

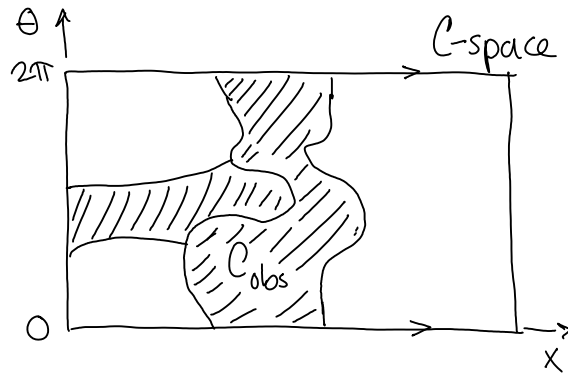
## Robotics II Final - Spring 2009

True / False Questions (14 pts total, 2 pts per question)

- ① Assume  $C = \mathbb{R}^2$  and  $C_{obs}$  is a polygon. Vertical decomposition of  $C_{free}$  will yield a simplicial complex covering  $C_{free}$ .
- ②  $C$ -space of a rigid body free to move in space ( $\mathbb{R}^3$ ) is not  $SE(3) = \mathbb{R}^3 \times SO(3)$ .
- ③ Some constraints of a Linear Complementarity problem are not linear in the unknowns.
- ④  $A^*$  search with cost-to-come function equal to zero, is equivalent to best-first search.
- ⑤ Semi-Algebraic sets are composed of a finite # of unions & intersections of polynomial inequalities.
- ⑥ Randomized potential field methods are

particular nice to use for a wide range of problems, because they do not require parameter tuning.

- ⑦ The  $\mathcal{C}$ -space at the right is  $I' \times S'$  ( $S' = [0, 2\pi] \setminus \sim$ ).  $\mathcal{C}_{\text{free}}$  has two components.



Short Answer Questions (21 pts total. 3 pts per question)

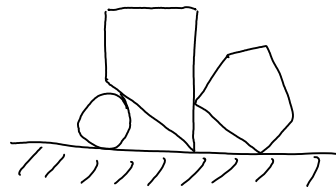
① (a) In words, what is the configuration space of a system of bodies?

(b) Assuming no obstacles, what does a curve in  $\mathcal{C}$ -space represent?

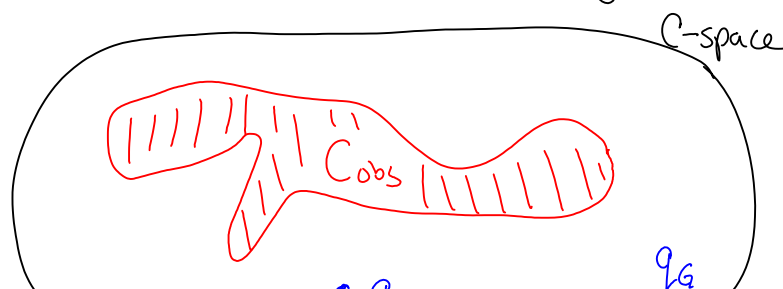
② Someone claiming to have a form closure grasp of a cylinder using 7 contacts.

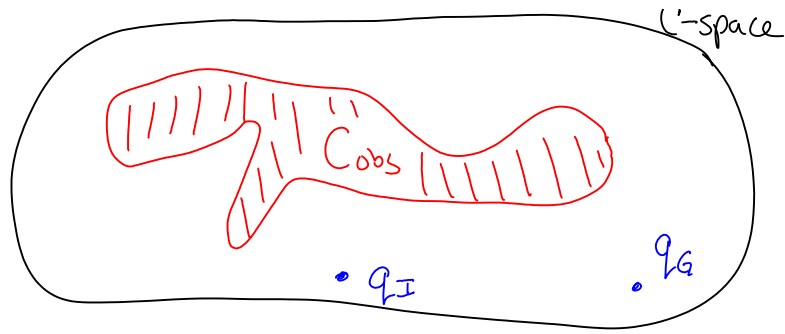
Is there a way to think about this problem such that the claim makes sense? Explain.

- ③ For the planar multibody system shown below, what are the unknown vectors and their sizes when applying the standard LCP (Stewart-Trinkle) time-stepper? (There are 5 unilateral contacts)

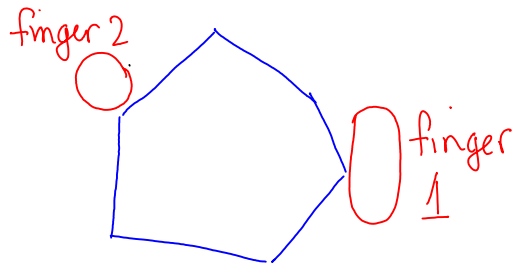


- ④ In the 2D C-space shown, sketch solutions from at least three different homotopy classes.



