Parallel SCC and Centrality Lecture 5

CSCI 4974/6971

15 Sep 2016

Today's Biz

- 1. Quick Review
- 2. Reminders
- 3. Parallel SCC
- 4. More Centrality
- 5. Even More MPI
- 6. More PageRank Tutorial

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Quick Review

- Structure of the Web
 - Directed graph SCCs and DAGs
 - Bowtie big SCC, in set, out set, tendrils, tubes, disconnected components
- PageRank
 - Centrality measure which pages hold highest influence
 - Random surfer PageRank equivalent to relative probability a random surfer visits a given page

More MPI functions

- MPI_Allgather(sbuf, scount, MPI_TYPE, rbuf, rcount, MPI_TYPE, MPI_COMM_WORLD)
- MPI_Alltoall(sbuf, scount, MPI_TYPE, rbuf, rcount, MPI_TYPE, MPI_COMM_WORLD)

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Reminders

- Assignment 1: Monday 19 Sept 16:00
- Assignment 2: Thursday 29 Sept 16:00 (posted soon)
- Project Proposal: Thursday 22 Sept 16:00
- Office hours: Tuesday & Wednesday 14:00-16:00 Lally 317
 - Or email me for other availability
- Class schedule:
 - Social net analysis methods
 - Bio net analysis methods
 - Random networks and usage
- Today: Leave advisor info for CCI at end of class

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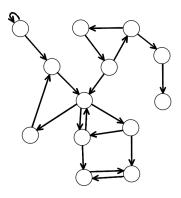
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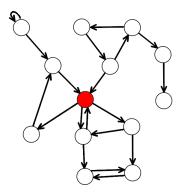
Parallel Strongly Connected Components Algorithms

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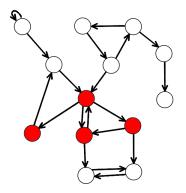
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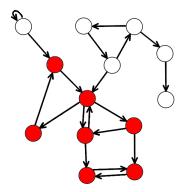
Select pivot



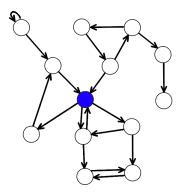
- Select pivot
- Find all vertices that can be reached from the pivot (descendant (D))



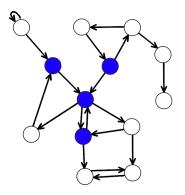
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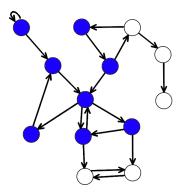
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- Find all vertices that can reach the pivot (**predecessor** (P))



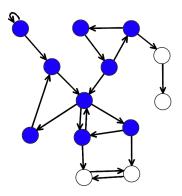
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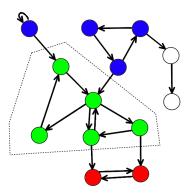
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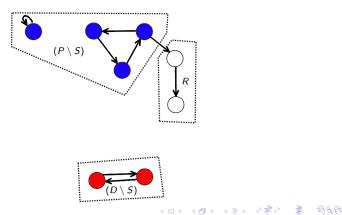
- Select pivot
- Find all vertices that can be reached from the pivot (descendant (D))
- Find all vertices that can reach the pivot (**predecessor** (P)) Intersection of those two sets is an SCC $(S = P \cap D)$



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- Select pivot
- Find all vertices that can be reached from the pivot (descendant (D))

- Find all vertices that can reach the pivot (predecessor (P))
 Intersection of those two sets is an SCC (S = P ∩ D)
 Now have three distinct sets leftover (D \ S), (P \ S), and remainder (R)



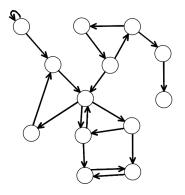
Forward-Backward (FW-BW) Algorithm

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1: procedure FW-BW(V)2:3:4:5:6:7:8: if $V = \emptyset$ then return \emptyset Select a pivot $u \in V$ $D \leftarrow BFS(G(V, E(V)), u)$ $P \leftarrow BFS(G(V, E'(V)), u)$ $R \leftarrow (V \setminus (P \cup D))$ $S \leftarrow (P \cap D)$ 9: **new task do** FW-BW $(D \setminus S)$ 10:new task do FW-BW $(P \setminus S)$ 11: new task do FW-BW(R)

