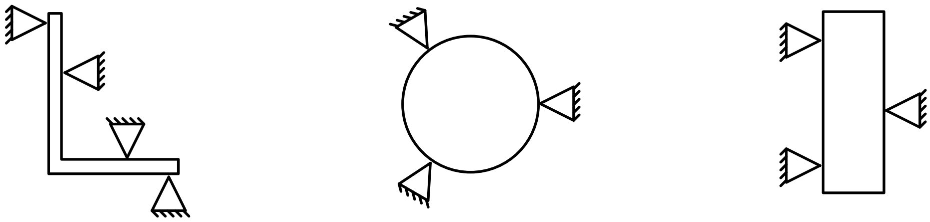
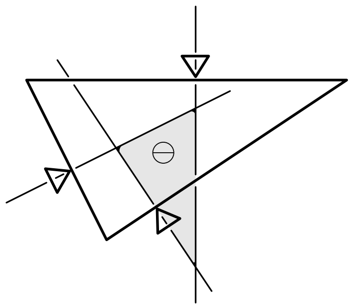
16-741 Mechanics of Manipulation
Spring term, 2004

Assignment 2, Out: Groundhog's Day, 2004 — Due: 9 February 2003

1. Exercise 2.8 from the text: apply Reuleaux's method to the problems drawn in the figure, identifying all possible motions of the constrained rigid body.



2. Exercise 2.9 from the text: Suppose we have placed three fingers on a triangle as in the figure. Before picking up the triangle, we want the triangle immobilized relative to the hand. Find a placement for a fourth finger that immobilizes the triangle. Apply Reuleaux's technique to prove your placement works, assuming that the fingers are perfectly stiff.



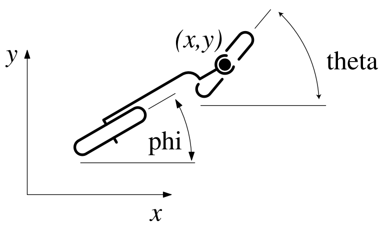
3. Exercise 2.10 from the text: Apply Frobenius's theorem to show that a rigid body in the plane with two independent Pfaffian constraints must be holonomic.

4. In the text and in lecture 5 we looked at the unicycle. If we were to have the unicycle pull a trailer, we would need an additional element in our configuration, ϕ, to indicate the angle of the trailer. The robot configuration can now be represented as $q = (x, y, \theta, \phi)$.

   (a) Give a common sense argument that the system is nonholonomic.

   (b) Use Lie brackets to show that the system is nonholonomic. The additional freedom obeys the equation

$$\dot{\phi} = \frac{1}{l}\sin(\theta - \phi)v$$



where v is the forward velocity of the unicycle, and l is the length of the trailer..

Your expression for $\dot{q}$ will therefore have one more row than that for the unicycle covered in the lecture notes. The vector fields g_1 and g_2 will also now have a fourth component.

(c) Give a *brief* physical description of the net motion described by the Lie bracket.

5. Exercise 3.16 from the text:
For this question you will conduct an experiment to answer the question: What is the average angle of rotations in $\mathbf{E}^3$?

   (a) Write code to generate uniformly distributed unit quaternions.

   (b) Write code to produce the smallest rotation angle for a given quaternion, in the range from zero to π.

   (c) Write code to generate a lot of unit quaternions, uniformly distributed, and take the mean angle.

   How does one produce a uniform distribution on the surface of a sphere in $\mathbf{E}^4$? One easy way is to generate four real numbers uniformly in the interval $[-1,1]$. That defines a uniform distribution in a cube. If we discard every quadruple with magnitude greater than 1, we will have a uniform distribution in the interior of the sphere. Normalize to get a uniform distribution on the surface of the sphere.