

Homework 4

CSCI-4967: Three-Dimensional Computer Graphics

Fall 2004

Due: Thursday, November 4, 2004

Homeworks are due at the **beginning** of lecture on Thursday, November 4. **Late homeworks will receive no credit.** Homeworks are to be done individually and will be graded on the basis of correctness, clarity, and legibility. Show the steps in your work where appropriate. Each question is worth **10 points**, for a total of **50 points**.

Be sure to write your **name, section number, and RPI email address** on your homework submission.

1. Given an array $\mathbf{p}_{j,k}$ of control points, $0 \leq j \leq m$, $0 \leq k \leq n$, the Bezier surface patch is given by

$$\mathbf{P}(u, v) = \sum_{j=0}^m \sum_{k=0}^n \mathbf{p}_{j,k} \text{Bez}_{j,m}(u) \text{Bez}_{k,n}(v)$$

for $0 \leq u, v \leq 1$, where $\text{Bez}_{j,m}(u)$ and $\text{Bez}_{k,n}(v)$ denote Bezier blending functions of degree m and n respectively. Consider a bicubic Bezier surface patch, that is, a patch with $m = n = 3$.

- (a) Given any fixed u_0 , $0 \leq u_0 \leq 1$, define the u_0 -slice to be the curve $\mathbf{c}(v) = \mathbf{P}(u_0, v)$. Show that $\mathbf{c}(v)$ is a Bezier curve of degree 3.
 - (b) What are the four control points for the curve $\mathbf{c}(v)$?
 - (c) What is the normal vector to the Bezier surface at $\mathbf{P}(u = 0, v = 0)$?
2.
 - (a) What are the advantages of using a spline based representation of objects such as the Utah teapot instead of a polygonal mesh representation?
 - (b) You want to know whether a given point $P = (x, y, z)$ lies on a surface. With which surface representation is it easier to answer this query: parametric or implicit? Justify your answer.
 - (c) Give an implicit surface representation of a (right circular) cone with its vertex at the origin and with its central axis along the positive Z axis. That is, give an implicit function $f(x, y, z) = 0$ that describes the surface of the cone. Assume the cone makes an angle of α with the Z axis.

3. (a) Consider drawing an ellipsoid using OpenGL. Write a snippet of code to draw an axis-aligned ellipsoid centered at $(1, 3, 5)$ such that its axis parallel to the X axis has length 2 units, its axis parallel to the Y axis has length 8 units, and its axis parallel to the Z axis has length 6 units. After completion, the modelview matrix should be unchanged from its original value.
(b) Write the parametric representation of this ellipsoid, indicating the ranges of the parameters.
4. Consider a scene with l light sources that is to be ray traced. Let the maximum depth of the ray tree be d_{max} (where a ray tree of depth of 1 has a single reflected ray and a single transmitted ray after the first intersection with a surface). Assume a single ray is generated per pixel.
 - (a) Compute the maximum number of reflected and transmitted rays generated.
 - (b) Compute the maximum number of shadow rays generated.
 - (c) Compute the maximum total number of rays generated if the image resolution is 1024×1024 .
5. Compute the intersection points of a ray $P_0 + s\hat{u}$ with the infinite cylinder of radius 2 centered at the origin and with its axis along the Y axis. Assume $P_0 = (4, 0, 3)$ and $\hat{u} = \frac{1}{13}(-4, 12, -3)$.