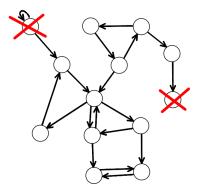
Previous Algorithms Trimming

Used to find trivial SCCs



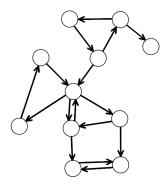
Previous Algorithms

- Used to find trivial SCCs
- Detect and prune all vertices that have an in/out degree of 0 or an in/out degree of 1 with a self loop (simple trimming)



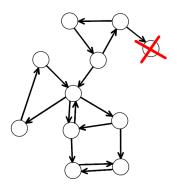
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Previous Algorithms Trimming

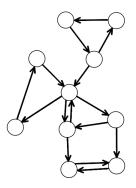
- Used to find trivial SCCs
- Detect and prune all vertices that have an in/out degree of 0 or an in/out degree of 1 with a self loop (simple trimming)
- Repeat iteratively until no more vertices can be removed (complete trimming)



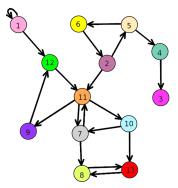
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Previous Algorithms Trimming

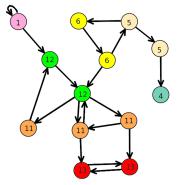
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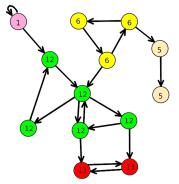
Consider vertex identifiers as colors



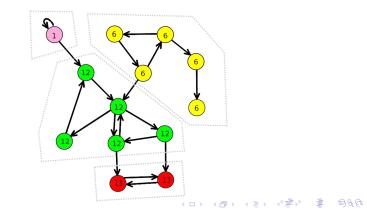
- Consider vertex identifiers as colors
- Highest colors are propagated **forward** through the network to create sets



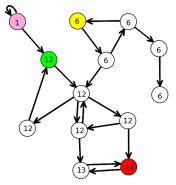
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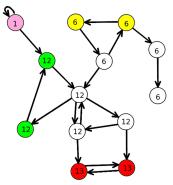
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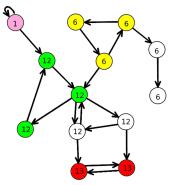
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- Consider the original vertex of each color to be the *root* of a new SCC



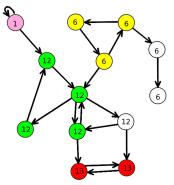
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- Each SCC is all vertices (of the same color as the root) reachable **backward** from each root.



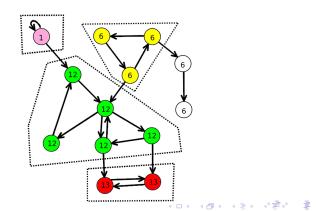
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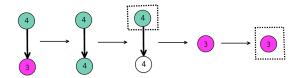


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- Remove found SCCs, reset colors, and repeat until no vertices remain



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Coloring Algorithms

```
1:
     procedure COLORSCC(G(V, E))
 2:
3:
          while G \neq \emptyset do
              for all u \in V do Colors(u) \leftarrow u
 4:5:6:7:8:
              while at least one vertex has changed colors do
                  for all u \in V in parallel do
                      for all \langle u, v \rangle \in E do
                          if Colors(u) > Colors(v) then
                              Colors(v) \leftarrow Colors(u)
 9:
              for all unique c \in Colors in parallel do
10:
                  V_c \leftarrow \{u \in V : Colors(u) = c\}
11:
                  SCCV_c \leftarrow BFS(G(V_c, E'(V_c)), u)
12:
                  V \leftarrow (V \setminus SCCV_c)
```

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Network Centrality Slides from Ahmed Louri, University of Arizona

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Network Centrality

Based on materials by Lada Adamic, UMichigan

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Network Centrality

Which nodes are most 'central'?

Definition of 'central' varies by context/purpose.

Local measure: degree

Relative to rest of network: closeness, betweenness, eigenvector (Bonacich power centrality)

How evenly is centrality distributed among nodes? centralization...

Applications:

Friedkin: Interpersonal Influence in Groups Baker: The Social Organization of Conspiracy

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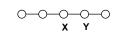
Centrality: Who's Important Based On Their Network Position

In each of the following networks, X has higher centrality than Y according to a particular measure









indegree

outdegree

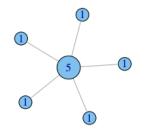
betweenness

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closeness

Degree Centrality (Undirected)

He or she who has many friends is most important.



