2008_final_soln

Tuesday, May 06, 2008
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Robotics II Final - Spring 2008

True / False Questions (14 points, 2 per question)

1. Cylindrical algebraic cell decomposition is equivalent to vertical cell decomposition.

   False

2. C-space of a triangle free to move in space ($\mathbb{R}^3$) is not $SE(3) = \mathbb{R}^3 \times SO(3)$.

   False

3. Some constraints of a Linear Complementarity problem are not linear in the unknowns.

   True, $z^T(Mz+b) = 0$ is quadratic in z

4. A^* search with cost-to-go function equal to zero, is equivalent to Dijkstra's algorithm.

   True

5. Semi-Algebraic sets are composed of a

finite # of unions & intersections of polynomial inequalities.

True

⑥ Randomized potential field methods were developed because deterministic potential field methods get stuck.

True

⑦ Sampling-based planning methods are particularly effective when C-space contains narrow passages between C-obstacles

False

Short Answer Questions (24 points, 3 per question)

① In words, what is the configuration space of a system of bodies? (Hint: how would you decide if you had enough parameters and what is the dimension of C-space?)

The C-space is the space of parameter values used to describe the position & orientation of every body in the system. The dimension of C-space is equal to the number of degrees of freedom in the system.

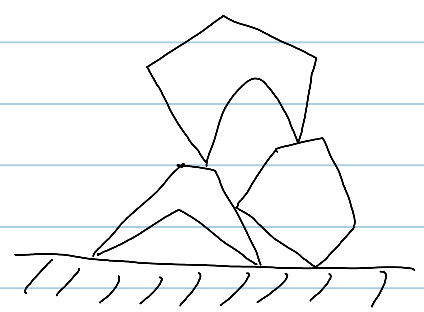
② Someone claims to have a form closure grasp of a sphere using only 4 contacts.

Is there a way to think about the C-space of a sphere such that this claim is reasonable?

If one assumes that it is impossible to detect changes in the orientation of the sphere, then its only degrees of freedom are translational, then only $3+1=4$ contacts are needed for form closure.

③ What is the size of the LCP needed to predict the motion of the planar system of bodies piled on the right?

The three bodies are moveable and there are 6 contact points.



The unknown appearing in the LCP are:

$\underbrace{\nu_1}, \underbrace{\nu_2}, \underbrace{\nu_3}, \underbrace{\lambda_n}, \underbrace{\lambda_f}, \underbrace{\sigma}$

$$\underbrace{\nu_1, \nu_2, \nu_3}_{9}, \underbrace{\lambda_n}_{6\times 1}, \underbrace{\lambda_f}_{12\times 1}, \underbrace{\sigma}_{6\times 1}$$

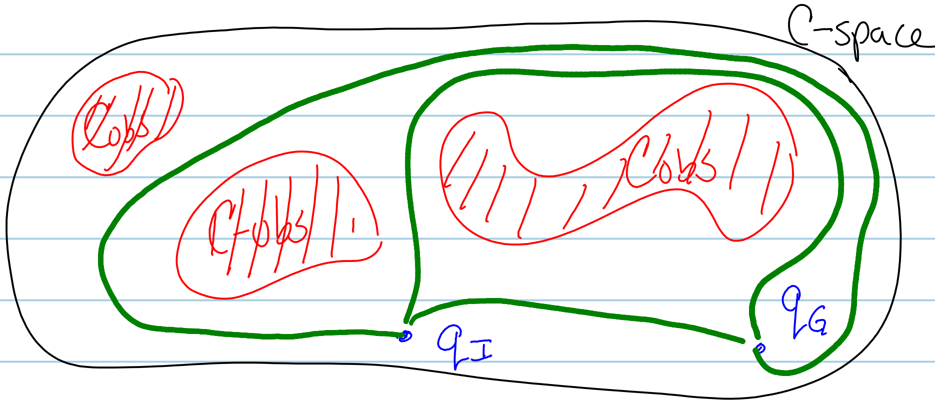
The mixed LCP is of size $\underline{\underline{33}}$.
(I accepted 24 if it was for $\lambda_n, \lambda_f, \sigma$)

4\. Why is the mobius strip a manifold (with boundary)?

