# Parallel SCC and Centrality <br> Lecture 5 

CSCI 4974/6971

15 Sep 2016

## Today's Biz

1. Quick Review
2. Reminders
3. Parallel SCC
4. More Centrality
5. Even More MPI
6. More PageRank Tutorial

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## Quick Review

- Structure of the Web
- Directed graph - SCCs and DAGs
- Bowtie - big SCC, in set, out set, tendrils, tubes, disconnected components
- PageRank
- Centrality measure - which pages hold highest influence
- Random surfer - PageRank equivalent to relative probability a random surfer visits a given page
More MPI functions
- MPI_Allgather(sbuf, scount, MPI_TYPE, rbuf, rcount, MPI_TYPE, MPI_COMM_WORLD)
- MPI_Alltoall(sbuf, scount, MPI_TYPE, rbuf, rcount, MPI_TYPE, MPI_COMM_WORLD)


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## Reminders

- Assignment 1: Monday 19 Sept 16:00
- Assignment 2: Thursday 29 Sept 16:00 (posted soon)
- Project Proposal: Thursday 22 Sept 16:00
- Office hours: Tuesday \& Wednesday 14:00-16:00 Lally 317
- Or email me for other availability
- Class schedule:
- Social net analysis methods
- Bio net analysis methods
- Random networks and usage
- Today: Leave advisor info for CCI at end of class


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## Parallel Strongly Connected Components Algorithms

## Previous Algorithms

 Forward-Backward (FW-BW)

## Previous Algorithms Forward－Backward（FW－BW）

－Select pivot


## Previous Algorithms Forward-Backward (FW-BW)

- Select pivot
- Find all vertices that can be reached from the pivot (descendant $(D)$ )



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- Intersection of those two sets is an SCC $(S=P \cap D)$



## Previous Algorithms Forward－Backward（FW－BW）

－Select pivot
－Find all vertices that can be reached from the pivot（descendant $(D)$ ）
－Find all vertices that can reach the pivot（predecessor $(P)$ ）
－Intersection of those two sets is an SCC（ $S=P \cap D$ ）
－Now have three distinct sets leftover（ $D \backslash S$ ），（ $P \backslash S$ ），and remainder（ $R$ ）


## Forward-Backward (FW-BW) Algorithm

```
    1: procedure FW-BW(V)
2: if V=\varnothing then
                return \varnothing
    Select a pivot u\inV
    D\leftarrow\operatorname{BFS}(G(V,E(V)),u)
        P}\leftarrow\operatorname{BFS}(G(V,\mp@subsup{E}{}{\prime}(V)),u
        R\leftarrow(V\(P\cupD)
        S\leftarrow(P\capD)
        new task do FW-BW (D\S)
        new task do FW-BW (P\S)
        new task do FW-BW(R)
```


## Previous Algorithms Trimming

－Used to find trivial SCCs


## Previous Algorithms

Trimming

- Used to find trivial SCCs
- Detect and prune all vertices that have an in/out degree of 0 or an in/out degree of 1 with a self loop (simple trimming)



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- Repeat iteratively until no more vertices can be removed (complete trimming)



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## Previous Algorithms

 Coloring- Consider vertex identifiers as colors



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- Remove found SCCs, reset colors, and repeat until no vertices remain



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## Coloring Algorithms

1: procedure $\operatorname{ColorSCC}(G(V, E))$
2: $\quad$ while $G \neq \varnothing$ do
for all $u \in V$ do $\operatorname{Colors}(u) \leftarrow u$
while at least one vertex has changed colors do
for all $u \in V$ in parallel do
for all $\langle u, v\rangle \in E$ do
if Colors $(u)>\operatorname{Colors}(v)$ then
Colors $(v) \leftarrow$ Colors $(u)$
for all unique $c \in$ Colors in parallel do
$10:$
11:
12 :

$$
V_{c} \leftarrow\{u \in V: \operatorname{Colors}(u)=c\}
$$

$$
S C C V_{c} \leftarrow \operatorname{BFS}\left(G\left(V_{c}, E^{\prime}\left(V_{c}\right)\right), u\right)
$$

$$
V \leftarrow\left(V \backslash S C C V_{c}\right)
$$

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## Network Centrality

Slides from Ahmed Louri, University of Arizona

## Network Centrality

Based on materials by Lada Adamic, UMichigan

## Network Centrality

Which nodes are most 'central' ?
Definition of 'central' varies by context/purpose.
Local measure:
degree
Relative to rest of network:
closeness, betweenness, eigenvector (Bonacich power centrality)

How evenly is centrality distributed among nodes? centralization...

