Homework 3
CSCI-4961/6961: 3D Computer Graphics
Fall 2004

Due: Wednesday, October 18, 2006

Homeworks are due by **3:00pm** on Wednesday, October 18. **Late homeworks will receive no credit.** Homeworks are to be done individually and will be graded on the basis of correctness, clarity, and legibility. Show the steps in your work where appropriate. Each question is worth **10 points**, for a total of **50 points**.

Be sure to write your **name**, **section number**, and **RPI email address** on your homework submission.

1. (a) The combined illumination model can be described by the following equation:
   $I = k_e + k_a I_a + \sum_i \frac{1}{(a_0 + a_1 d_i + a_2 d_i^2)} [I_{l_i} [k_d (N.L_i) + k_s (V.R_i)^{n_s}]]$.
   The variables used are: $I, a_0, a_1, a_2, d_i, k_e, k_a, k_d, k_s, n_s, I_a, I_{l_i}, L_i, R_i, N, V$.
   Which of the quantities above are affected if :
   - the viewer position changes?
   - the object material changes?

   (b) Under what circumstances can L and V be assumed constant? How does the illumination model using the halfway vector H simplify the shading equations?

   (c) Describe a scene where the difference between Gouraud and Phong shading would be noticeable.

2. Suppose that a viewer is at coordinates (0, 0, 1) (at the tip of the Z unit vector) in 3-space, and is looking at the infinite XY coordinate plane, which is being illuminated by a single light source lying somewhere above this plane. Assume that we are using the Phong illumination model, there is no emission from the plane and there is no ambient reflection. We assume that the diffuse and specular coefficients of the plane are both nonzero.

   (a) Suppose that the light source lies at the point (3, 0, 4). What are the coordinates of the point on the XY plane with the brightest diffuse reflection?

   (b) What are the coordinates of the point on the XY plane with the brightest specular reflection?

3. Consider computing the normal vectors to a surface, as required for lighting calculations.

   (a) The surface of an ellipsoid centered at the origin is described by the equation
   $f(x, y, z) = (x/r_x)^2 + (y/r_y)^2 + (z/r_z)^2 - 1 = 0$.
   Compute and express the outward normal to this ellipsoid in terms of x, y, z.

(b) In parametric form, the ellipsoidal surface can be written:
$x = r_x \cos\phi \cos\theta,\ -\pi/2 \leq \phi \leq \pi/2,\ -\pi \leq \theta \leq \pi$
$y = r_y \cos\phi \sin\theta,\ -\pi/2 \leq \phi \leq \pi/2,\ -\pi \leq \theta \leq \pi$
$z = r_z \sin\phi,\ -\pi/2 \leq \phi \leq \pi/2$
Compute the outward normal to this surface.

4. Consider a scene with l light sources that is to be ray traced. Let the maximum depth of the ray tree be d_{max} (where a ray tree of depth of 1 has a single reflected ray and a single transmitted ray after the first intersection with a surface). Assume a single ray is generated per pixel.

   (a) Compute the maximum number of reflected and transmitted rays generated.

   (b) Compute the maximum number of shadow rays generated.

   (c) Compute the maximum total number of rays generated if the image resolution is 1024×1024.

5. Compute the intersection points of a ray $P_0 + s\hat{u}$ with the infinite cylinder of radius 2 centered at the origin and with its axis along the Y axis. Assume $P_0 = (4, 0, 3)$ and $\hat{u} = \frac{1}{13}(-4, 12, -3)$.