Homework 3
CSCI-4962: Three-Dimensional Computer Graphics
Fall 2002

Due: Tuesday, October 15, 2002

Homeworks are due at the **beginning** of lecture on Tuesday, October 15. **Late homeworks will receive no credit.** Homeworks are to be done individually and will be graded on the basis of correctness, clarity, and legibility. Show the steps in your work where appropriate. Each question is worth **10 points**, for a total of **50 points**.

Be sure to write your **name**, **section number**, and **RPI email address** on your homework submission.

1. (a) The combined illumination model can be described by the following equation:
   $I = k_e + k_a I_a + \sum_i \frac{1}{(a_0 + a_1 d_i + a_2 d_i^2)} [I_{l_i} [k_d (N.L_i) + k_s (V.R_i)^{n_s}]]$.
   The variables used are: $I, a_0, a_1, a_2, d_i, k_e, k_a, k_d, k_s, n_s, I_a, I_{l_i}, L_i, R_i, N, V$.
   Which of the quantities above are affected if :
   - the viewer position changes?
   - the object material changes?

   (b) Under what circumstances can L and V be assumed constant? How does the illumination model using the halfway vector H simplify the shading equations?

   (c) Describe a scene where the difference between Gouraud and Phong shading would be noticeable.

2. Suppose that a viewer is at coordinates (0, 0, 1) (at the tip of the Z unit vector) in 3-space, and is looking at the infinite XY coordinate plane, which is being illuminated by a single light source lying somewhere above this plane. Assume that we are using the Phong illumination model, there is no emission from the plane and there is no ambient reflection. We assume that the diffuse and specular coefficients of the plane are both nonzero.

   (a) Suppose that the light source lies at the point (3, 0, 4). What are the coordinates of the point on the XY plane with the brightest diffuse reflection?

   (b) What are the coordinates of the point on the XY plane with the brightest specular reflection?

3. Consider computing the normal vectors to a surface, as required for lighting calculations.

   (a) The surface of an ellipsoid centered at the origin is described by the equation
   $f(x, y, z) = (x/r_x)^2 + (y/r_y)^2 + (z/r_z)^2 - 1 = 0$.
   Compute and express the outward normal to this ellipsoid in terms of x, y, z.

(b) In parametric form, the ellipsoidal surface can be written:
$x = r_x \cos\phi \cos\theta,\ -\pi/2 \leq \phi \leq \pi/2,\ -\pi \leq \theta \leq \pi$
$y = r_y \cos\phi \sin\theta,\ -\pi/2 \leq \phi \leq \pi/2,\ -\pi \leq \theta \leq \pi$
$z = r_z \sin\phi,\ -\pi/2 \leq \phi \leq \pi/2$
Compute the outward normal to this surface.

4. (a) Prove that for the Bezier curve with $n+1$ control points $p_0, p_1, \ldots, p_n$, the derivative at the first control point is $n(p_1 - p_0)$.

   (b) Suppose that we join two Bezier curves of degree 2, using the control point sequences p_0, p_1, p_2 and p_2, p_3, p_4 respectively. What conditions must be satisfied by these five points for the combined curve to have C^1 parametric continuity at the point at which they are joined?

   (c) Determine the Bezier blending functions for a Bezier curve with five control points.

5. (a) Let the number of control points for a B-spline curve be 10, and the number of knot points be 15. What is the order of the resulting B-spline curve? What is its degree? How many control points determine the shape of the curve at any point along it?

   (b) B-splines possess a property called local support. What is local support, and why is this property desirable?

   (c) What is a rational parameterization? Give an example of a shape that has a rational parameterization, but no polynomial parameterization.