

# 18. Force Dual

## *Mechanics of Manipulation*

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# Outline.

Finish planar sliding.

Review representation of polyhedral convex cones in wrench/twist space.

Duality between points and lines.

Extension to oriented plane.

Examples.

# Planar sliding, so far

We derived force and torque for planar sliding:

$$\mathbf{f}_f = -\mu \operatorname{sgn}(\dot{\theta}) \hat{\mathbf{k}} \times \int_R \frac{\mathbf{r} - \mathbf{r}_{\text{IC}}}{|\mathbf{r} - \mathbf{r}_{\text{IC}}|} p(\mathbf{r}) dA$$
$$n_{fz} = -\mu \operatorname{sgn}(\dot{\theta}) \int_R \mathbf{r} \cdot \frac{\mathbf{r} - \mathbf{r}_{\text{IC}}}{|\mathbf{r} - \mathbf{r}_{\text{IC}}|} p(\mathbf{r}) dA$$

We noted a simpler expression for translational sliding:

$$\mathbf{f}_f = -\mu \frac{\mathbf{v}}{|\mathbf{v}|} f_0$$
$$\mathbf{n}_f = \mathbf{r}_0 \times \mathbf{f}_f$$

where  $\mathbf{r}_0$  is the **center of friction**

We observed that force and torque are undetermined when  $p(x)$  is undetermined.

# Planar sliding: limit surface

To explore mapping of planar sliding motion to force we use the **Limit Surface**.

Assume pressure distribution is known, and not necessarily finite.

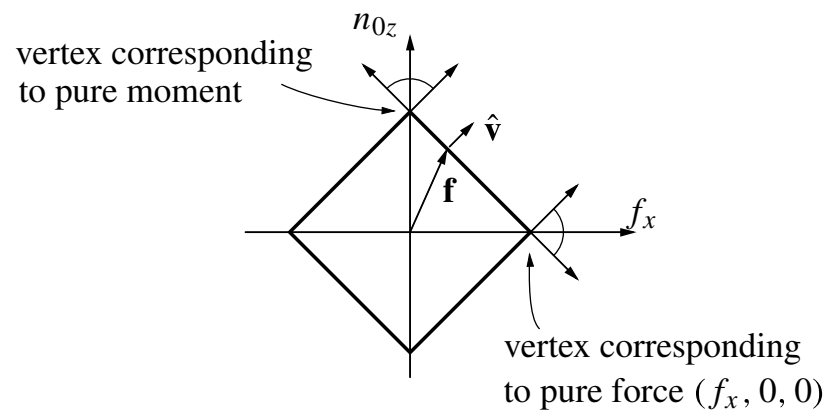
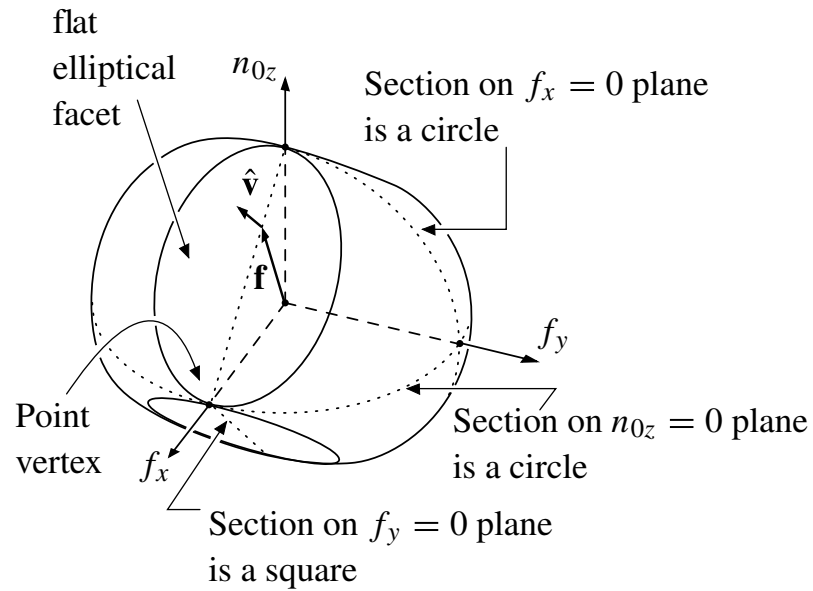
Define **frictional load** as wrench applied by slider to ground.

Define **Limit Surface** as boundary of set of all possible load wrenches  $\mathbf{p}^*$ , constrained only to satisfy Coulomb's law locally.

Derive **maximum power inequality**: the frictional load wrench yields maximum power over all wrenches in the limit surface.

Equivalently: during slip the total fric-

# Barbell Limit Surface



# LS properties

The barbell LS illustrates some properties that hold generally:

Closed, convex, enclosing the origin of wrench space.

Symmetric when reflected through origin.

Orthogonal projection onto the  $f_x, f_y$  plane is a circle of radius  $\sum \mu f_n$ .

Each discrete point of support yields two antipodal flat facets. On each facet several loads map to one motion (rotation about the support point.)

(No discrete points: LS is strictly convex and load-motion mapping is one-to-one.)

