

# 15. Moment Labeling

## *Mechanics of Manipulation*

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# Outline.

Review: where are we? Where are we going?

Moment labeling: practical introduction.

Moment labeling: a more formal approach.

Relation to PCC's in wrench space.

Examples.

# Where are we?

Wrench and twist spaces.

- Chasles' theorem: rigid body motion is a twist.
- Poincot's theorem: forces on rigid body are a wrench

Polyhedral convex cones.

- Edge representation:  $\text{pos}(\{\mathbf{e}_i\})$
- Face representation:  $\cap\{\text{half}(\mathbf{n}_i)\}$
- Supplementary cone:  $\text{pos}(\{\mathbf{n}_i\})$  or  $\cap\{\text{half}(\mathbf{e}_i)\}$

Oriented plane.

- Rays in planar homogeneous coordinate space  $(x, y, w)$ .
- Project to  $w = 1$  plane, with  $+$  or  $-$  to remember sign of  $w$ .
- Convexity: interior and exterior line segments.

# Where are we going?

Graphical techniques for planar contact problems:

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	<b>Project cone to oriented plane</b>	<b>Project supplementary cone to oriented plane</b>
single wrench	(acc'n center)	line of action
single diff'l twist	IC	?
wrench cone	force dual	moment labeling
diff'l twist cone	Reuleaux	?

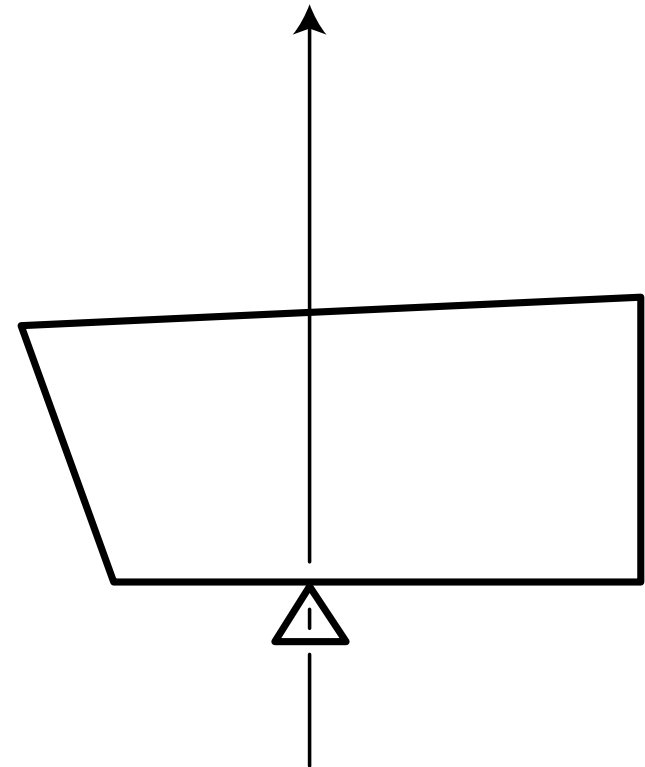
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# Possible resultants for one contact.

For a frictionless contact, force is along contact normal, magnitude is indeterminate.

I.e., the (directed) line of force is determined.

Another definition of line of force: locus of zero moment points.



# Another way of drawing the line of force.

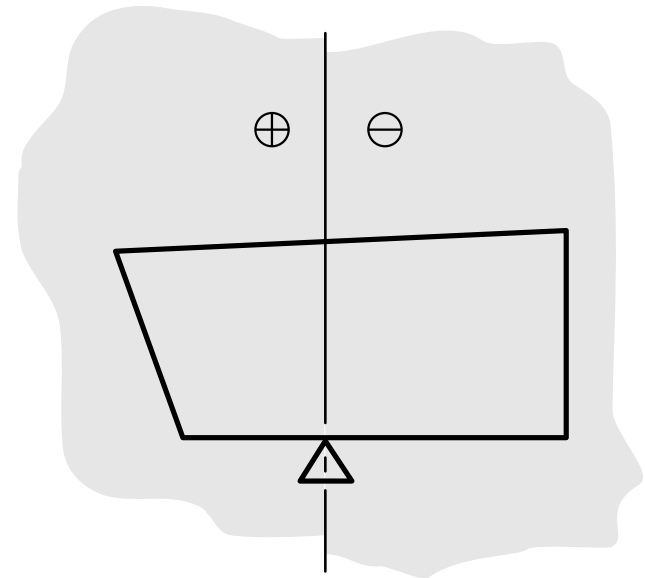
If we indicate sign of moment at each point ...

