18. Force Dual

Mechanics of Manipulation

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Outline.

Finish planar sliding.

Review representation of polyhedral convex cones in wrench/twist space.

Duality between points and lines.

Extension to oriented plane.

Examples.

Planar sliding, so far

We derived force and torque for planar sliding:

$$\mathbf{f}_f = -\mu \ \mathrm{sgn}(\dot\theta)\,\hat{\mathbf{k}} \times \int_R \frac{\mathbf{r}-\mathbf{r}_{\mathrm{IC}}}{|\mathbf{r}-\mathbf{r}_{\mathrm{IC}}|} p(\mathbf{r})\, dA$$

$$n_{fz} = -\mu \ \mathrm{sgn}(\dot\theta) \int_R \mathbf{r} \cdot \frac{\mathbf{r}-\mathbf{r}_{\mathrm{IC}}}{|\mathbf{r}-\mathbf{r}_{\mathrm{IC}}|} p(\mathbf{r})\, dA$$

We noted a simpler expression for translational sliding:

$$\mathbf{f}_f = -\mu \frac{\mathbf{v}}{|\mathbf{v}|} f_0$$

$$\mathbf{n}_f = \mathbf{r}_0 \times \mathbf{f}_f$$

where $\mathbf{r}_0$ is the **center of friction**

We observed that force and torque are undetermined when $p(x)$ is undetermined.

Planar sliding: limit surface

To explore mapping of planar sliding motion to force we use the **Limit Surface**.

Assume pressure distribution is known, and not necessarily finite.

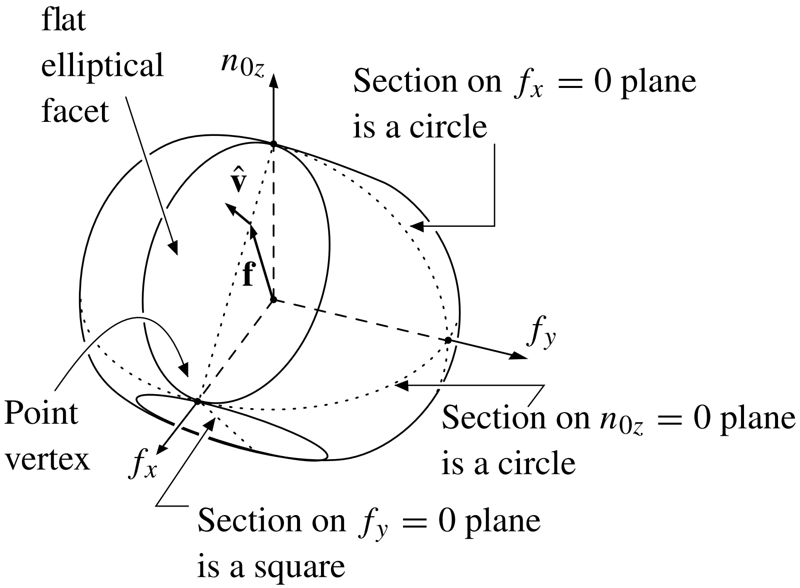
Define **frictional load** as wrench applied by slider to ground.

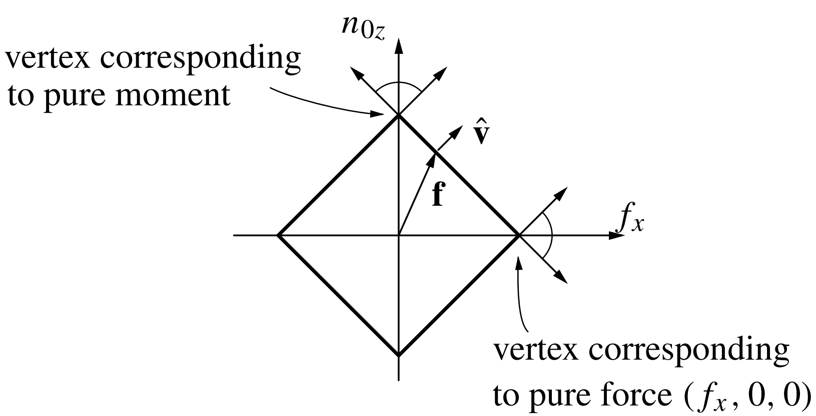
Define **Limit Surface** as boundary of set of all possible load wrenches $\mathbf{p}^*$, constrained only to satisfy Coulomb's law locally.

