Homework 4
CSCI-4961/6961: 3D Computer Graphics
Fall 2006

Due: Thursday, November 9, 2006

Homeworks are due at the **beginning** of lecture on Thursday, November 9. **Late homeworks will receive no credit.** Homeworks are to be done individually and will be graded on the basis of correctness, clarity, and legibility. Show the steps in your work where appropriate. Each question is worth **10 points**, for a total of **50 points**.

Be sure to write your **name**, **section number**, and **RPI email address** on your homework submission.

1. (a) Prove that for the Bezier curve with $n+1$ control points $p_0, p_1, \ldots, p_n$, the derivative at the first control point is $n(p_1 - p_0)$.

   (b) Suppose that we join two Bezier curves of degree 2, using the control point sequences p_0, p_1, p_2 and p_2, p_3, p_4 respectively. What conditions must be satisfied by these five points for the combined curve to have C^1 parametric continuity at the point at which they are joined?

   (c) Determine the Bezier blending functions for a Bezier curve with five control points.

2. (a) Let the number of control points for a B-spline curve be 10, and the number of knot points be 15. What is the order of the resulting B-spline curve? What is its degree? How many control points determine the shape of the curve at any point along it?

   (b) B-splines possess a property called local support. What is local support, and why is this property desirable?

   (c) What is a rational parameterization? Give an example of a shape that has a rational parameterization, but no polynomial parameterization.

3. Given an array $\mathbf{p}_{j,k}$ of control points, $0 \le j \le m$, $0 \le k \le n$, the Bezier surface patch is given by

$$\mathbf{P}(u,v) = \sum_{j=0}^{m} \sum_{k=0}^{n} \mathbf{p}_{j,k} Bez_{j,m}(u) Bez_{k,n}(v)$$

   for $0 \le u, v \le 1$, where $Bez_{j,m}(u)$ and $Bez_{k,n}(v)$ denote Bezier blending functions of degree m and n respectively. Consider a bicubic Bezier surface patch, that is, a patch with $m = n = 3$.

   (a) Given any fixed u_0, $0 \le u_0 \le 1$, define the u_0-slice to be the curve $\mathbf{c}(v) = \mathbf{P}(u_0, v)$. Show that $\mathbf{c}(v)$ is a Bezier curve of degree 3.

(b) What are the four control points for the curve $\mathbf{c}(v)$?

(c) What is the normal vector to the Bezier surface at $\mathbf{P}(u = 0, v = 0)$?

4. (a) What are the advantages of using a spline based representation of objects such as the Utah teapot instead of a polygonal mesh representation?

   (b) You want to know whether a given point $P = (x, y, z)$ lies on a surface. With which surface representation is it easier to answer this query: parametric or implicit? Justify your answer.

   (c) Give an implicit surface representation of a (right circular) cone with its vertex at the origin and with its central axis along the positive Z axis. That is, give an implicit function $f(x, y, z) = 0$ that describes the surface of the cone. Assume the cone makes an angle of α with the Z axis.

5. (a) Consider drawing an ellipsoid using OpenGL. Write a snippet of code to draw an axis-aligned ellipsoid centered at (1, 3, 5) such that its axis parallel to the X axis has length 2 units, its axis parallel to the Y axis has length 8 units, and its axis parallel to the Z axis has length 6 units. After completion, the modelview matrix should be unchanged from its original value.

   (b) Write the parametric representation of this ellipsoid, indicating the ranges of the parameters.