Applications:
Friedkin: Interpersonal Influence in Groups
Baker: The Social Organization of Conspiracy

## Centrality: Who's Important Based On Their Network Position

In each of the following networks, X has higher centrality than Y according to a particular measure

indegree

outdegree

betweenness

closeness

## Degree Centrality (Undirected)

He or she who has many friends is most important.


When is the number of connections the best centrality measure?

- people who will do favors for you
- people you can talk to / have coffee with


## Degree: Normalized Degree Centrality

divide by the max. possible, i.e. ( $\mathrm{N}-1$ )


## Centralization: How Equal Are The Nodes?

How much variation is there in the centrality scores among the nodes?

Freeman' s general formula for centralization (can use other metrics, e.g. gini coefficient or standard deviation):

$$
C_{D}=\frac{\sum_{i=1}^{g}\left[C_{D}\left(n^{*}\right)-C_{D}(i)\right]}{[(N-1)(N-2)]}
$$

## Degree Centralization Examples

$C_{D}=1.0$


$$
C_{D}=0.167
$$



## Degree Centralization Examples

example financial trading networks

high centralization: one node trading with many others
low centralization: trades are more evenly distributed

## When Degree Isn't Everything

In what ways does degree fail to capture centrality in the following graphs?


In What Contexts May Degree Be Insufficient To Describe Centrality?

- ability to broker between groups
- likelihood that information originating anywhere in the network reaches you...


## Betweenness: Another Centrality Measure

- Intuition: how many pairs of individuals would have to go through you in order to reach one another in the minimum number of hops?
- Who has higher betweenness, X or Y ?



## Betweenness Centrality: Definition

$$
C_{B}(i)=\sum_{j<k} g_{j k}(i) / g_{j k}
$$

Where $g_{j k}=$ the number of geodesics connecting $j k$, and $g_{j k}(i)=$ the number of geodesics that actor $i$ is on.

Usually normalized by:

$$
C_{B}^{\prime}(i)=C_{B}(i) /[(n-1)(n-2) / 2]
$$

number of pairs of vertices excluding the vertex itself

## Example

Example facebook network：nodes are sized by degree， and colored by betweenness．


## Betweenness Example (Continued)

Can you spot nodes with high betweenness but relatively low degree?

Explain how this might arise.

What about high degree but relatively low betweenness?

## Betweenness On Toy Networks

- non-normalized version:

- A lies between no two other vertices
- $B$ lies between $A$ and 3 other vertices: $C, D$, and $E$
- C lies between 4 pairs of vertices (A,D),(A,E),(B,D),(B,E)
- note that there are no alternate paths for these pairs to take, so C gets full credit


## Betweenness On Toy Networks

 non-normalized version:

## Betweenness On Toy Networks

non-normalized version:


## Betweenness On Toy Networks

- non-normalized version:

- why do C and D each have betweenness 1?
- They are both on shortest paths for pairs (A,E), and (B,E), and so must share credit:
- $1 / 2+1 / 2=1$
- Can you figure out why B has betweenness 3.5 while E has betweenness 0.5 ?


## All-pairs shortest paths...

"Floyd-Warshall algorithm"


## All-pairs shortest paths...

$$
\begin{aligned}
& \mathrm{D}^{0}=\left(\mathrm{d}_{\mathrm{ij}}^{0}\right) \\
& \begin{array}{l}
\mathrm{A} \\
\mathrm{~B} \\
\mathrm{C} \\
\mathrm{D} \\
\mathrm{E}
\end{array}\left(\begin{array}{ccccc}
0 & 8 & 13 & - & 1 \\
- & 0 & - & 6 & 12 \\
- & 9 & 0 & - & - \\
7 & - & 0 & 0 & - \\
- & - & - & 11 & 0
\end{array}\right) \\
& \mathrm{d}_{\mathrm{ij}}^{\mathrm{k}}=\text { shortest distance from } \mathrm{i} \text { to } \mathrm{j} \\
& \text { through }\{1, \ldots, k\} \\
& \mathrm{D}^{1}=\left(\mathrm{d}_{\mathrm{ij}}^{1}\right) \\
& \begin{array}{l}
\text { A } \\
\text { B } \\
\text { C } \\
\text { D } \\
\text { E }
\end{array}\left(\begin{array}{ccccc}
0 & 8 & 13 & - & 1 \\
- & 0 & - & 6 & 12 \\
- & 9 & 0 & - & - \\
7 & 15 & 0 & 0 & 8 \\
- & - & - & 11 & 0
\end{array}\right)
\end{aligned}
$$

