + id`**
$\Rightarrow \underline{expr}$ **`* id + id`**
$\Rightarrow$ **`id * id + id`**

  - A right-most derivation. At each step we replace the right-most nonterminal

Question

- What kind of derivation is this:

$$
\begin{aligned}
\mathit{expr} &\Rightarrow \mathit{expr}\ \mathit{op}\ \underline{\mathit{expr}} \\
&\Rightarrow \mathit{expr}\ \underline{\mathit{op}}\ \mathbf{id} \\
&\Rightarrow \underline{\mathit{expr}}\ \mathbf{+}\ \mathbf{id} \\
&\Rightarrow \underline{\mathit{expr}}\ \mathit{op}\ \mathit{expr}\ \mathbf{+}\ \mathbf{id} \\
&\Rightarrow \mathbf{id}\ \mathit{op}\ \underline{\mathit{expr}}\ \mathbf{+}\ \mathbf{id} \\
&\Rightarrow \mathbf{id}\ \underline{\mathit{op}}\ \mathbf{id}\ \mathbf{+}\ \mathbf{id} \\
&\Rightarrow \mathbf{id}\ \mathbf{*}\ \mathbf{id}\ \mathbf{+}\ \mathbf{id}
\end{aligned}
$$

  - Neither left-most nor right-most

Parse

Recall our context-free grammar for expressions:

expr → **id** | **(** *expr* **)** | *expr op expr*

op → **+** | *****

- A parse is the reverse of a derivation

id * id + id ⇒ *expr* <u>*****</u> **id + id**

⇒ *expr op* <u>**id**</u> **+ id**

⇒ <u>*expr op expr*</u> **+ id**

⇒ *expr* <u>**+**</u> **id**

⇒ *expr op* <u>**id**</u>

⇒ <u>*expr op expr*</u>

⇒ *expr*

Parse

- A parse starts with the string of terminals, and at each step, replaces the right-hand-side (rhs) of a production with the left-hand-side (lhs) of that production. E.g.,

  … $\Rightarrow$ <u>*expr op expr*</u> **`+ id`**

  $\Rightarrow$ *expr* **`+ id`**

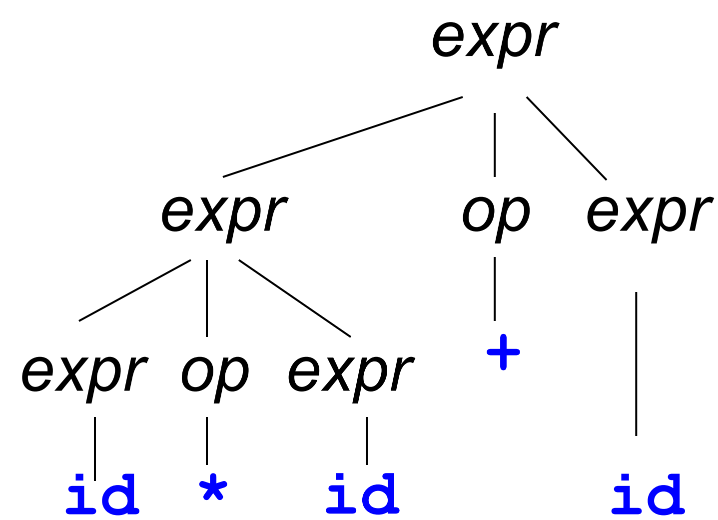
  Here we replaced *expr op expr* (the rhs of production *expr* $\rightarrow$ *expr op expr*) with *expr* (the lhs of the production)

Parse Tree

expr → **id** | **(** *expr* **)** | *expr op expr*

op → **+** | *****

$$\begin{aligned} expr &\Rightarrow expr\ op\ \underline{expr} \\ &\Rightarrow expr\ \underline{op}\ \mathbf{id} \\ &\Rightarrow \underline{expr}\ \mathbf{+}\ \mathbf{id} \\ &\Rightarrow expr\ op\ \underline{expr}\ \mathbf{+}\ \mathbf{id} \\ &\Rightarrow expr\ \underline{op}\ \mathbf{id}\ \mathbf{+}\ \mathbf{id} \\ &\Rightarrow \underline{expr}\ \mathbf{*}\ \mathbf{id}\ \mathbf{+}\ \mathbf{id} \\ &\Rightarrow \mathbf{id}\ \mathbf{*}\ \mathbf{id}\ \mathbf{+}\ \mathbf{id} \end{aligned}$$



Internal nodes are nonterminals. Children are the rhs of a rule for that nonterminal.
Leaf nodes are terminals.