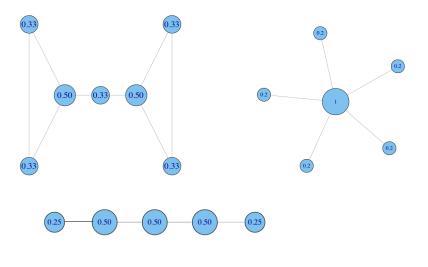
When is the number of connections the best centrality measure?

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- o people who will do favors for you
- \circ people you can talk to / have coffee with

Degree: Normalized Degree Centrality

divide by the max. possible, i.e. (N-1)



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Centralization: How Equal Are The Nodes?

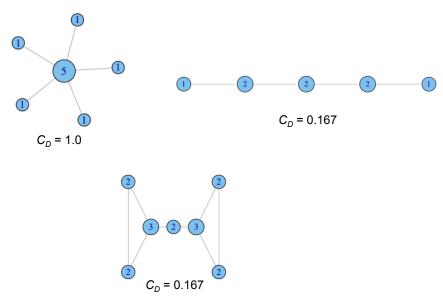
How much variation is there in the centrality scores among the nodes?

Freeman's general formula for centralization (can use other metrics, e.g. gini coefficient or standard deviation):

$$C_{D} = \frac{\sum_{i=1}^{g} \left[C_{D}(n^{*}) - C_{D}(i) \right]}{\left[(N-1)(N-2) \right]}$$

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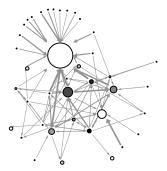
Degree Centralization Examples

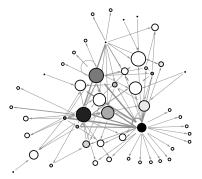


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Degree Centralization Examples

example financial trading networks





high centralization: one node trading with many others

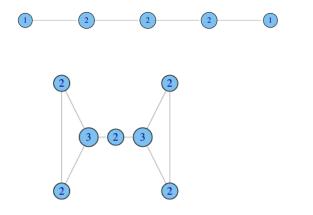
low centralization: trades are more evenly distributed

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When Degree Isn't Everything

In what ways does degree fail to capture centrality in the following graphs?

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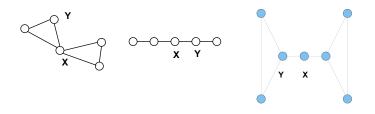
In What Contexts May Degree Be Insufficient To Describe Centrality?

- ability to broker between groups
- likelihood that information originating anywhere in the network reaches you...

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Betweenness: Another Centrality Measure

- Intuition: how many pairs of individuals would have to go through you in order to reach one another in the minimum number of hops?
- Who has higher betweenness, X or Y?



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Betweenness Centrality: Definition

$$C_B(i) = \sum_{j < k} g_{jk}(i) / g_{jk}$$

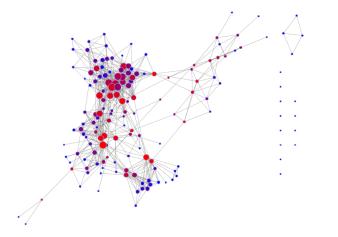
Where g_{jk} = the number of geodesics connecting *jk*, and $g_{jk}(i)$ = the number of geodesics that actor *i* is on.

Usually normalized by:

$$C'_{B}(i) = C_{B}(i) / [(n-1)(n-2)/2]$$
number of pairs of vertices
excluding the vertex itself
adapted from a slide by James Moody

Example

Example facebook network: nodes are sized by degree, and colored by betweenness.

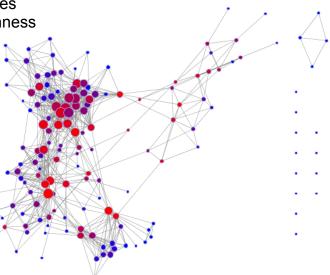


Betweenness Example (Continued)

Can you spot nodes with high betweenness but relatively low degree?

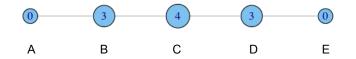
Explain how this might arise.

What about high degree but relatively low betweenness?