## All-pairs shortest paths...

$$
\mathrm{D}^{5}=\left(\mathrm{d}_{\mathrm{ij}}^{5}\right)
$$

$$
\begin{aligned}
& \text { A } \\
& \text { B } \\
& \text { C } \\
& \text { D } \\
& \mathrm{E}
\end{aligned}\left(\begin{array}{ccccc}
0 & 8 & 12 & 12 & 1 \\
13 & 0 & 6 & 6 & 12 \\
22 & 9 & 0 & 15 & 21 \\
7 & 9 & 0 & 0 & 8 \\
18 & 20 & 11 & 11 & 0
\end{array}\right)
$$

$$
\begin{aligned}
& D^{3}=\left(d_{i j}^{3}\right) \\
& \begin{array}{l}
\text { A } \\
\text { B } \\
\text { C } \\
\text { D } \\
\text { E }
\end{array}\left(\begin{array}{ccccc}
0 & 8 & 13 & 14 & 1 \\
- & 0 & - & 6 & 12 \\
- & 9 & 0 & 15 & 21 \\
7 & 9 & 0 & 0 & 8 \\
- & - & - & 11 & 0
\end{array}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{D}^{2}=\left(\mathrm{d}_{\mathrm{ij}}^{2}\right) \\
& \begin{array}{l}
\text { A } \\
\text { B } \\
\text { C } \\
\text { D } \\
\text { E }
\end{array}\left[\begin{array}{ccc|cc}
0 & 8 & 13 & 14 & 1 \\
- & 0 & - & 6 & 12 \\
- & 9 & 0 & 15 & 21 \\
7 & 15 & 0 & 0 & 8 \\
- & - & - & 11 & 0
\end{array}\right)
\end{aligned}
$$

## Floyd-Warshall Pseudocode

Input: $\quad D^{0}=\left(d_{i j}^{0}\right) \quad$ (the initial edge-cost matrix)
Output: $D^{\mathrm{n}}=\left(\mathrm{d}_{\mathrm{ij}}^{\mathrm{n}}\right) \quad$ (the final path-cost matrix)


## Closeness: Another Centrality Measure

- What if it's not so important to have many direct friends?
- Or be "between" others
- But one still wants to be in the "middle" of things, not too far from the center


## Closeness Centrality: Definition

Closeness is based on the length of the average shortest path between a vertex and all vertices in the graph

Closeness Centrality:

$$
C_{c}(i)=\left[\sum_{j=1}^{N} d(i, j)\right]^{-1}
$$

Normalized Closeness Centrality

$$
C_{C}^{\prime}(i)=\left(C_{C}(i)\right) /(N-1)
$$

## Closeness Centrality: Toy Example

$$
\begin{gathered}
\mathrm{A} \\
C_{c}^{\prime}(A)=\left[\frac{\sum_{j=1}^{N} d(A, j)}{N-1}\right]^{-1}=\left[\frac{1+2+3+4}{4}\right]^{-1}=\left[\frac{10}{4}\right]^{-1}=0.4
\end{gathered}
$$

Closeness Centrality: More Toy Examples


## How Closely Do Degree And Betweenness Correspond To Closeness?

- degree (number of connections)
denoted by size
- closeness (length of shortest path to all others) denoted by color


## Centrality: Check Your Understanding

- generally different centrality metrics will be positively correlated
- when they are not, there is likely something interesting about the network
- suggest possible topologies and node positions to fit each square

|  | Low <br> Degree | Low <br> Closeness | Low <br> Betweenness |
| :--- | :--- | :--- | :--- |
| High Degree |  |  |  |
| High Closeness |  |  |  |
| High <br> Betweenness |  |  |  |