... half plane of  $\oplus$  to the left;

... half plane of  $\ominus$  to the right;

zero moment points are the boundary.

Gives line of force *and* its direction.

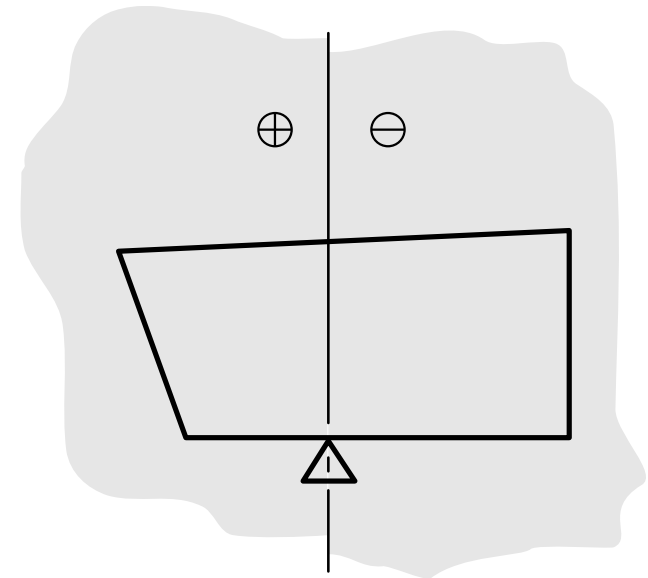


# Reuleaux's method has *two* interpretations!

The cone of differential twists reciprocal or repelling to a given wrench.

The cone of wrenches in the positive linear span of a given wrench.

*Reciprocal or repelling* turns any wrench-PCC rep'n into a twist-PCC rep'n.





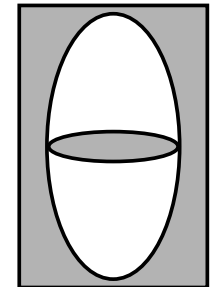
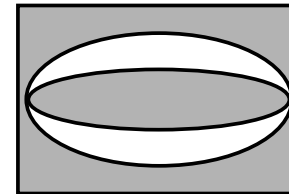
# Superiority of wrench interpretation.

Actually the wrench interpretation is more useful.

Differential twist cone is only first order approximation.

Wrench interpretation extends to include friction.

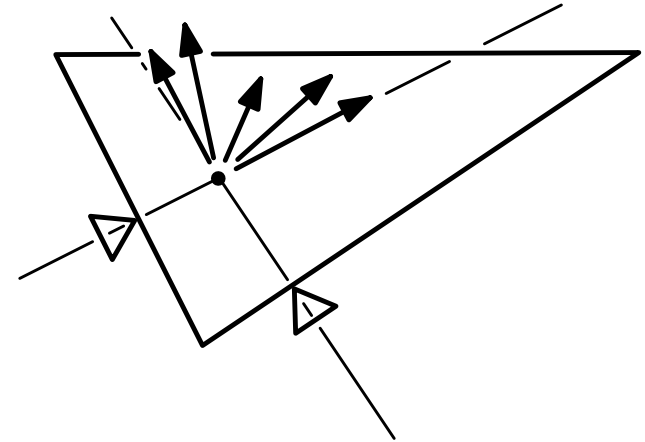
Wrench interpretation extends beyond convex cones.



# Possible resultants of *two* contacts.

Remember how to construct resultant of two forces in the plane?

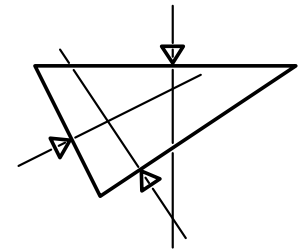
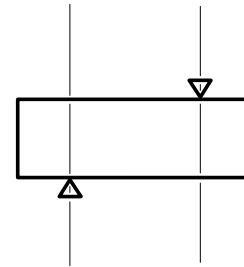
We can represent this set of resultants by a cone drawn in the plane.