Derive **maximum power inequality**: the frictional load wrench yields maximum power over all wrenches in the limit surface.

Equivalently: during slip the total fric-

Barbell Limit Surface





LS properties

The barbell LS illustrates some properties that hold generally: Closed, convex, enclosing the origin of wrench space.

Symmetric when reflected through origin.

Orthogonal projection onto the f_x,f_y plane is a circle of radius $\sum \mu f_n$.

Each discrete point of support yields two antipodal flat facets. On each facet several loads map to one motion (rotation about the support point.)

(No discrete points: LS is strictly convex and load-motion mapping is one-to-one.)

Collinear discrete support is even weirder: vertices on LS where one load maps to several velocities (rotation about point collinear with support).

Revisiting representation of PCCs of wrenches ar

Why polyhedral convex cones in wrench or twist space?

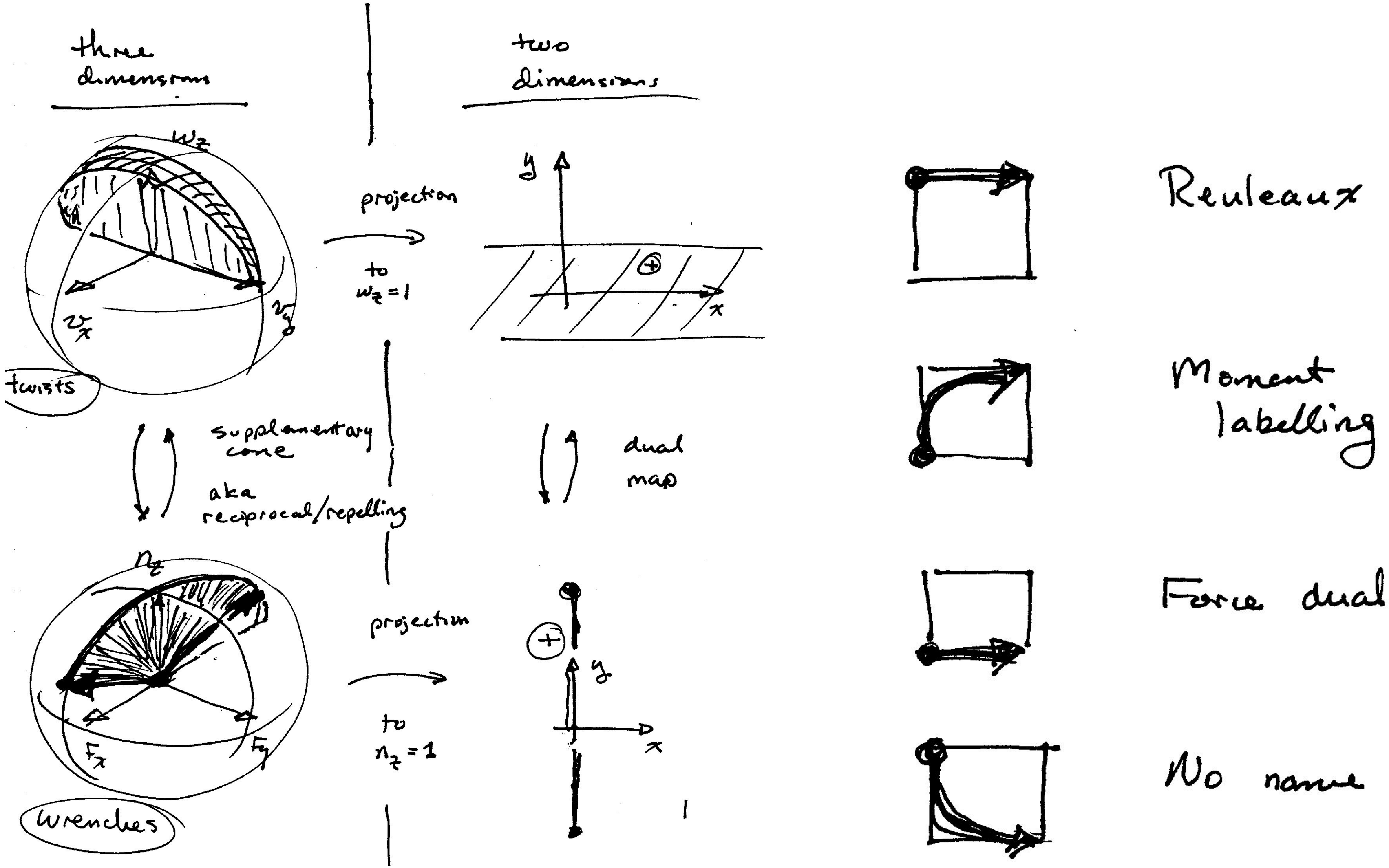
- Possible wrenches resulting from frictional or frictionless contacts. (Positive linear span $\mathrm{pos}(\{\mathbf{w}_i\})$. Edge representation of a cone.)
- Twists consistent with constraints. (Intersection of half spaces reciprocal or repelling to the constraint $\Cap\, \mathrm{half}(\mathbf{c}_i)$. Face representation of a cone.)