## Centrality: Check Your Understanding

- generally different centrality metrics will be positively correlated
- when they are not, there is likely something interesting about the network
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|  | Low <br> Degree | Low <br> Closeness | Low <br> Betweenness |
| :--- | :--- | :--- | :--- |
| High Degree |  | Embedded in cluster <br> that is far from the <br> rest of the network | Ego's connections <br> are redundant - <br> communication <br> bypasses him/her |
| High Closeness | Key player tied to <br> important/active <br> players | Ego's few ties are <br> crucial for network <br> flow | Very rare cell. <br> Would mean that <br> ego monopolizes <br> the ties from a small <br> number of people to <br> many others. |
| High in the <br> Between | many people, but so <br> are many others |  |  |

## Extending Betweenness Centrality To Directed Networks

- We now consider the fraction of all directed paths between any two vertices that pass through a node

- Only modification: when normalizing, we have $(\mathrm{N}-1)^{*}(\mathrm{~N}-2)$ instead of $(\mathrm{N}-1)^{*}(\mathrm{~N}-2) / 2$, because we have twice as many ordered pairs as unordered pairs

$$
C_{B}^{\prime}(i)=C_{B}(i) /[(N-1)(N-2)]
$$

## Directed Geodesics

－A node does not necessarily lie on a geodesic from $j$ to $k$ if it lies on a geodesic from $k$ to $j$


## Extensions Of Undirected Degree Centrality - Prestige

## - degree centrality

- indegree centrality
- a paper that is cited by many others has high prestige
- a person nominated by many others for a reward has high prestige



## Extensions Of Undirected Closeness Centrality

- closeness centrality usually implies

■ all paths should lead to you and unusually not:

- paths should lead from you to everywhere else
- usually consider only vertices from which the node $i$ in question can be reached



## Influence Range

- The influence range of $i$ is the set of vertices who are reachable from the node $i$



## Wrap Up

## Centrality

$\square$ many measures：degree，betweenness， closeness，．．．
$\square$ may be unevenly distributed
－measure via centralization
$■$ extensions to directed networks：
－prestige
－influence
－PageRank

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## Even More MPI - Alltoallv <br> Slides from Lori Pollock, University of Delaware

## MPI AlltoAllv Function Outline

int MPI_Alltoallv ( void *sendbuf, int *sendents, int *sdispls,
MPI_Datatype sendtype,
void *recvbuf, int *recvents, int *rdispls,
MPI_Datatype recvtype,
MPI_Comm comm )

## Input Parameters

sendbuf starting address of send buffer (choice)
sendcounts integer array equal to the group size specifying the number of
elements to send to each processor
sdispls integer array (of length group size). Entry j specifies the displacement (relative to sendbuf from which to take the outgoing data destined for process j
recvcounts integer array equal to the group size specifying the maximum number of elements that can be received from each processor
rdispls integer array (of length group size). Entry i specifies the
displacement (relative to recvbuf at which to place the incoming data from process i

Each node in parallel community has


## Example of Send for Proc 0

| 0 | A |
| :---: | :---: |
| 1 | B |
| 2 | C |
| 3 | D |
| 4 | E |
| 5 | F |
| 6 | G |


index $\uparrow$
Proc 0 send buffer

## Example of Send for Proc 0

| 0 | A |
| :---: | :---: |
| 1 | B |
| 2 | C |
| 3 | D |
| 4 | E |
| 5 | F |
| 6 | $G$ |


index $\uparrow$
Proc 0 send buffer

## Example of Send for Proc 0


index $\uparrow$
Proc 0 send buffer

## Example of Send for Proc 0

| 0 | AB |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |
| 2 | C | 0 | 2 | 0 |
| 3 | D | 1 | 3 | 2 |
| 4 | E |  |  |  |
| 5 | F | 2 | 2 | 5 |
| 6 | G |  | ay | spl |