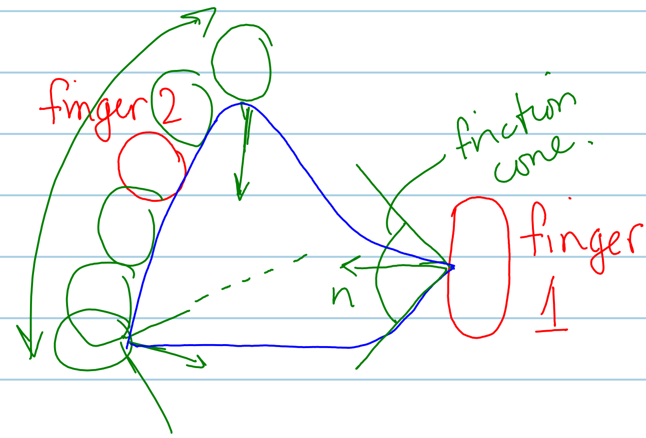
Every point on the interior appears locally as a point in $\mathbb{R}^2$. Every bndry point appears locally as a pt on bndry of a half plane in $\mathbb{R}^2$

5\. In the 2D C-space shown, sketch solutions from at least three different homotopy classes.

Path are shown in bold green lines



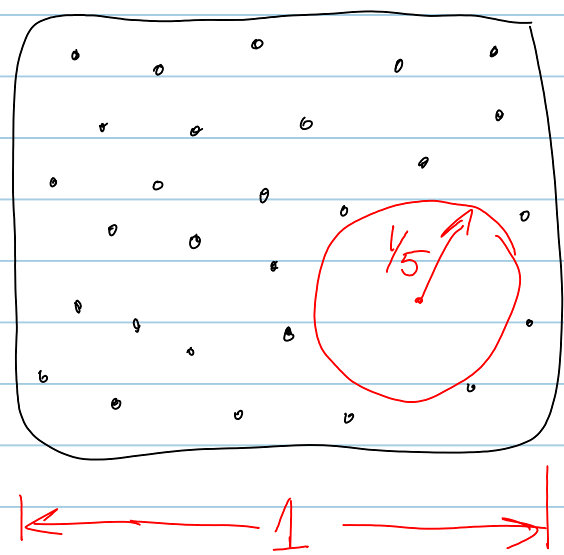
6\. For the two-finger grasp of the object below, determine an approx determine an approx range of placements of finger 2 (finger 1 remains fixed), such that the grasp has frictional form closure.



$\boxed{\text{Assume } \mu = 1.0}$

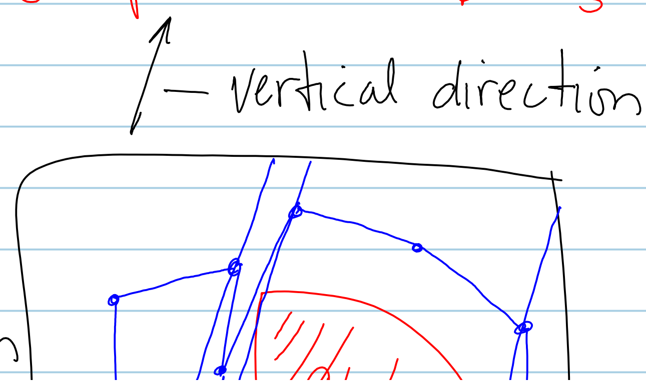
range of placements of finger 2 shown in green

⑦ For the samples shown in the unit "square" on the right, what is the approximate dispersion corresponding to the L_2 norm?

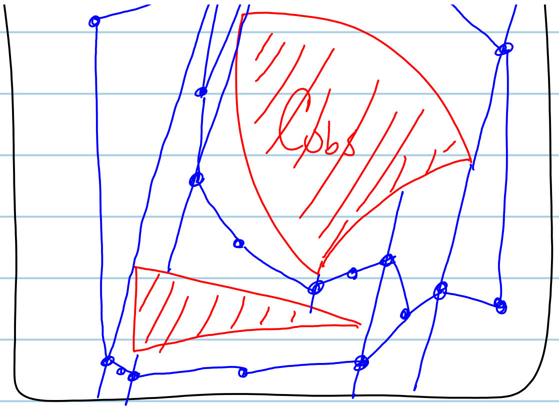


The dispersion is the radius of the largest disc not containing samples. It is about $\frac{1}{5}$.