# Some more challenging problems

How do we deal with these cases:

- Possible resultants of two *parallel* contacts?
- Possible resultants of three contacts?

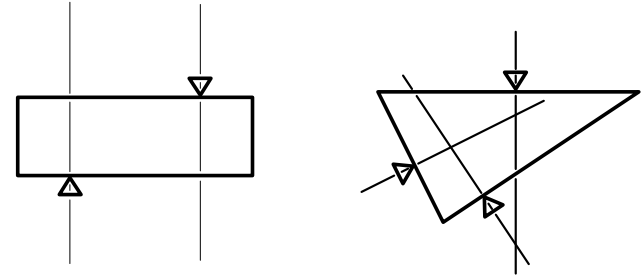


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Let's try Reuleaux's method and reinterpret.

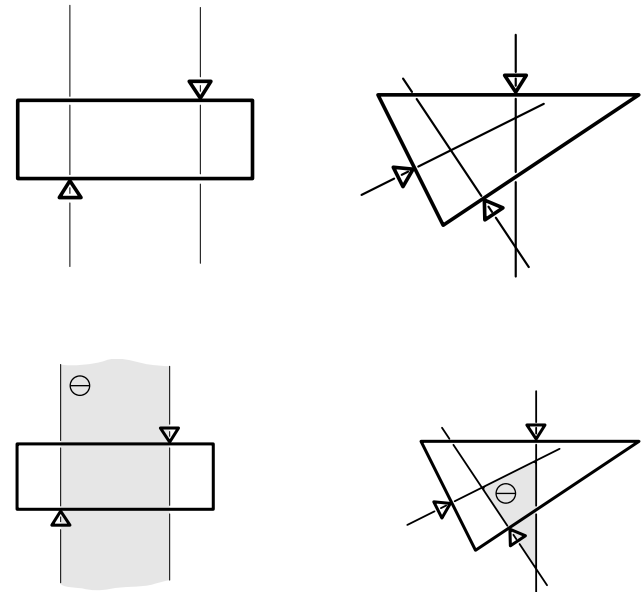


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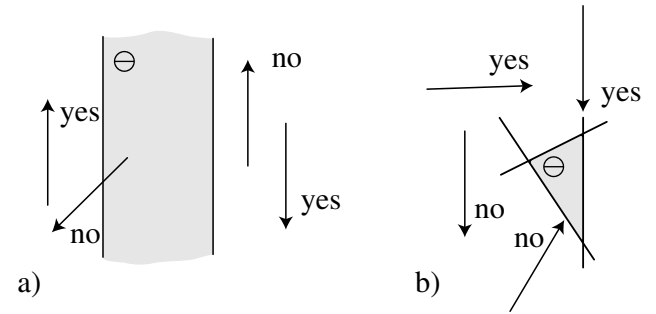
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# Interpreting the labeled regions.

The set of all forces that go between  $\oplus$  and  $\ominus$ , in the right direction.

They can graze, but cannot go through the interior.



# More formally

Given a set of contact wrenches  $\{w_i\}$ ,

Let  $\oplus_i$  and  $\ominus_i$  be the points of nonnegative and nonpositive moments, respectively, for contact  $w_i$ ,

Let  $\oplus$  be the intersection of all nonnegative regions

$$\oplus = \bigcap \oplus_i$$

and let  $\ominus$  be the intersection of all nonpositive regions

$$\ominus = \bigcap \ominus_i$$

Then the possible resultants  $\text{pos}(\{w_i\})$  is the set of all wrenches making nonnegative moments with all points in  $\oplus$  and nonpositive moments with all points in  $\ominus$ .

# Why does it work?

If, for example, all  $w_i$  give positive moments with respect to some point,

Then so does any wrench of the form

$$\sum k_i w_i$$

if all the  $k_i$  are nonnegative.

So, represent every  $w_i$  by where it *can't* go,

Intersect to determine where *none* of the  $w_i$  can go,

That's where wrenches  $\text{pos}(\{w_i\})$  cannot go.



# Examples

Sliding friction (preview).

Two facing cones.

Disk in concrete.

GOAT against step.

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