index $\uparrow$
Proc 0 send buffer

## Example of Send for Proc 0

| 0 | A |
| :---: | :---: |
| 1 | B |
| 2 | C |
| 3 | D |
| 4 | E |
| 5 | F |
| 6 | $G$ |



## Example of Send for Proc 0


index $\uparrow$
Proc 0 send buffer

## Example of Send for Proc 0

| 0 | A |
| :--- | :--- |
| 1 | B |
| 2 | C |
| 3 | D |
| 4 | E |
| 5 | F |
| 6 | G |



## Example of Send for Proc 0


index $\uparrow$
Proc 0 send buffer

## Example of Send for Proc 0

| 0 | A |
| :--- | :--- |
| 1 | B |
| 2 | C |
| 3 | D |
| 4 | E |
| 5 | F |
| 6 | $G$ |

for final send, start at index 5


## Example of Send for Proc 0


index $\quad \uparrow$
Proc 0 send buffer

## Example of Send for Proc 0

| 0 | A |
| :--- | :--- |
| 1 | B |
| 2 | C |
| 3 | D |
| 4 | E |
| 5 | F |
| 6 | G |

send to receive buffer of proc 2

index ${ }^{\dagger}$
Proc 0 send buffer

## Example of Send for Proc 0

| 0 | A |
| :---: | :---: |
| 1 | B |
| 2 | C |
| 3 | D |
| 4 | E |
| 5 | F |
| 6 | G |

this process
occurs for each
node in the community

sdispl Array
proc 0
$\sigma z m \omega$
proc 1
proc 2

proc 0

|  | 0 |  |
| :--- | :--- | :--- |
|  |  |  |
|  | 1 |  |
| $b$ | 2 |  |
| $u$ | 3 |  |
| $f$ | 4 |  |
|  | 4 |  |
| $r$ | 5 |  |
|  | 6 |  |
|  | 7 |  |
|  | 8 |  |

proc 1

| 0 |  |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |

proc 2

| 0 |  |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |


| 2 | 0 |
| :--- | :--- |
| 1 | 2 |
| 4 | 3 |

proc 0
proc 1
proc 2

proc 0

|  | 0 | $A$ |
| :--- | :--- | :--- |
| $r$ | 1 | $B$ |
| $b$ | 2 |  |
| $u$ |  |  |
|  | 3 |  |
|  | 4 |  |
|  |  |  |
|  | 5 |  |
|  | 6 |  |
|  | 7 |  |
|  | 8 |  |

proc 1

| 0 |  |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |

proc 2

| 0 |  |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |


| 2 | 0 |
| :--- | :--- |
| 1 | 2 |
| 4 | 3 |

proc 0
proc 1
proc 2
$\nabla \geq m \omega$

| 0 | A |  |  |
| :--- | :--- | :--- | :--- |
| 1 |  |  |  |
| 1 | B |  |  |
| 2 | C |  |  |
| 2 | 2 | 0 |  |
| 3 | D |  |  |
| 4 | E | 2 |  |
| 5 | F |  |  |
| 6 |  |  | 5 |
|  |  |  |  |


| 0 | $H$ |
| :---: | :---: |
| 1 | I |
| 2 | $J$ |
| 3 | $K$ |
| 4 | $L$ |
| 5 | $M$ |
| 6 | $N$ |


| 3 | 0 |
| :--- | :--- |
| 3 | 3 |
| 1 | 6 |


| 0 |
| :--- |
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

6
proc 0

|  | 0 | $A$ |
| :--- | :--- | :--- |
| $r$ | 1 | $B$ |
| $b$ | 2 |  |
| u |  |  |
|  | 3 |  |
| f | 4 |  |
| e | 5 |  |
| $r$ | 5 |  |
|  | 6 |  |
|  | 7 |  |
|  | 8 |  |

proc 1

| 0 | $C$ |
| :---: | :---: |
| 1 | $D$ |
| 2 | $E$ |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |

proc 2

| 0 |  |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |


| 2 | 0 |
| :--- | :--- |
| 1 | 2 |
| 4 | 3 |

proc 0
$\sigma z m \omega$
proc 1

| 0 | $H$ |
| :---: | :---: |
| 1 | I |
| 2 | $J$ |
| 3 | $K$ |
| 4 | $L$ |
| 5 | $M$ |
| 6 | $N$ |


| 3 | 0 |
| :--- | :--- |
| 3 | 3 |
| 1 | 6 |

proc 2

proc 0

|  | 0 | $A$ |
| :--- | :--- | :--- |
| $r$ | 1 | $B$ |
| $b$ | 2 |  |
| $u$ | 3 |  |
| $f$ | 3 |  |
|  | 4 |  |
|  |  |  |
| $r$ | 5 |  |
|  | 6 |  |
|  | 7 |  |
|  | 8 |  |