*Collinear* discrete support is even weirder: vertices on LS where one load maps to several velocities (rotation about point collinear with support).

# Revisiting representation of PCCs of wrenches and twists

Why polyhedral convex cones in wrench or twist space?

Possible wrenches resulting from frictional or frictionless contacts. (Positive linear span  $\text{pos}(\{\mathbf{w}_i\})$ . Edge representation of a cone.)

Twists consistent with constraints. (Intersection of half spaces reciprocal or repelling to the constraint  $\bigcap \text{half}(\mathbf{c}_i)$ . Face representation of a cone.)

For 3 space (6D wrench or twist space) represent them by the edges or by the face normals.

For the plane (3D wrench or twist space) we can use 2D graphical techniques:

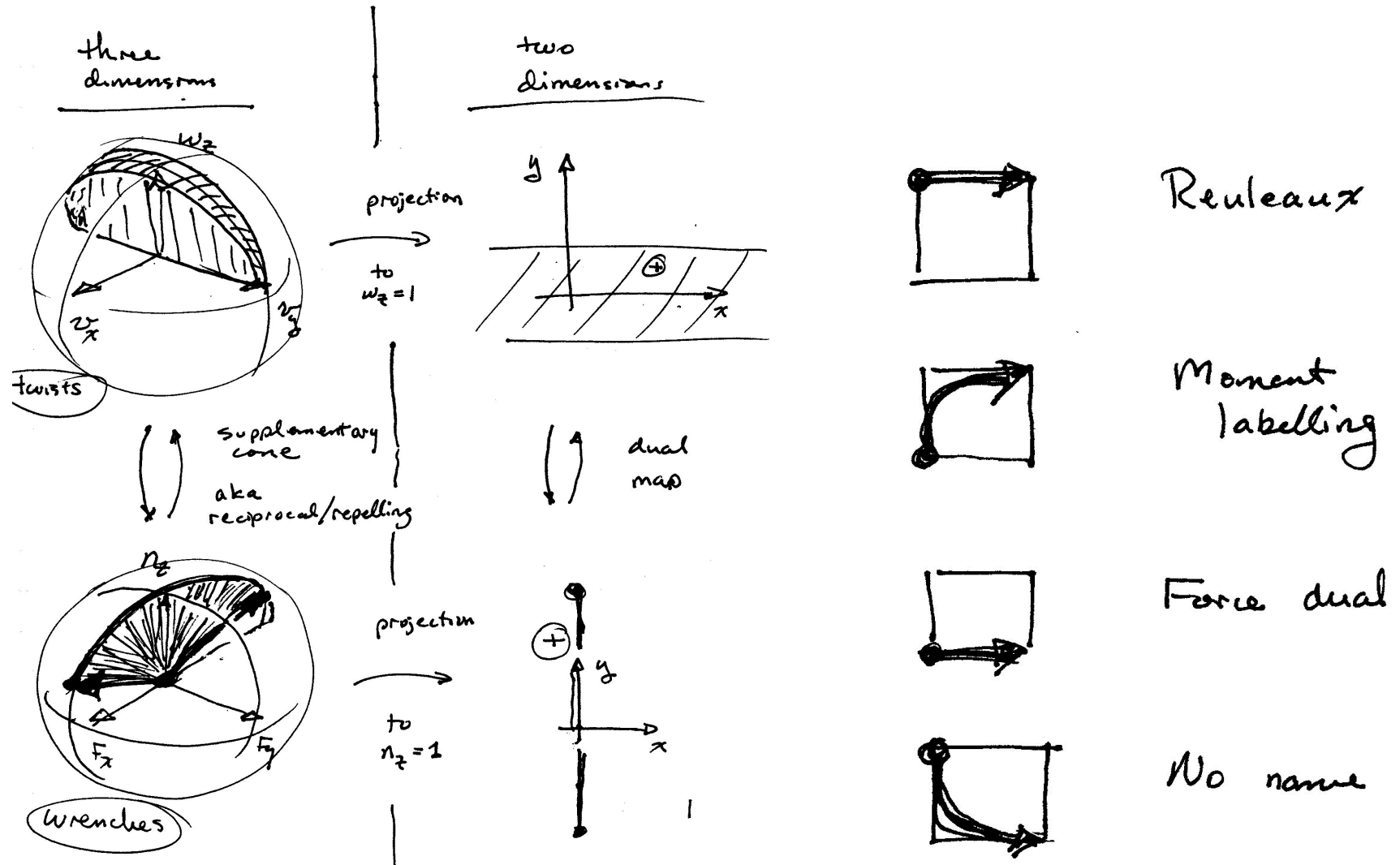
Reuleaux's method. Label rotation centers. Equivalent to projection of twists to oriented plane.

Moment labeling. Label moments. Equivalent to ...

Force dual ...



# Roadmap to graphical techniques



# Duality in the projective plane

Recall that for the projective plane there is a duality between *point* and *line*

We can make that concrete by defining a mapping  $D$ .

Define  $D(l)$  of a line  $l$  to be the point  $p$  such that  $O_p \dots$

Define  $D(p)$  of a point  $p$  to be the locus of  $D(l)$  for all  $l$  through  $p$ .