Ambiguity

- Ambiguity
  - A grammar is ambiguous if some string can be generated by two or more distinct parse trees
  - There is no algorithm that can tell if an arbitrary context-free grammar is ambiguous
- Ambiguity arises in programming language grammars
  - Arithmetic expressions
  - If-then-else: the dangling else problem
- Ambiguity is bad

Ambiguity

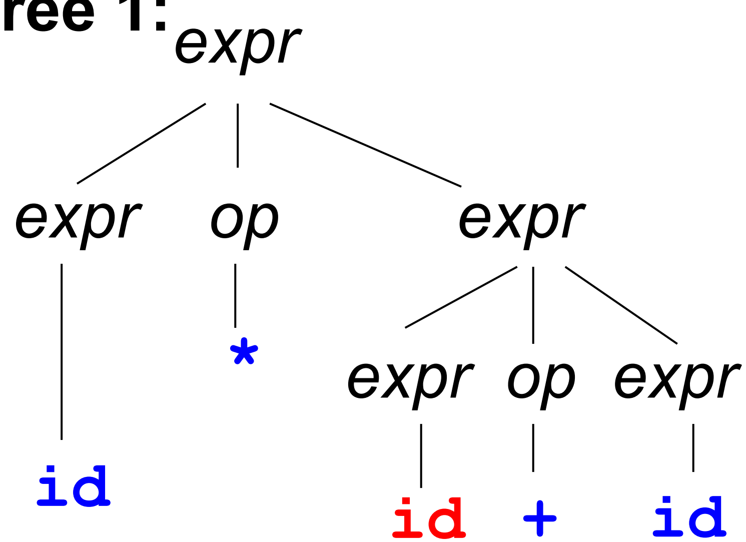
$t_1 = id_1 * id_2$
$t_2 = t_1 + id_3$

expr → **id** | **(** *expr* **)** | *expr op expr*

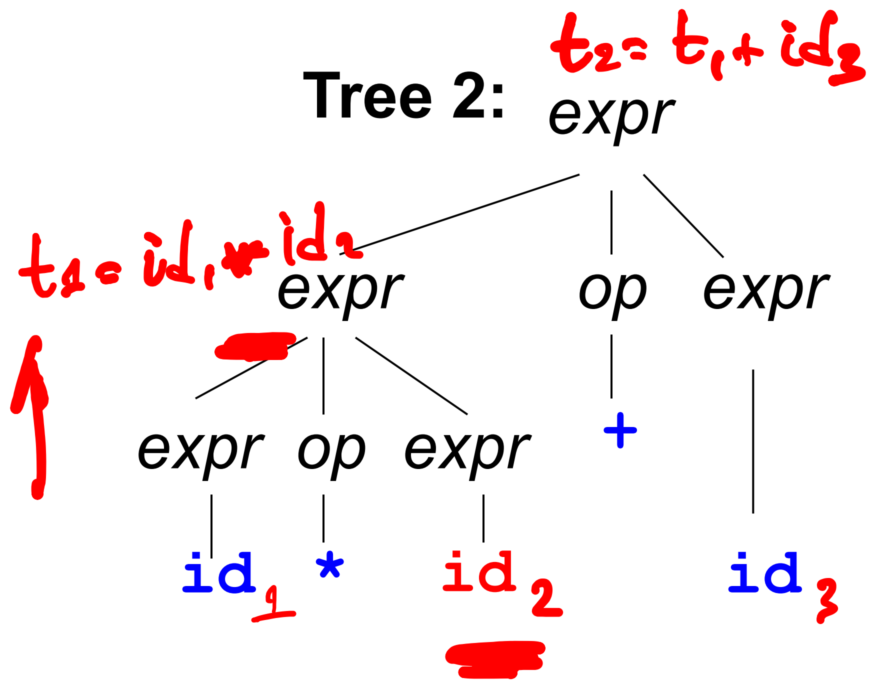
op → **+** | *****

- How many parse trees for **id * id + id** ?

Tree 1:



Tree 2:



- Which one is "correct"?