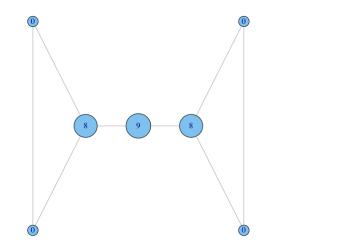
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non-normalized version:

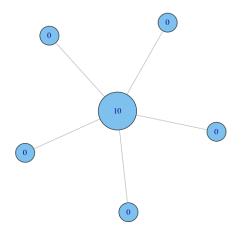


- A lies between no two other vertices
- B lies between A and 3 other vertices: C, D, and E
- C lies between 4 pairs of vertices (A,D),(A,E),(B,D),(B,E)
- note that there are no alternate paths for these pairs to take, so C gets full credit

non-normalized version:

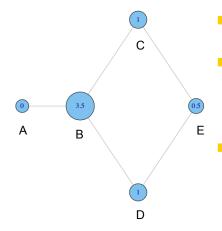


non-normalized version:



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non-normalized version:



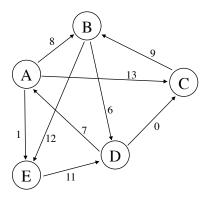
- why do C and D each have betweenness 1?
- They are both on shortest paths for pairs (A,E), and (B,E), and so must share credit:

 $\frac{1}{2} + \frac{1}{2} = 1$

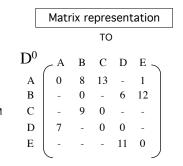
Can you figure out why B has betweenness 3.5 while E has betweenness 0.5?

All-pairs shortest paths...

"Floyd-Warshall algorithm"

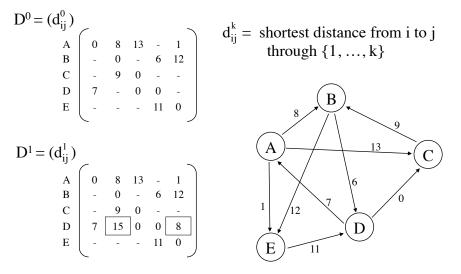


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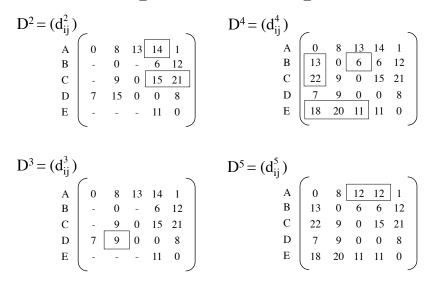
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All-pairs shortest paths...



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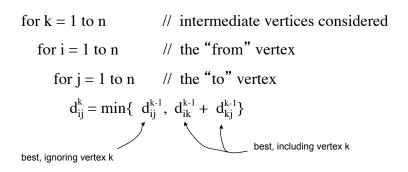
All-pairs shortest paths...



to store the path, another matrix can track the <u>last intermediate vertex</u>

Floyd-Warshall Pseudocode

Input: $D^0 = (d^0_{ij})$ (the initial edge-cost matrix) Output: $D^n = (d^n_{ii})$ (the final path-cost matrix)



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Closeness: Another Centrality Measure

- What if it's not so important to have many direct friends?
- Or be "between" others
- But one still wants to be in the "middle" of things, not too far from the center

Closeness Centrality: Definition

Closeness is based on the length of the average shortest path between a vertex and all vertices in the graph

Closeness Centrality:

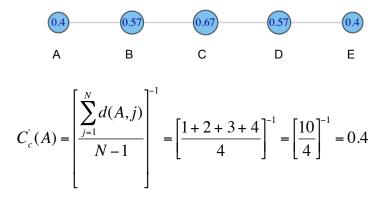
$$C_{c}(i) = \left[\sum_{j=1}^{N} d(i,j)\right]^{-1}$$

Normalized Closeness Centrality

$$C_{C}(i) = (C_{C}(i))/(N-1)$$

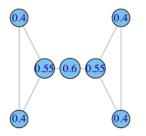
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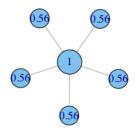
Closeness Centrality: Toy Example



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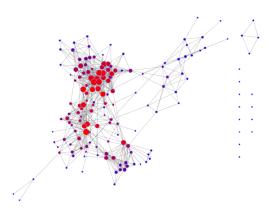
Closeness Centrality: More Toy Examples





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How Closely Do Degree And Betweenness Correspond To Closeness?



