2012_mdtrm_soln

Wednesday, March 07, 2012
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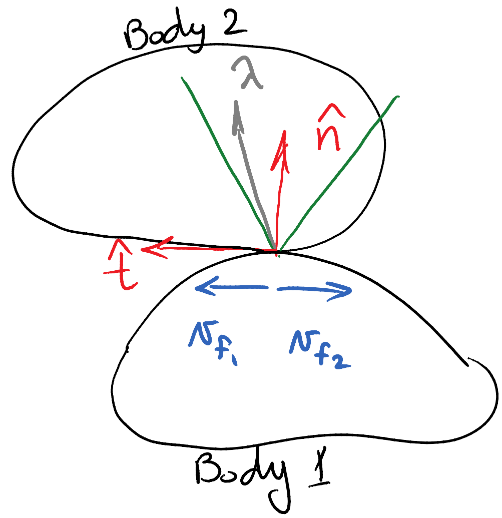

(1) Two bodies in the plane touch at a contact point with Coulomb friction.

Let $\lambda = \begin{bmatrix} \lambda_n \\ \lambda_t \end{bmatrix}$ be the contact force applied to body 2 by body 1.

The relative velocity of the contact point of body 2 w.r.t. body 1 is $\begin{bmatrix} v_n \\ v_t \end{bmatrix}$.

Assume $v_n = 0$, $\mu > 0$, $\lambda_n > 0$.

Let v_t be represented by the difference of its positive and negative parts, i.e.,

$$v_t = v_{f_1} - v_{f_2}, \quad v_{f_1} \geq 0, \quad v_{f_2} \geq 0$$

I claim that the following pair of linear complementarity conditions model planar Coulomb friction:

$$0 \leq \mu\lambda_n + \lambda_t \perp v_{f_1} \geq 0$$

$$0 \leq \mu\lambda_n - \lambda_t \perp v_{f_2} \geq 0$$

Demonstrate that I am right or wrong.

The model must allow the contact force to be anywhere inside the friction cone when $\nu_{f_1} = \nu_{f_2} = 0$. If $\nu_{f_1} > 0$ & $\nu_{f_2} = 0$, then λ_t must equal $-\mu\lambda_n$. If $\nu_{f_2} > 0$ and $\nu_{f_1} = 0$, then λ_t must equal $\mu\lambda_n$.

Case 1: $\nu_{f_1} = \nu_{f_2} = 0 \Rightarrow \underbrace{\mu\lambda_n + \lambda_t \geq 0 \text{ \& } \mu\lambda_n - \lambda_t \geq 0}_{\text{These define the friction cone.}}$

Case 2: $\nu_{f_1} = 0$, $\underbrace{\mu\lambda_n - \lambda_t = 0}_{\Downarrow\ \lambda_t = \mu\lambda_n} \Rightarrow \underbrace{\nu_{f_2} \geq 0}_{\text{right sliding}}$, $\underbrace{\mu\lambda_n + \lambda_t \geq 0}_{\text{half space defined by right edge of cone}}$

λ_t is to the left which is opposite sliding direction

$\lambda_t = \mu\lambda_n$ satisfies so all is good.

Case 3: $\nu_{f_2} = 0$, $\mu\lambda_n + \lambda_t = 0$

Analysis same as case 2, but with signs flipped.

Case 4: $\underbrace{\mu\lambda_n + \lambda_t = 0 \text{ \& } \mu\lambda_n - \lambda_t = 0}_{\Downarrow} \Rightarrow \underbrace{\nu_{f_1}, \nu_{f_2} \geq 0}_{\text{sticking or sliding in}}$

$\lambda_n = \lambda_t = 0$

$\Downarrow$

$\lambda_t = \lambda_n = 0$

sticking or sliding in either direction

Degenerate case.

Cases 1, 2, 3, properly model friction, case 4 does not hurt anything, just defines a rare case.

The claim is true!

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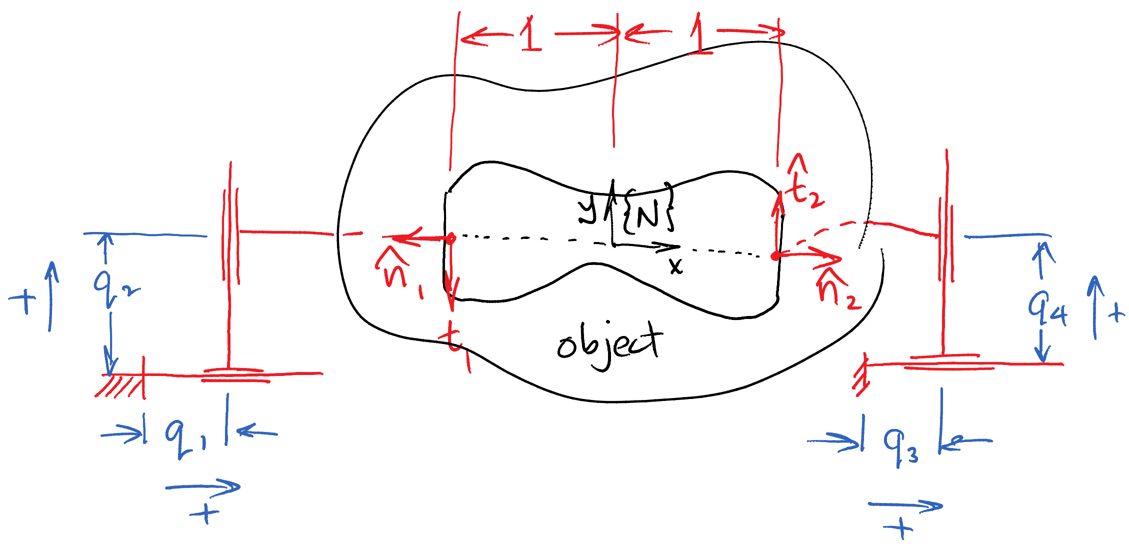