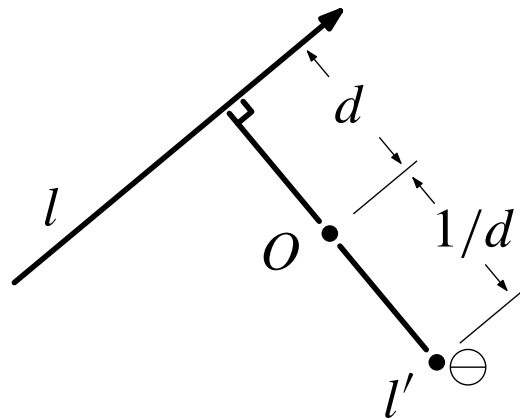
Note  $D(p)$  is a line, and  $D(D(p))$  is  $p$ .

Note what happens at infinity.

Note it depends totally on choice of scale and origin.

Check out the movies.

# Construction of force dual



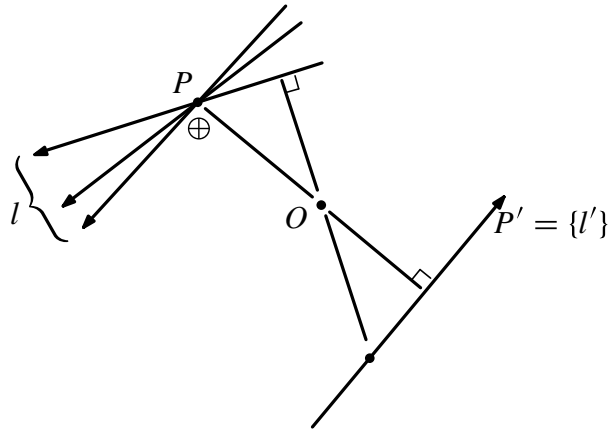
Given a (directed) line of force, and an origin;

Construct perpendicular through origin;

Take point on perpendicular, at distance inverse to moment arm;

Note the sign of the moment.

# Dual of a signed point



We defined map of directed line to a signed point.

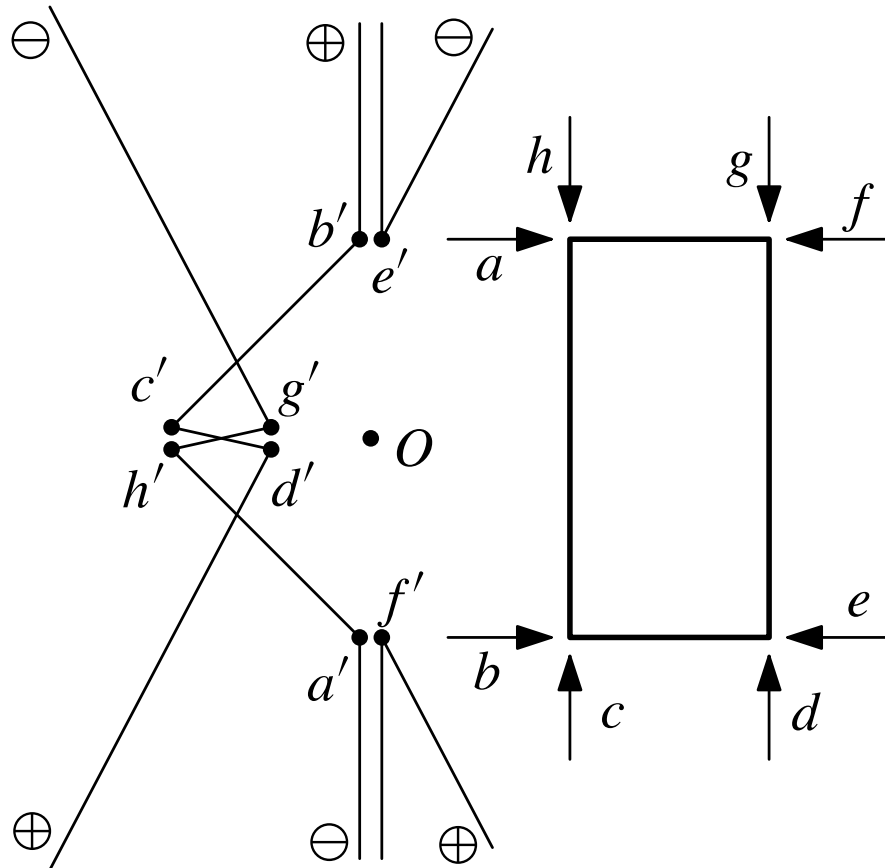
Extend definition to map signed points to something.

- Given signed point  $P$ , let  $\{l\}$  be the set of directed lines through  $P$ .
- Define  $P'$  is defined to be  $\{l'\}$ , with a direction determined by the sign of  $P$ .
- Note that A simple geometric
  - $P'$  is a directed line
  - $P'' = P$

Hence the transformation is *dual*.



# Zigzag locus



Force dual can represent *non-convex cones!*

Example: The set of contact normals.

Also known as the set of frictionless contact forces.

Force dual is called the *zigzag locus*.

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