For 3 space (6D wrench or twist space) represent them by the edges or by the face normals.

For the plane (3D wrench or twist space) we can use 2D graphical techniques:

- Reuleaux's method. Label rotation centers. Equivalent to projection of twists to oriented plane.
- Moment labeling. Label moments. Equivalent to . . .
- Force dual . . . .

Roadmap to graphical techniques



Duality in the projective plane

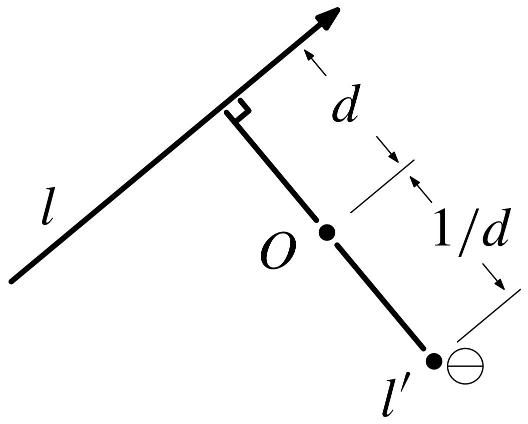
Recall that for the projective plane there is a duality between *point* and *line*

We can make that concrete by defining a mapping D.

> Define $D(l)$ of a line l to be the point p such that $Op \ldots$
>
> Define $D(p)$ of a point p to be the locus of $D(l)$ for all l through p.
>
> Note $D(p)$ is a line, and $D(D(p))$ is p.
>
> Note what happens at infinity.
>
> Note it depends totally on choice of scale and origin.

Check out the movies.

Construction of force dual



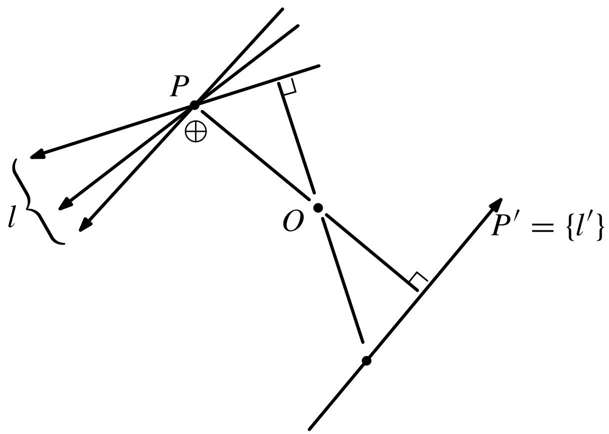
Given a (directed) line of force, and an origin;

Construct perpendicular through origin;

Take point on perpendicular, at distance inverse to moment arm;

Note the sign of the moment.

Dual of a signed point



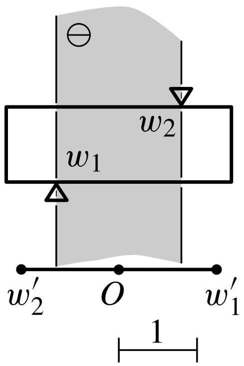
We defined map of directed line to a signed point.