- degree (number of connections)
 denoted by size
- closeness (length of shortest path to all others) denoted by color

(D) (B) (B)

Centrality: Check Your Understanding

- · generally different centrality metrics will be positively correlated
- when they are not, there is likely something interesting about the network
- · suggest possible topologies and node positions to fit each square

	Low Degree	Low Closeness	Low Betweenness
High Degree			
High Closeness			
High Betweenness			

adapted from a slide by James Moody

Centrality: Check Your Understanding

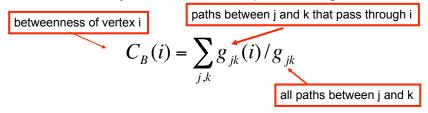
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- · suggest possible topologies and node positions to fit each square

	Low Degree	Low Closeness	Low Betweenness
High Degree		Embedded in cluster that is far from the rest of the network	Ego's connections are redundant - communication bypasses him/her
High Closeness	Key player tied to important/active players		Probably multiple paths in the network, ego is near many people, but so are many others
High Betweenness	Ego's few ties are crucial for network flow	Very rare cell. Would mean that ego monopolizes the ties from a small number of people to many others.	

adapted from a slide by James Moody

Extending Betweenness Centrality To Directed Networks

We now consider the fraction of all directed paths between any two vertices that pass through a node



 Only modification: when normalizing, we have (N-1)*(N-2) instead of (N-1)*(N-2)/2, because we have twice as many ordered pairs as unordered pairs

$$C'_{B}(i) = C_{B}(i) / [(N-1)(N-2)]$$

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Directed Geodesics

A node does not necessarily lie on a geodesic from j to k if it lies on a geodesic from k to j

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Extensions Of Undirected Degree Centrality - Prestige

degree centrality

- indegree centrality
 - a paper that is cited by many others has high prestige
 - a person nominated by many others for a reward has high prestige

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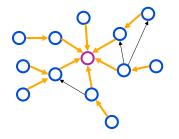
Extensions Of Undirected Closeness Centrality

closeness centrality usually implies

- all paths should lead to you and unusually not:
- paths should lead from you to everywhere else
- usually consider only vertices from which the node *i* in question can be reached

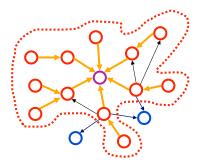
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Influence Range

The influence range of *i* is the set of vertices who are reachable from the node *i*



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Wrap Up

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Centrality

many measures: degree, betweenness, closeness, ...

may be unevenly distributed

measure via centralization

- extensions to directed networks:
 - prestige
 - influence
 - PageRank

Today's Biz

- 1. Quick Review
- 2. Reminders
- 3. Parallel SCC
- 4. More Centrality
- 5. Even More MPI
- 6. More PageRank Tutorial

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Even More MPI – Alltoallv Slides from Lori Pollock, University of Delaware

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MPI_AlltoAllv Function Outline

int MPI_Alltoallv (void *sendbuf, int *sendcnts, int *sdispls,

MPI_Datatype sendtype,

void *recvbuf, int *recvcnts, int *rdispls,

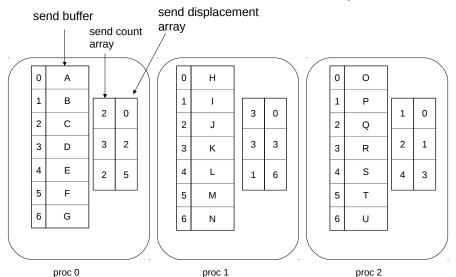
MPI_Datatype recvtype,

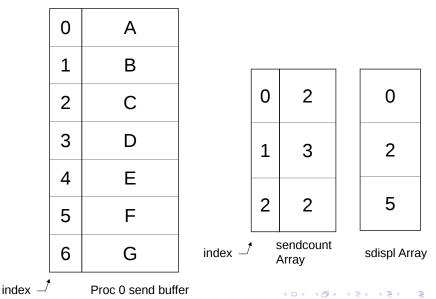
MPI_Comm comm)