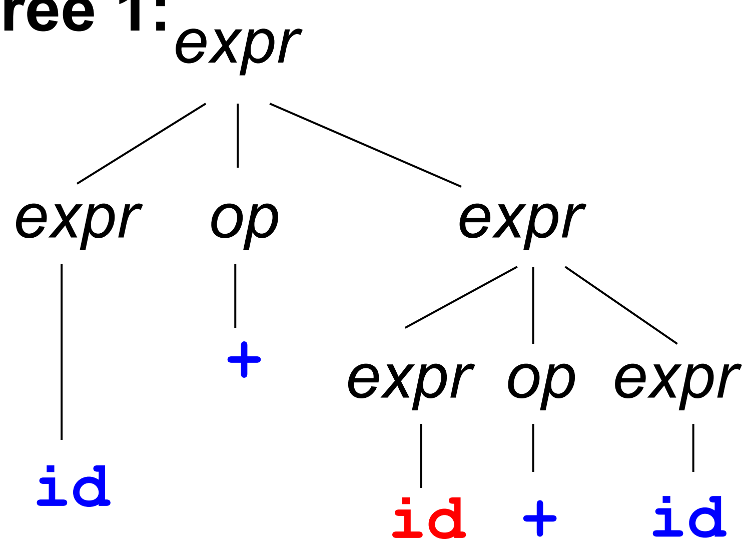
Ambiguity

expr → **id** | **(** *expr* **)** | *expr op expr*

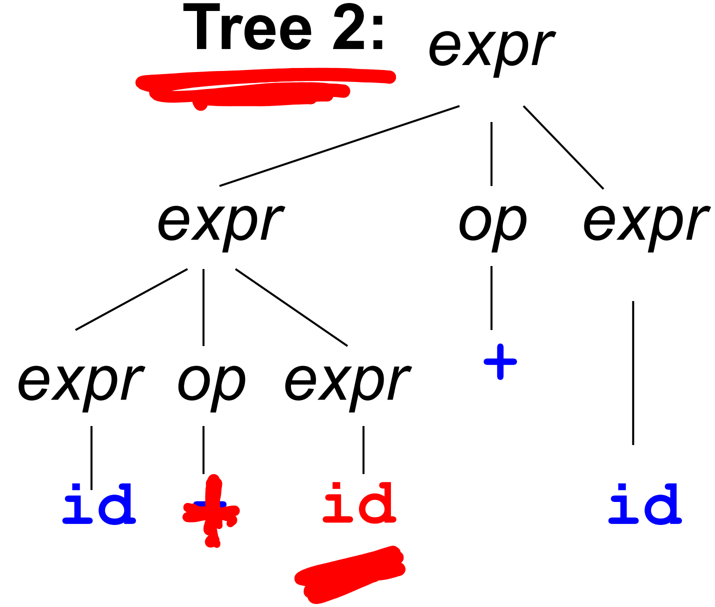
op → **+** | *****

- How many parse trees for **id + id + id** ?

Tree 1:



Tree 2:



- Which one is "correct"?

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Expression Grammars

- Generate expressions
  - Arithmetic expressions
  - Regular expressions
  - Other

- Terminals: operands, operators, and parentheses

$$expr \rightarrow \mathbf{id} \mid \mathbf{(}\ expr\ \mathbf{)} \mid expr\ op\ expr$$

$$op \rightarrow \mathbf{+} \mid \mathbf{*}$$

Handling Ambiguity

Our ambiguous grammar, slightly simplified:

expr → **id** | **(** *expr* **)** | *expr* **+** *expr* | *expr* ***** *expr*

- Rewrite the grammar into unambiguous one:

  expr → *expr* **+** *term* | *term*

  term → *term* ***** *factor* | *factor*

  factor → **id** | **(** *expr* **)**

- Forces left associativity of **+** and *****
- Forces higher precedence of ***** over **+**