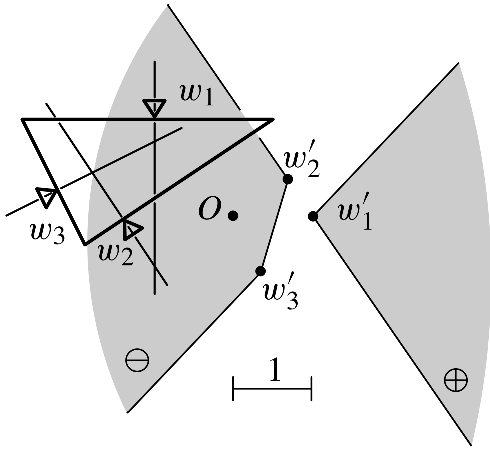
Extend definition to map signed points to something.

- Given signed point P, let $\{l\}$ be the set of directed lines through P.
- Define P' is defined to be $\{l'\}$, with a direction determined by the sign of P.
- Note that A simple geometric
  - P' is a directed line
  - $P'' = P$

  Hence the transformation is *dual*.

Representing wrench cones

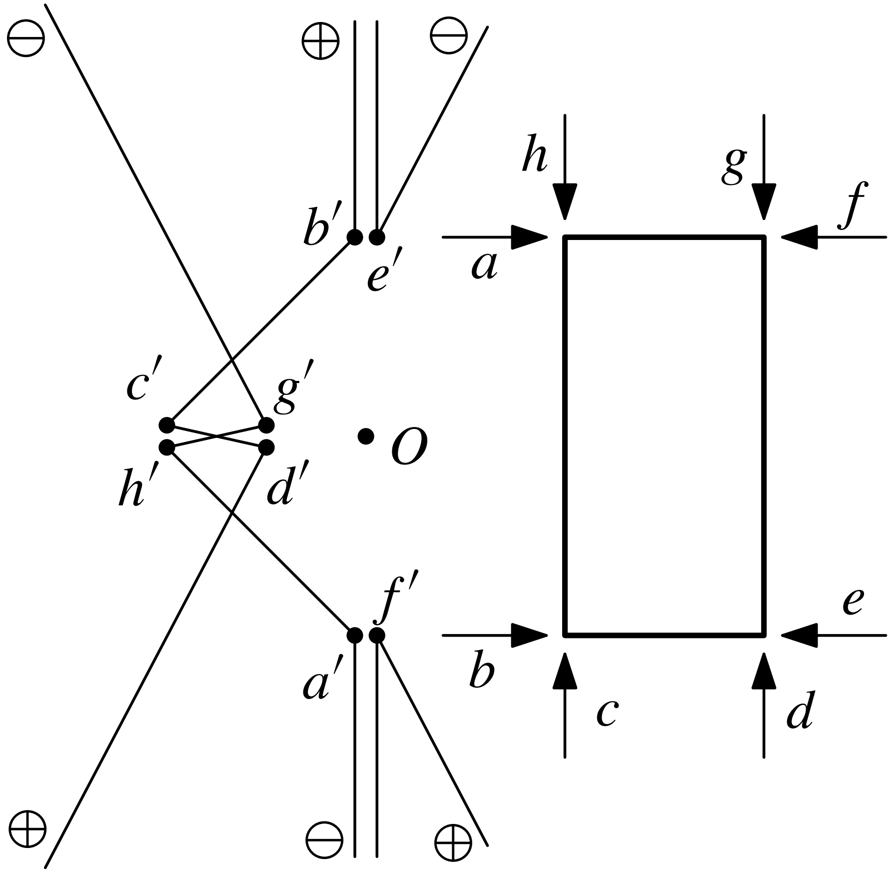




The method:

1. Choose origin and unit length.
2. Construct dual of each line of action.
3. Take the convex hull.

Zigzag locus



Force dual can represent *non-convex cones*!

Example: The set of contact normals.

Also known as the set of frictionless contact forces.

Force dual is called the *zigzag locus*.