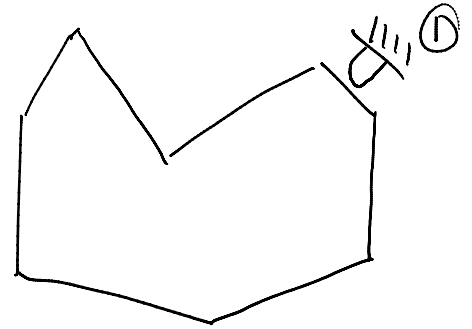


(20 pts)

1.) A planar object is grasped with two hard fingers. The coefficient of friction at both contact points is 1.0.

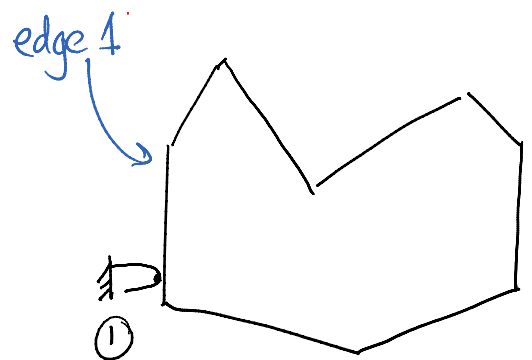
(12 pts)

a.) Find a location for contact ② such that a 2-fingered grasp has frictional form closure.



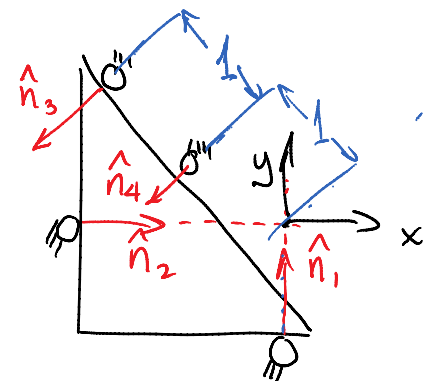
(8 pts)

b.) You know contact ① is somewhere on edge 1, but its precise location is not known. Find a finite region on the polygon such that placing contact ② any where in that region, will form a grasp with frictional form closure.



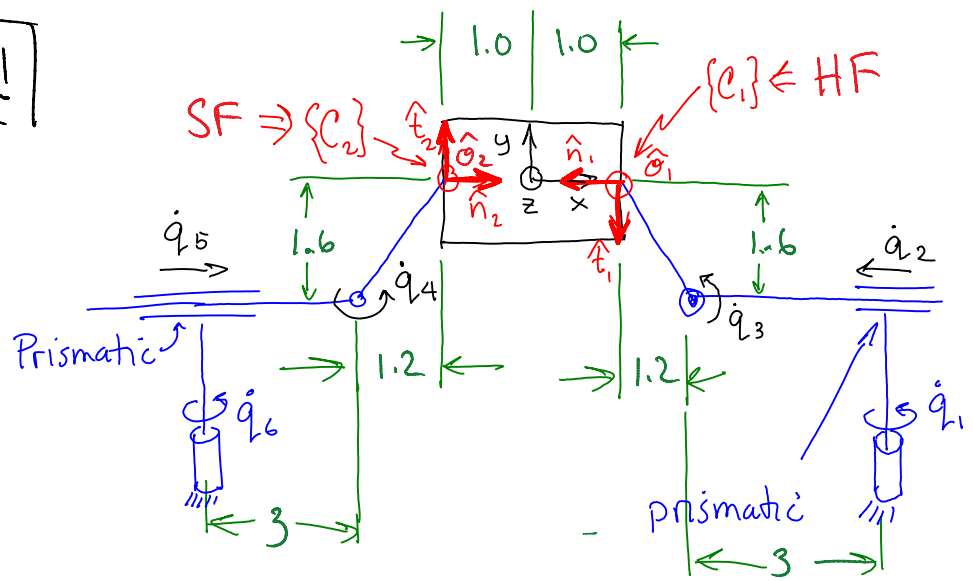
10 points

2.) Show analytically that the grasp shown on the right does not have form closure.



30pts

3.) 3D Problem!



Contact 1 (on left) is a soft finger contact.
 Contact 2 (on right) is a hard finger contact.

10pts

a.) Construct G & J using the (x-y-z) reference frame shown.

10 pts

b.) For the correct G and J , bases of the four null

spaces are:

$$N(G) = \begin{bmatrix} -1 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