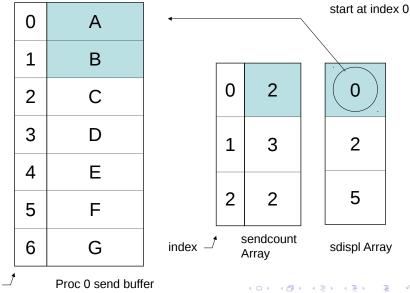
Input Parameters

sendbuf starting address of send buffer (choice) **sendcounts** integer array equal to the group size specifying the number of elements to send to each processor **sdispls** integer array (of length group size). Entry j specifies the displacement (relative to sendbuf from which to take the outgoing data destined for process j **recvcounts** integer array equal to the group size specifying the maximum number of elements that can be received from each processor **rdispls** integer array (of length group size). Entry i specifies the displacement (relative to recvbuf at which to place the incoming data from process i

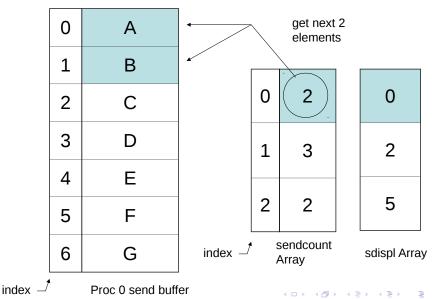
Each node in parallel community has



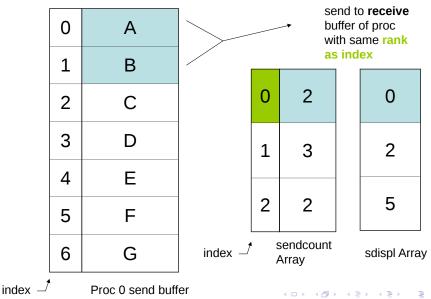




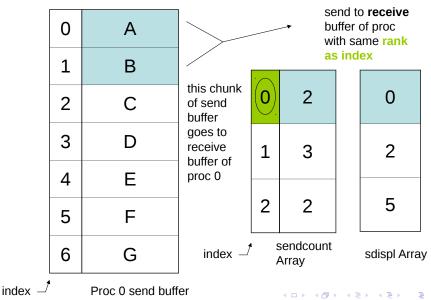
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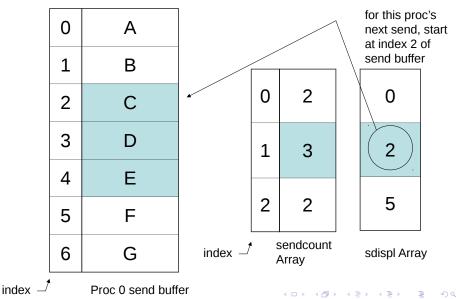
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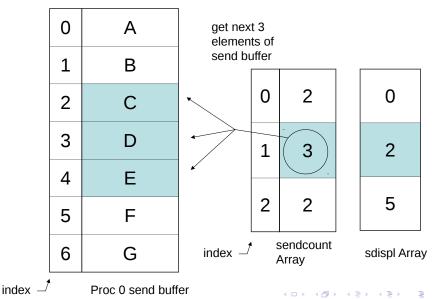


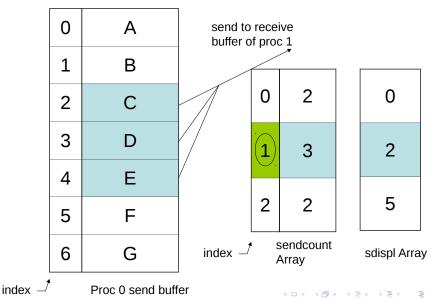
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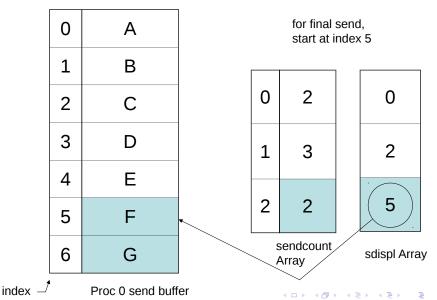
SAR