⑧ For the C-space shown, apply the vertical cell decomposition



vertical cell decomposition method, then construct a roadmap of C-free.



The decomposition and one possible roadmap is shown in blue.

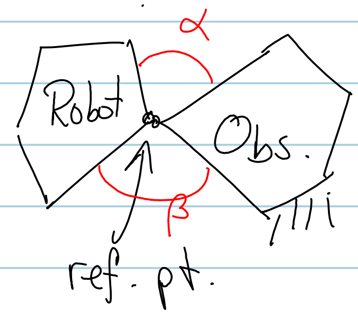
Analysis Questions (62 points)

15 pts (5 pts for each part)

1. Let P_1 and P_2 be convex polygons in a plane. Let n_1 and n_2 be the number of edges of P_1 & P_2, respectively. Assume one polygon is a fixed obstacle and the other is moveable.

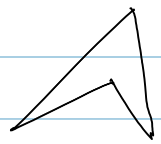
The C-space of the system is $SE(2) = \mathbb{R}^2 \times S^1$

a.) Determine the number of 2-dimensional facets of C_{obs} in $SE(2)$. (Hint: 2-d facets arise from EV and VE contacts) b.) Suppose the reference point on the robot is one of the vertices.



Derive a 1-D edge of Cobs corresponding to the ref. pt. in contact with a vertex of the obstacle

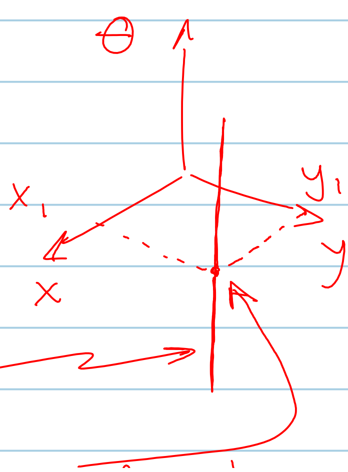
c.) Suppose one of the polygons in nonconvex with shape shown →



Determine lower and upper bounds on the # of 2-D facets of Cobs.

a.) EV contacts = $n_1 n_2$
VE contacts = $n_1 n_2$ $\Rightarrow$ $\boxed{2n_1 n_2}$ 2D facets

b.) The reference point remains fixed in the world at a given (x,y). Robot has only one D.O.F. — rotation

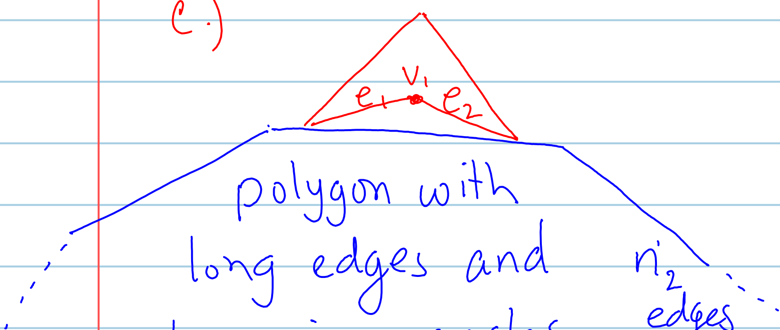


the edge of

the edge of the C-obstacle is a vertical line segment of length $\alpha+\beta$, passing thru (x_1, y_1) which is the location of the fixed vertex.

position of vertex on obstacle.

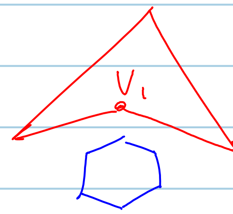
c.)



polygon with long edges and large inner angles so that no vertex can touch e_1 or e_2. Note that no edge can touch v_1. Contact is possible w/ 3 vertices and 2 edges. Therefore

$$3n_2 + 2n_2 = 5n_2$$

VE EV



all vertices of small polygon can touch all edges of quadrilateral. But edges of polygon cannot touch v_1. Therefore