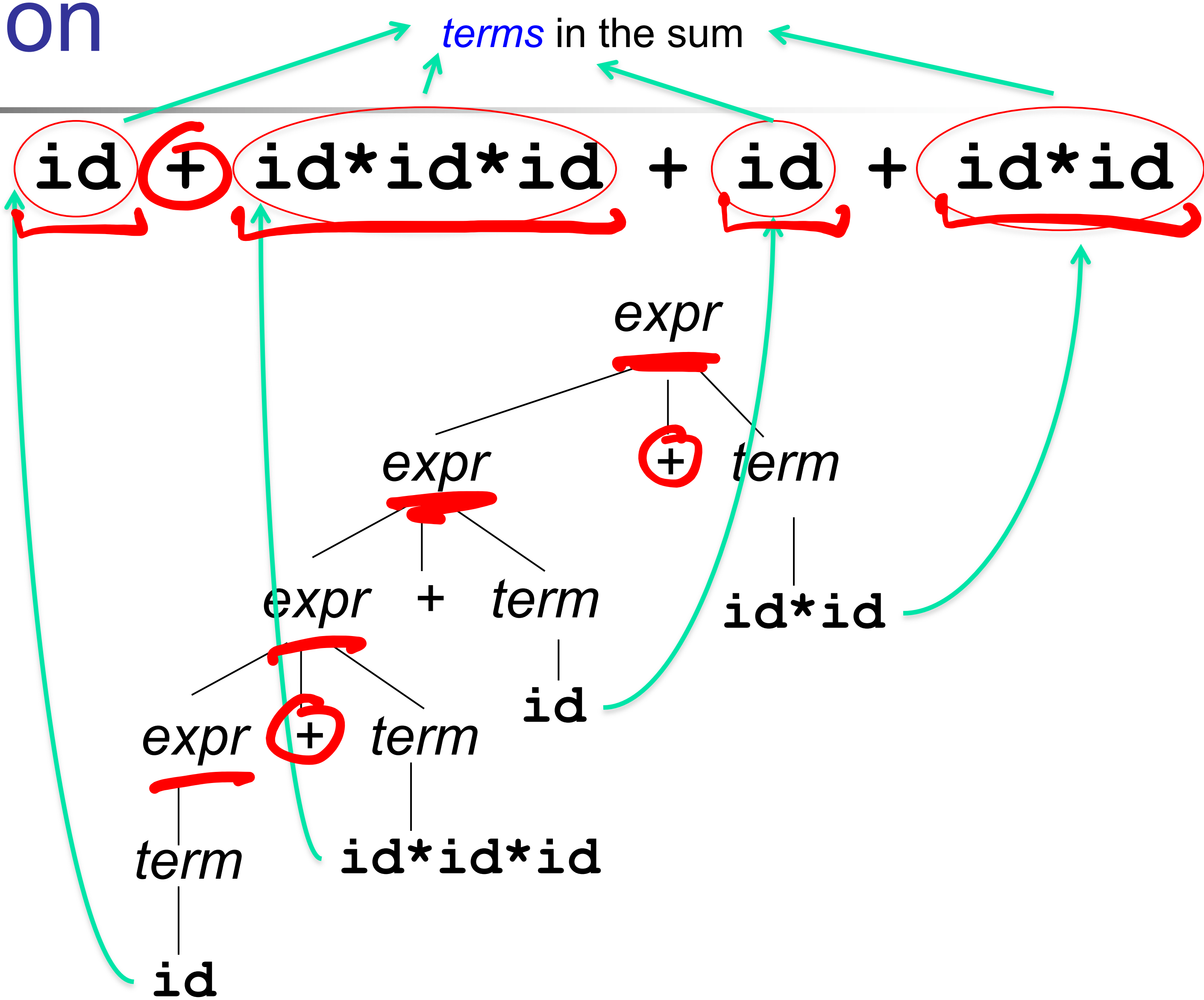
Rewriting Expression Grammars: Intuition

expr → **id** | **(** *expr* **)** | *expr* **+** *expr* | *expr* ***** *expr*

- A new nonterminal, *term*
- *expr* ***** *expr* becomes *term*. Thus, ***** gets pushed down the tree, forcing higher precedence of *****
- *expr* **+** *expr becomes expr* **+** *term*. Pushes leftmost **+** down the tree, forcing operand to associate with **+** on its left
  - *expr* → *expr* **+** *expr* becomes *expr* → *expr* **+** *term* | *term*

Rewriting Expression Grammars: Intuition

E.g., look at `id + id*id*id + id + id*id`



Rewriting Expression Grammars: Intuition

- Another new nonterminal, *factor* and productions:
  - *term* → *term* `*` *factor* | *factor*
  - *factor* → **`id`** | (*expr*)

Exercise

$expr \rightarrow expr$ **×** $expr$ | $expr$ **^** $expr$ | **`id`**

- How many parse trees for **`id×id^id×id`** ?
  - No need to draw them all
- Rewrite this grammar into an equivalent unambiguous grammar where
  - ^ has higher precedence than ×
  - ^ is right-associative
  - × is left-associative

The End