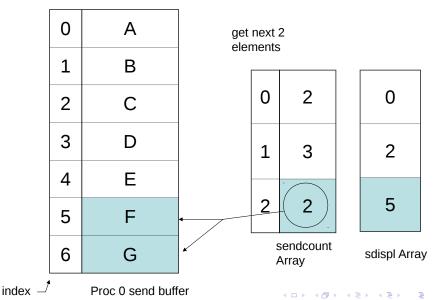


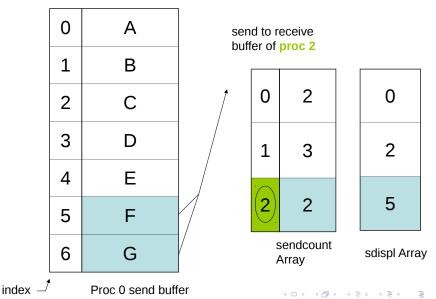


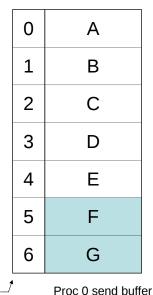
SAR



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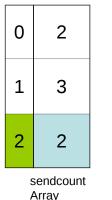






index

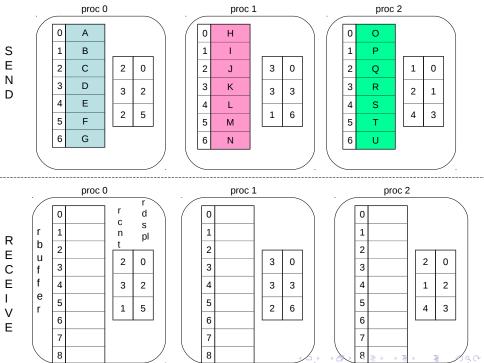
this process occurs for each node in the community

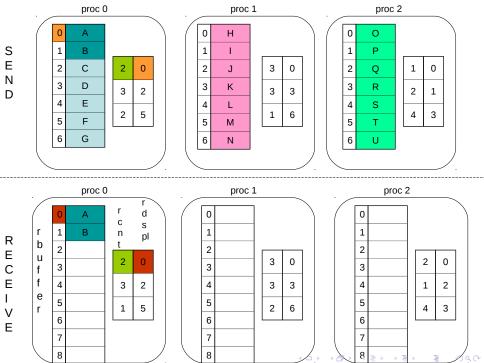


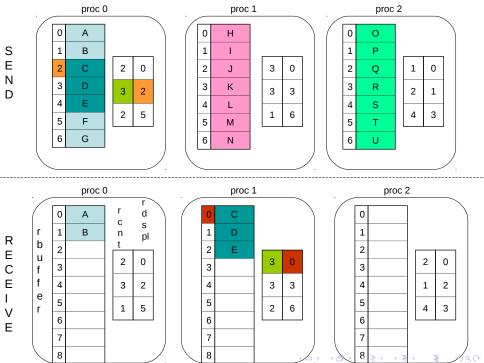


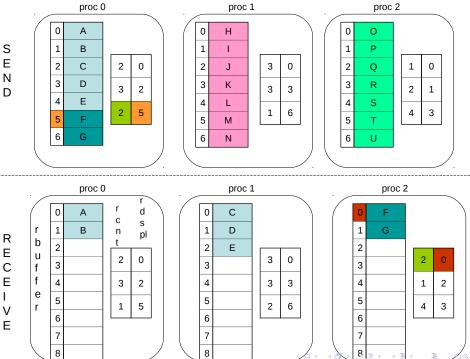
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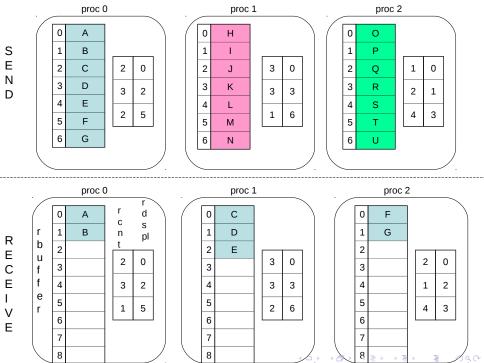