$$4n_2 + 3n_2 = 7n_2$$

$$\boxed{5n_2 \leq \#\text{facets} \leq 7n_2}$$

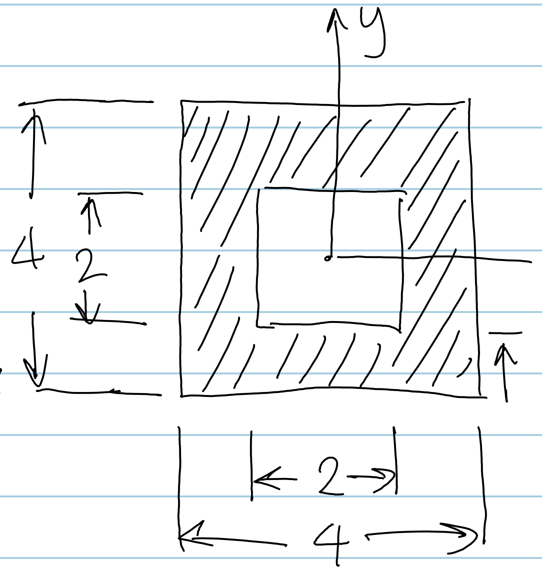
15pts (2.) Let X be a space and let $x, x', x'' \in X$ be points.

Is the following a metric on $\underline{X}$?

$$\rho(x, x') = \begin{cases} 1; & \forall \; x \neq x' \\ 0; & \text{if } x = x' \end{cases}$$

Yes.
Satisfies all properties: non-negativity, reflexivity, symmetry, triangle inequality

15 pts ③ Derive primitives from linear inequalities and combine them with intersections and unions to represent the shaded area.



outer box $(x \leq 2 \;\wedge\; x \geq -2 \;\wedge\; y \leq 2 \;\wedge\; y \geq -2)$

$\wedge$

$(x \geq 1 \;\vee\; x \leq -1 \;\vee\; y \geq 1 \;\vee\; y \leq -1)$

17 pts ④ For the LCP (M, b), with $M = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$, determine the values of b for which the LCP has no solution.

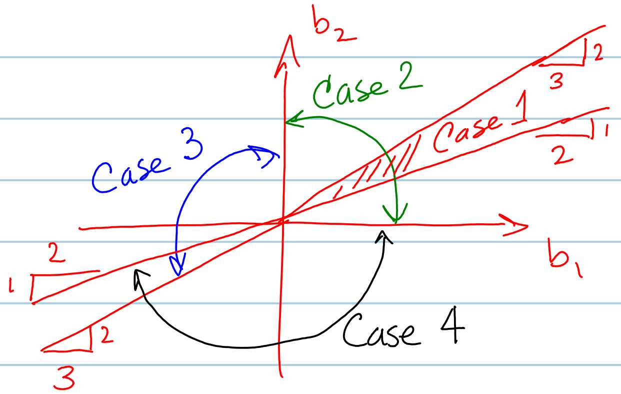
$$0 \le \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \perp \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \ge 0$$

Case 1: $\begin{bmatrix} + & 0 \\ + & 0 \end{bmatrix}$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \boxed{\begin{matrix} -b_1 + 2b_2 \ge 0 \\ 2b_1 - 3b_2 \ge 0 \end{matrix}}$$

Case 2: $\begin{bmatrix} 0 & + \\ 0 & + \end{bmatrix}$

$$\Rightarrow \boxed{\begin{matrix} b_1 \ge 0 \\ b_2 \ge 0 \end{matrix}}$$



Case 3: $\begin{bmatrix} + & 0 \\ 0 & + \end{bmatrix}$

$$3z_1 + \cancel{2z_2} + b_1 = 0 \Rightarrow b_1 = -3z_1 \Rightarrow z_1 = -\frac{b_1}{3} \ge 0$$

$$z_2 = 0 \quad \& \quad 2z_1 + \cancel{z_2} + b_2 \ge 0 \Rightarrow -\frac{2}{3} b_1 + b_2 \ge 0$$

$$0 \quad \Rightarrow \boxed{b_2 \ge \frac{2}{3} b_1} \qquad \boxed{b_1 \le 0}$$

Case 4: $\begin{bmatrix} 0 & + \\ + & 0 \end{bmatrix}$

$z_1 = 0 \Longrightarrow 3\cancel{z_1} + 2z_2 + b_1 \geq 0 \Longrightarrow -2b_2 + b_1 \geq 0 \Longrightarrow \boxed{b_2 \leq \frac{b_1}{2}}$

$2\cancel{z_1} + z_2 + b_2 = 0 \Longrightarrow z_2 = -b_2 \Longrightarrow \boxed{b_2 \leq 0}$

Note that each case has 2 inequalities in b_1, b_2.