proc 1

| 0 | $C$ |
| :--- | :--- |
| 1 | $D$ |
| 2 | $E$ |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |


| 3 | 0 |
| :--- | :--- |
| 3 | 3 |
| 2 | 6 |

proc 2

| 0 | $F$ |  |
| :--- | :--- | :--- |
| 1 |  |  |
|  |  |  |

proc 0
$\nabla z m \omega$
proc 1
proc 2

proc 0

|  | 0 | $A$ |
| :--- | :--- | :--- |
| $r$ | 1 | $B$ |
| $b$ | 2 |  |
| $u$ | 3 |  |
| $f$ | 3 |  |
|  | 4 |  |
|  |  |  |
| $r$ | 5 |  |
|  | 6 |  |
|  | 7 |  |
|  | 8 |  |

proc 1

| 0 | $C$ |
| :--- | :--- |
| 1 | $D$ |
| 2 | $E$ |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |


| 3 | 0 |
| :--- | :--- |
| 3 | 3 |
| 2 | 6 |

proc 2

| 0 | $F$ |
| :--- | :--- |
| 1 | $G$ |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |


| 2 | 0 |
| :--- | :--- |
| 1 | 2 |
| 4 | 3 |

proc 0
$\nabla \geq m \omega$
proc 1
proc 2

proc 0

|  | 0 | $A$ |
| :--- | :---: | :---: |
| $r$ | 1 | $B$ |
| $b$ | 2 | $H$ |
| $u$ |  |  |
|  | 3 | 1 |
| $f$ | 4 | $J$ |
|  |  |  |
|  | 5 |  |
|  | 6 |  |
|  | 7 |  |
|  | 8 |  |

proc 1

| 0 | $C$ |
| :--- | :--- |
| 1 | $D$ |
| 2 | $E$ |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |


| 3 | 0 |
| :--- | :--- |
| 3 | 3 |
| 2 | 6 |

proc 2

| 0 | $F$ |
| :--- | :--- |
| 1 | $G$ |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |


| 2 | 0 |
| :--- | :--- |
| 1 | 2 |
| 4 | 3 |

proc 0
$\nabla z m \omega$
proc 1
proc 2

proc 0

proc 1

| 0 | $C$ |
| :--- | :--- |
| 1 | $D$ |
| 2 | $E$ |
| 3 | K |
| 4 | L |
| 5 | M |
| 6 |  |
| 7 |  |
| 8 |  |


| 3 | 0 |
| :--- | :--- |
| 3 | 3 |
| 2 | 6 |

proc 2

| 0 | $F$ |
| :--- | :--- |
| 1 | $G$ |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |


| 2 | 0 |
| :--- | :--- |
| 1 | 2 |
| 4 | 3 |

proc 0
$\nabla z m \omega$
proc 1
proc 2

proc 0

proc 1

| 0 | $C$ |
| :---: | :---: |
| 1 | $D$ |
| 2 | $E$ |
| 3 | $K$ |
| 4 | $L$ |
| 5 | $M$ |
| 6 |  |
| 7 |  |
| 8 |  |


| 3 | 0 |
| :--- | :--- |
| 3 | 3 |
| 2 | 6 |

proc 2

| 0 | $F$ |
| :--- | :--- |
| 1 | $G$ |
| 2 | $N$ |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |


| 2 | 0 |
| :--- | :--- |
| 1 | 2 |
| 4 | 3 |

proc 0
$\nabla z m \omega$
proc 1
proc 2

| 0 | $O$ |
| :---: | :---: |
| 1 | $P$ |
| 2 | $Q$ |
| 3 | $R$ |
| 4 | S |
| 5 | T |
| 6 | U |


| 1 | 0 |
| :--- | :--- |
| 2 | 1 |
| 4 | 3 |

proc 0

proc 1

| 0 | $C$ |
| :---: | :---: |
| 1 | $D$ |
| 2 | $E$ |
| 3 | $K$ |
| 4 | $L$ |
| 5 | $M$ |
| 6 |  |
| 7 |  |
| 8 |  |


| 3 | 0 |
| :--- | :--- |
| 3 | 3 |
| 2 | 6 |

proc 2

| 0 | $F$ |
| :--- | :--- |
| 1 | $G$ |
| 2 | N |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |