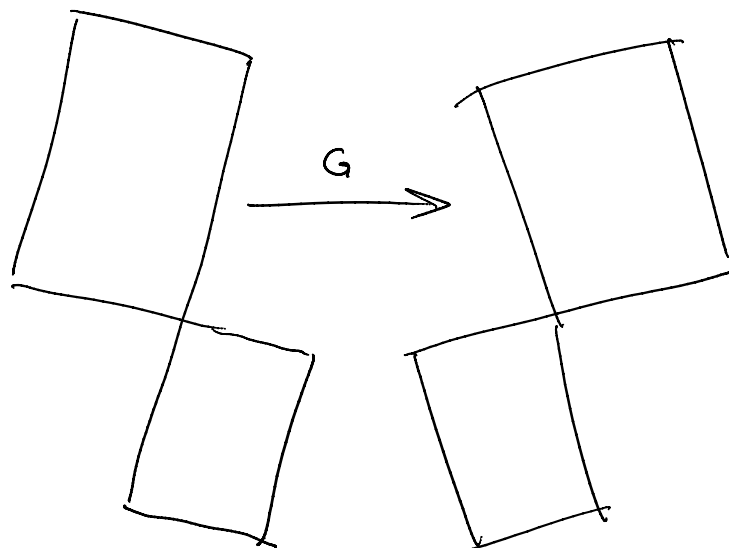
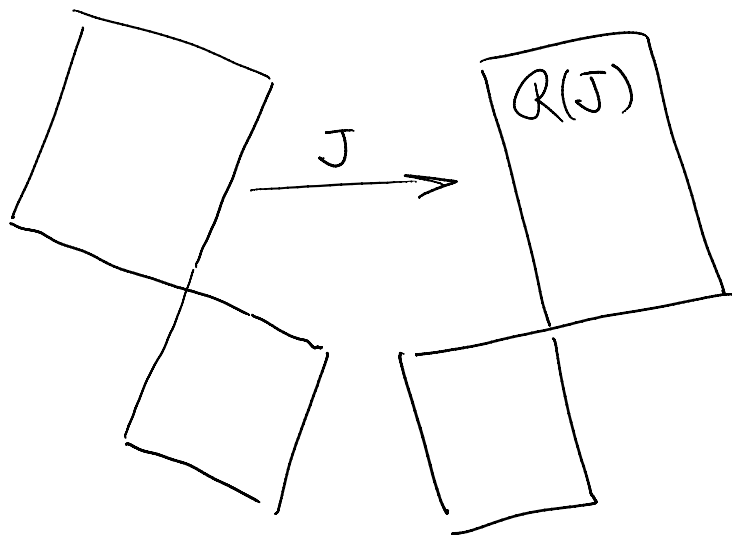
$$N(G^T) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$N(J) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$N(J^T) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

Complete the picture below, i.e. identify the dimensions of the various subspaces.

$q, r \in \mathbb{R}$



$v, g_{app} \in \mathbb{R}$

For the next two problems you might find the following quantities helpful.

$$J^T G^+ = \begin{bmatrix} 0 & 0 & 2.1 & 0 & -2.1 & 0 \\ -0.5 & 0 & 0 & 0 & 0 & 0 \\ -0.8 & -0.6 & 0 & 0 & 0 & -0.6 \\ -0.8 & 0.6 & 0 & 0 & 0 & -0.6 \\ 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2.1 & 0 & -2.1 & 0 \end{bmatrix} \quad G N(J^T) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$G(J^T)^+ = \begin{bmatrix} 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 4/3 & -5/6 & 5/6 & 4/3 & 0 \\ 5/21 & 0 & 0 & 0 & 0 & -5/21 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -5/21 & 0 & 0 & 0 & 0 & -5/21 \\ 0 & 4/3 & -5/6 & -5/6 & -4/3 & 0 \end{bmatrix} \quad J^T N(G) = \begin{bmatrix} 0 \\ -\sqrt{2}/2 \\ -1.131 \\ 1.131 \\ -\sqrt{2}/2 \\ 0 \end{bmatrix}$$

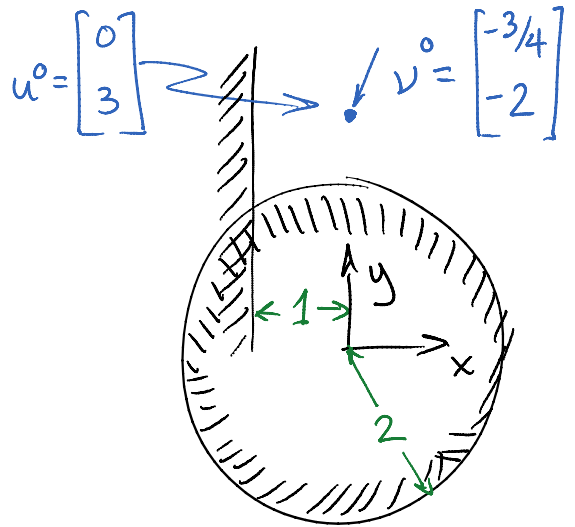
e.) ^{5 pts} Use the relationships $\tau = J^T \lambda$ and $g_{app} = G \lambda$ to determine which joint torques do not change in response to changes in the internal wrench.

5 pts

d.) Use the relationships $\tau = J^T \lambda$ and $g_{app} = G \lambda$ to determine which component of the external wrench cannot be controlled by adjusting joint torques.

40 pts

4.) A particle is close to circular and linear obstacles ($x^2 + y^2 \geq R^2$ and $x \geq -1$).



30 pts

a.) Assume $\mu = 0$, $m = h = 1$.

Determining u , v , and p_n at $t=1$ and $t=2$.

5 pts

b.) Assuming $\mu_1, \mu_2 \neq 0$, give the definitions of G_n , G_f , E , U , M , and $\frac{\partial \Psi_n}{\partial t}$, for the first time step.

5 pts

c.) What is the size of the LCP if both obstacles are incorporated and friction is not zero?