

# Final Exam

Tuesday, May 06, 2008

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## Robotics II Final - Spring 2008

### True / False Questions

- ① Cylindrical algebraic cell decomposition is equivalent to vertical cell decomposition.
- ② C-space of a triangle free to move in space ( $\mathbb{R}^3$ ) is not  $SE(3) = \mathbb{R}^3 \times SO(3)$ .
- ③ Some constraints of a Linear Complementarity problem are not linear in the unknowns.
- ④  $A^*$  search with cost-to-go function equal to zero, is equivalent to Dijkstra's algorithm.
- ⑤ Semi-Algebraic sets are composed of a finite # of unions & intersections of polynomial inequalities.

- ⑥ Randomized potential field methods were developed because deterministic potential field methods get stuck.
- ⑦ Sampling-based planning methods are particularly effective when C-space contains narrow passages between C-obstacles

### Short Answer Questions

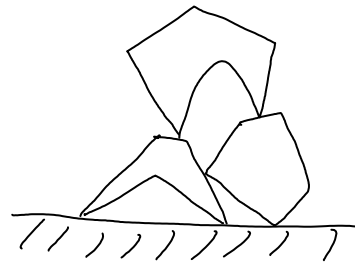
① In words, what is the configuration space of a system of bodies? (Hint: how would you decide if you had enough parameters and what is the dimension of C-space?)

② Someone claims to have a form closure grasp of a sphere using only 4 contacts.

Is there a way to think about the C-space of a sphere such that this claim is reasonable?

③ What is the size of the LCP needed to predict the motion of the <sup>planar</sup> system of bodies piled on the right?

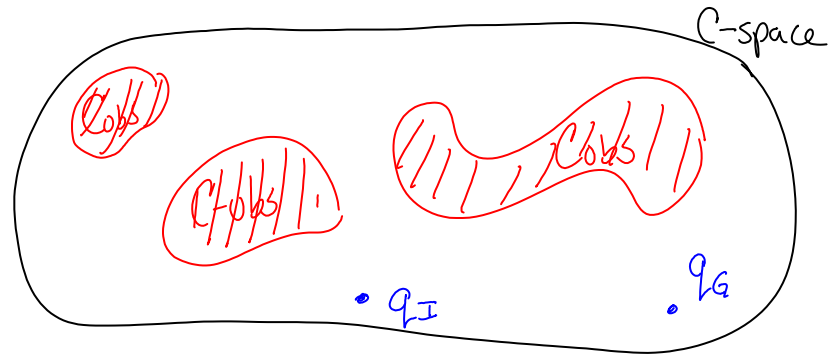
The three bodies are moveable and there are 6 contact points.



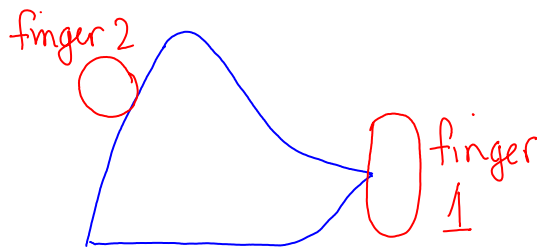
④ Why is the mobius strip a manifold (with boundary)?

⑤ In the 2D C-space shown, sketch solutions

from at least three different homotopy classes.

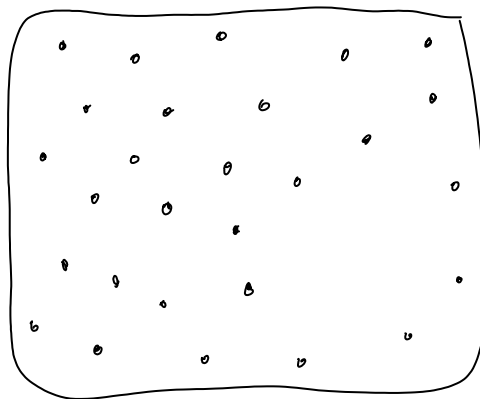


- ⑥ For the two-finger grasp of the object below, determine an approx range of placements of finger 2 (finger 1 remains fixed), such that the grasp has frictional form closure. Assume  $\mu = 1.0$ .

