Homework 5
CSCI-4961/6961: 3D Computer Graphics
Fall 2006

Due: Monday, November 27, 2006

Homeworks are due at the **beginning** of lecture on Monday, November 27. **Late homeworks will receive no credit.** Homeworks are to be done individually and will be graded on the basis of correctness, clarity, and legibility. Show the steps in your work where appropriate. Each question is worth **10 points**, for a total of **50 points**.

Be sure to write your **name** and **RPI email address** on your homework submission.

1. Suppose that visible surface detection is performed using a 8-bit integer depth buffer. Suppose that the nearest object to be displayed is x units away from the viewer and the furthest object is $10x$ units away from the viewer. Assume the near clipping is set at a distance x, but the far clipping plane is set at distance $1000x$. What (if any) are some of the dangers of setting the far clipping plane at this unnecessarily large distance from the viewer?

2. Consider mapping a texture onto a cylinder. The cylinder is aligned with the Z axis and has a radius of r units and a height of h units with its base at $z = 0$. You are given a texture map whose s coordinate ranges from 0 to 1, and whose t coordinate ranges from 0 to 1. You are to wrap three copies of this texture around the cylinder (so they are not overlapping and their vertical join lines are parallel to the Z axis). Assume the first texture copy starts at the positive X axis. Give the inverse mapping function, which maps a point (x, y, z) on the cylinder to the corresponding point (s, t) on the texture.

3. (a) Texture mapping significantly increases the memory bandwidth required per rendered pixel. Consider a simplified model of the memory bandwidth required due to texture reads when we perform 2D texture mapping with multitexturing. Consider a display with 1280×1024 pixels, and assume each pixel has two textures applied to it, where each texel is represented as a 32 bit RGBA value. Additionally assume that bilinear interpolation is performed at each pixel (that is, a linear average of the 2×2 array of texels nearest the center of the pixel must be computed). Finally, assume an average depth complexity of 3. (The depth complexity refers to the number of times each pixel is rendered, and depends on the number of surfaces that cover each pixel).

   What is the average memory transfer required per frame for the texture reads (in Mbytes)? At 60 frames per second, what is the memory bandwidth required (in Gbytes per second)?

   (Note that practical techniques to reduce the memory bandwidth requirements include the use of texture caches and texture compression.)

(b) Why is mipmapping used?

(c) If a texture map is stored as a 512×512 image with 24-bit color per texel, what is the additional storage required to store all mipmap levels down to the smallest $1\times$ 1 texel image?

4. (a) Compute the fractal dimension of the Koch snowflake.

   (b) Mention an advantage of using shape grammars for object modeling.

   (c) Give a reason for introducing randomness in the application of rules when generating plants and trees using L-grammars.

5. Consider performing forward and inverse kinematics for a planar robot arm with links of length 1 unit (proximal link) and length 1.5 units (distal link). Let the base of the proximal link be at the origin (0, 0) and let the end-effector be at the tip of the distal link. Let θ_1 be the angle made by the proximal link with respect to the X axis and let θ_2 be the angle made by the distal link with respect to the proximal link. (See figure in class notes.)

   (a) Forward kinematics: Find the (x, y) coordinates of the end-effector when $\theta_1 = 60$ and $\theta_2 = 90$.

   (b) Inverse kinematics: Compute θ_1 and θ_2 when the end-effector is at the point (0.8660, 2).