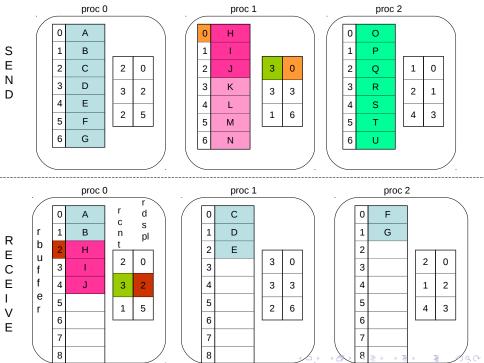


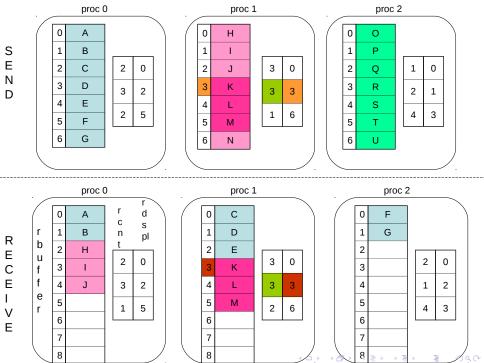


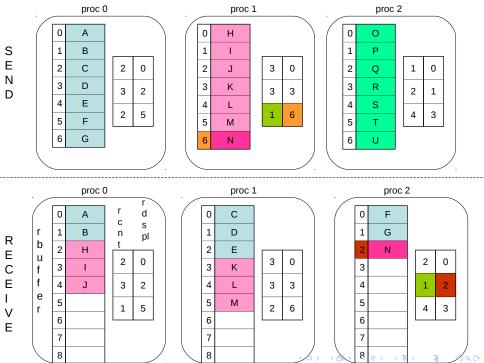
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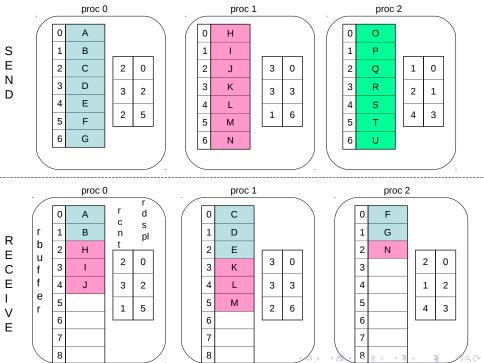
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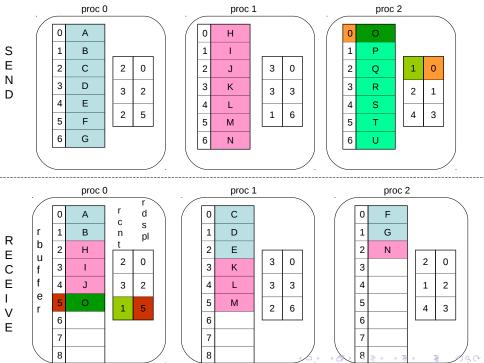


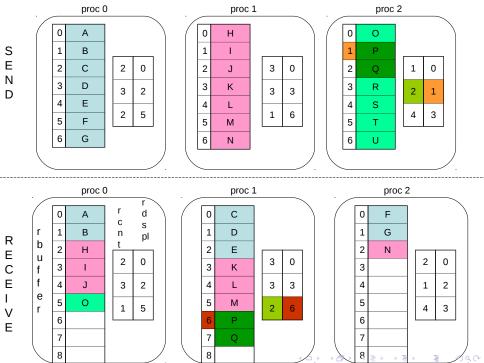


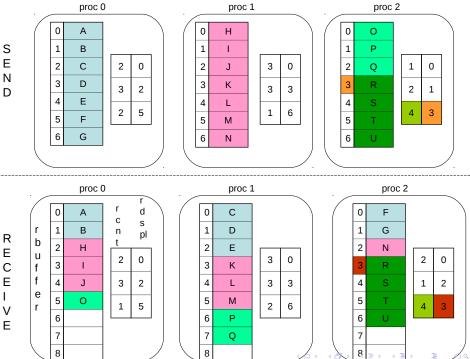




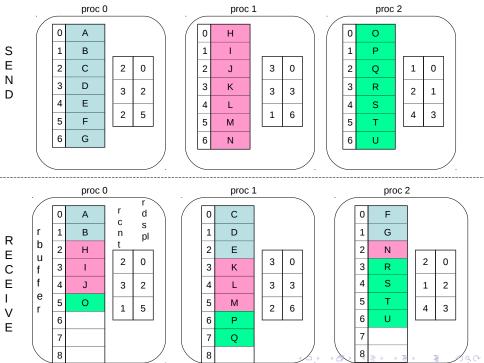








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Notes on AlltoAllv

- A receive buffer could potentially be as large as the sum of all send buffer sizes
- Care must be taken to coincide send counts with receive counts and displacements so data is not overwritten

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More PageRank Tutorial

- 1. OpenMP Work Queueing
- 2. MPI Alltoallv Communication

More PageRank Tutorial Blank code and data available on website (Lecture 5) www.cs.rpi.edu/~slotag/classes/FA16/index.html

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