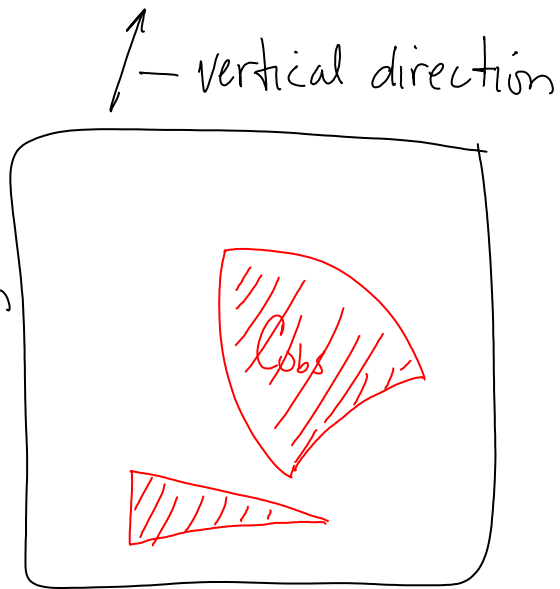


Assume  $\mu = 1.0$ .

- ⑦ For the samples shown in the unit "square" on the right, what is the approximate dispersion corresponding to the  $L_2$  norm?



⑧ For the C-space shown, apply the vertical cell decomposition method, then construct a roadmap of C-free.



## Analysis Questions

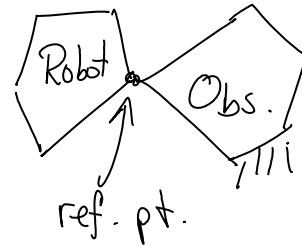
① Let  $P_1$  and  $P_2$  be convex polygons in a plane. Let  $n_1$  and  $n_2$  be the number of edges of  $P_1$  &  $P_2$ , respectively. Assume one polygon is a fixed obstacle and the other is moveable.

The C-space of the system is  $SE(2) = \mathbb{R}^2 \times S^1$

a.) Determine the number of 2-dimensional facets of  $C_{obs}$  in  $SE(2)$ . (Hint: 2-d facets arise from EV and VE contacts)

b.) Suppose the reference point on the robot is one of the vertices.

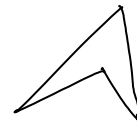
Derive a 1-D edge of  $C_{obs}$  corresponding to the ref. pt. in contact with a vertex of the obstacle



c.) Suppose one of the polygons is nonconvex with shape shown

Determine lower and upper

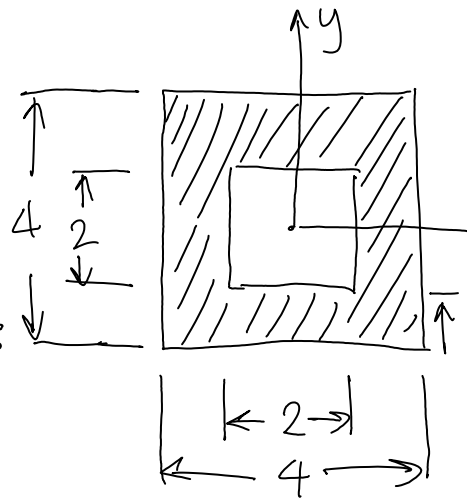
bounds on the # of 2D facets of  $C_{obs}$ .



- ② Let  $\bar{X}$  be a space and let  $x, x', x'' \in \bar{X}$  be points.  
Is the following a metric on  $\bar{X}$ ?

$$\rho(x, x') = \begin{cases} 1; & \forall x \neq x' \\ 0; & \text{if } x = x' \end{cases}$$

- ③ Derive primitives from linear inequalities and combine them with intersections and unions to represent the shaded area.



④ For the LCP  $(M, b)$ , with  $M = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$  and  $b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ , determine the values of  $b$  for which the LCP has no solution.

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