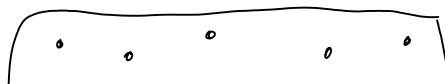


⑤ For the two-finger grasp of the object below, determine an approx range of placements of finger 2 (finger 1 remains fixed), such that the grasp has frictional form closure.

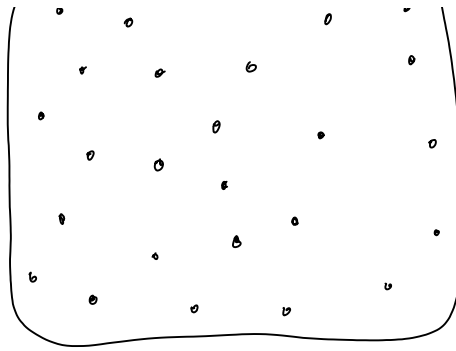


Assume  $\mu = 1.0$

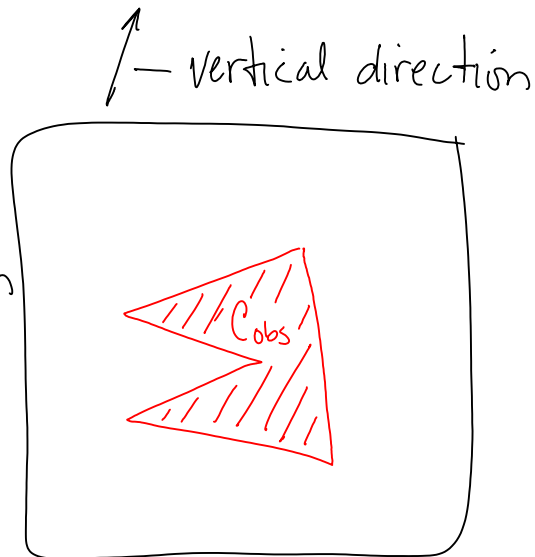
⑥ For the samples



6) For the samples shown in the unit "square" on the right, what is the approximate dispersion corresponding to the  $L_\infty$  norm?



7) (a) For the  $C$ -space shown, apply the vertical cell decomposition method,



(b) Add edges so that the decomposition of  $C_{free}$  is a simplicial complex.

Analysis Questions (65 pts total)

- ① Let  $P_1$  and  $P_2$  be convex polygons in a plane. Let  $n_1$  and  $n_2$  be the number of edges of  $P_1$  &  $P_2$ , respectively. Assume one polygon is a fixed obstacle and the other is moveable.

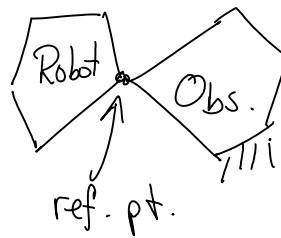
16  
pts

The C-space of the system is  $SE(2) = \mathbb{R}^2 \times S^1$

- a.) Determine the number of 2-dimensional facets of  $C_{obs}$  in  $SE(2)$ . (Hint: 2-d facets arise from EV and VE contacts)

- b.) Suppose the reference point on the robot is one of the vertices.

Derive a 1-D edge of  $C_{obs}$  corresponding to the ref. pt. in contact with a vertex of the obstacle



- c.) Suppose one of the polygons is nonconvex quadrilateral shown here →



Determine lower and upper bounds on the # of 2D facets of  $C_{obs}$ .

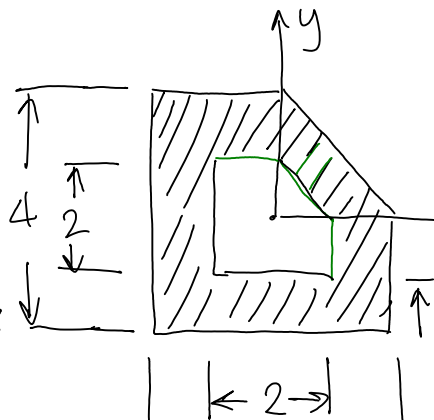
② Let  $\bar{X}$  be a space and let  $x, x', x'' \in \bar{X}$  be points.

③ Show that  $\rho(x, x')$  defined below is or is not a metric

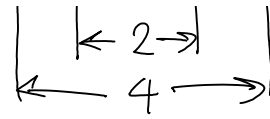
$$\rho(x, x') = (\Delta x)^2 + \Delta x$$

$$\text{where } \Delta x = x - x'$$

③ Derive primitives from linear inequalities and combine them with intersections and unions to represent the shaded area.



to represent the shaded area.



(4) For the LCP  $(M, b)$ , with  $M = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$  and  
(17 pts)  $b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ , determine the values of  $b$   
for which the LCP has no solution.