| 2 | 0 |
| :--- | :--- |
| 1 | 2 |
| 4 | 3 |

proc 0
proc 1
$\sigma z m \omega$
proc 2

proc 0

| < | $\infty$ | エ | - | $\square$ | 0 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bigcirc$ | $\checkmark$ | N | m | $\checkmark$ | ค | $\bullet$ | $\wedge$ | $\infty$ |

proc 1

| 0 | $C$ |
| :---: | :---: |
| 1 | $D$ |
| 2 | $E$ |
| 3 | $K$ |
| 4 | $L$ |
| 5 | $M$ |
| 6 |  |
| 7 |  |
| 8 |  |


| 3 | 0 |
| :--- | :--- |
| 3 | 3 |
| 2 | 6 |

proc 2

| 0 | $F$ |
| :--- | :--- |
| 1 | $G$ |
| 2 | $N$ |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |


| 2 | 0 |
| :--- | :--- |
| 1 | 2 |
| 4 | 3 |

proc 0
$\nabla z m \omega$
proc 1
proc 2

proc 0

| < | $\infty$ | エ | - | 7 | 0 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bigcirc$ | $\checkmark$ | N | m | $\checkmark$ | ค | $\bigcirc$ | $\wedge$ | $\infty$ |

proc 1

| 0 | $C$ |
| :---: | :---: |
| 1 | $D$ |
| 2 | $E$ |
| 3 | $K$ |
| 4 | $L$ |
| 5 | $M$ |
| 6 | $P$ |
| 7 | $Q$ |
| 8 |  |


| 3 | 0 |
| :--- | :--- |
| 3 | 3 |
| 2 | 6 |

proc 2

| 0 | $F$ |
| :--- | :--- |
| 1 | $G$ |
| 2 | N |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |


| 2 | 0 |
| :--- | :--- |
| 1 | 2 |
| 4 | 3 |

proc 0
$\nabla z m \omega$
proc 1

| 0 | $H$ |
| :---: | :---: |
| 1 | I |
| 2 | $J$ |
| 3 | $K$ |
| 4 | $L$ |
| 5 | $M$ |
| 6 | $N$ |

proc 2

proc 0

| < | $\infty$ | エ | - | 7 | 0 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bigcirc$ | $\checkmark$ | $\sim$ | ल | $\checkmark$ | ค | $\bigcirc$ | $\wedge$ | $\infty$ |

proc 1

| 0 | $C$ |
| :---: | :---: |
| 1 | $D$ |
| 2 | $E$ |
| 3 | $K$ |
| 4 | $L$ |
| 5 | $M$ |
| 6 | $P$ |
| 7 | $Q$ |
| 8 |  |


| 3 | 0 |
| :--- | :--- |
| 3 | 3 |
| 2 | 6 |

proc 2

| 0 | $F$ |
| :---: | :---: |
| 1 | $G$ |
| 2 | N |
| 3 | R |
| 4 | S |
| 5 | T |
| 6 | U |
| 7 |  |
| 8 |  |


| 2 | 0 |
| :--- | :--- |
| 1 | 2 |
| 4 | 3 |

proc 0
proc 1

proc 2

proc 0

| < | $\infty$ | エ | - | 7 | 0 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bigcirc$ | $\checkmark$ | $N$ | $\cdots$ | $\checkmark$ | 10 | $\bigcirc$ | $\wedge$ | $\infty$ |

proc 1

| 0 | $C$ |
| :---: | :---: |
| 1 | $D$ |
| 2 | $E$ |
| 3 | $K$ |
| 4 | $L$ |
| 5 | $M$ |
| 6 | $P$ |
| 7 | $Q$ |
| 8 |  |


| 3 | 0 |
| :--- | :--- |
| 3 | 3 |
| 2 | 6 |

proc 2


## Notes on AlltoAllv

- A receive buffer could potentially be as large as the sum of all send buffer sizes
- Care must be taken to coincide send counts with receive counts and displacements so data is not overwritten


## Today's Biz

1. Quick Review
2. Reminders
3. Parallel SCC
4. More Centrality
5. Even More MPI
6. More PageRank Tutorial

## More PageRank Tutorial

1. OpenMP - Work Queueing
2. MPI - Alltoallv Communication

More PageRank Tutorial Blank code and data available on website (Lecture 5)
www.cs.rpi.edu/~slotag/classes/FA16/index.html