A hand with two fingers is grasping an object with contact points in a hole.

a. Construct G and J (If the assumed order of ν_{cc} is $\nu_{cc} = [\nu_{1n} \; \nu_{1t} \; \nu_{2n} \; \nu_{2t}]^T$

$$G = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

is $\nu_{cc} = [v_{1n} \ v_{1t} \ v_{2n} \ v_{2t}]^T$ then a possible basis of $\mathcal{N}(G)$ is $[1 \ 0 \ 1 \ 0]^T$)

$$J = \begin{bmatrix} -1 & & & \\ & -1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

b. What are the dimensions of the four subspaces of G and the four of J?

$\dim(\mathcal{R}(J)) = \dim(\mathcal{R}(J^T)) = 4$

$\dim(\mathcal{N}(J)) = \dim(\mathcal{N}(J^T)) = 0$

$\dim(\mathcal{R}(G)) = \dim(\mathcal{R}(G^T)) = 3$

$\dim(\mathcal{N}(G)) = 1 \quad \dim(\mathcal{N}(G^T)) = 0$

c. Show that the grasp has frictional form closure for any $\mu > 0$.

Since $\hat{n}_1$ & $-\hat{n}_2$ are colinear, then the line segment joining contact points 1 & 2 will always be in the negative friction cones. q.e.d.

d. Show that the grasp has force closure.

In addition to frictional form closure, we must have $\mathcal{N}(G) \cap \mathcal{N}(J^T) = 0$.

Since $\mathcal{N}(J^T) = 0$, q.e.d.

e. You may permanently lock a single joint. Can you choose one which will cause the grasp to lose force closure? $\boxed{\text{No}}$. All possible J:

$$J_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad J_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad J_3 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad J_4 = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$N(J_1^T) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad N(J_2^T) = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad N(J_3^T) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad N(J_4^T) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathcal{N}(J_i^T) \cap \mathcal{N}(G) = \mathcal{N}(J_i^T) \cap \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{for all } i$$

f. You may permanently lock two joints. Can you choose two which will cause the grasp to lose force closure?

$\boxed{\text{Yes}}$ Joints 1 & 3. $\Rightarrow$ $J = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$

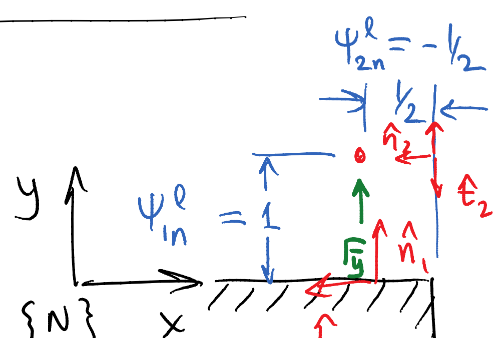
$N(J^T) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ $\quad N(G) = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

$\mathcal{N}(G) \cap \mathcal{N}(J^T) \neq 0$ $\quad$ i.e. $N(J^T)\begin{bmatrix} \alpha_1 \\ \alpha_1 \end{bmatrix} = N(G)\alpha_1$

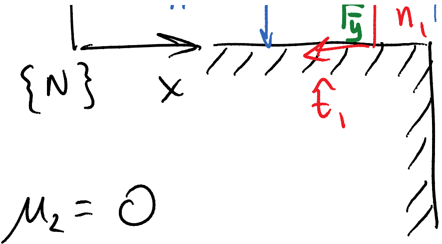
$\therefore$ Force closure is lost by locking jnts 1 & 3.

Remember, force closure does not require $\mathrm{Rank}(GJ) = n_\nu$. It requires $\mathrm{Rank}(G) = n_\nu$, frictional form closure, and $\mathcal{N}(G) \cap \mathcal{N}(J^T) = 0$.

(3.) A particle moving in the plane is near a corner



the plane is near a corner.



Assume mass $= 1$, $h = 1$, $\mu_1 = \mu_2 = 0$

$\nu_x^l = 0$, $\nu_y^l = -2$, $F_y = 1$

a. Set up the time-stepping LCP taking both edges into account.

We need the big matrix & vector that define the LCP.

$$\begin{bmatrix} M & -G_n & -G_f & 0 \\ G_n^T & & & 0 \\ G_f^T & & & E \\ 0 & U & -E^T & 0 \end{bmatrix}$$

$\mu_1 = \mu_2 = 0$

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \qquad G_n = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$G_f = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -M\nu^l - p_{ext} \\ \frac{\psi_n^l}{h} + \frac{\partial \psi_n}{\partial t} \\ \frac{\partial \psi_f}{\partial t} \\ 0 \end{bmatrix}$$

$$\nu^l = \begin{bmatrix} 0 \\ -2 \end{bmatrix} \qquad p_{ext} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\frac{\psi_n^l}{h} = \begin{bmatrix} 1 \\ -1/2 \end{bmatrix} \qquad \frac{\partial \psi_n}{\partial t} = 0 \qquad \frac{\partial \psi_f}{\partial t} = 0$$

b. Solve for v^{l+1}, u^{l+1}, p^{l+1}

$$\begin{bmatrix} 0 \\ \rho_n^{l+1} \end{bmatrix} = \begin{bmatrix} M & -G_n \\ G_n^T & 0 \end{bmatrix} \begin{bmatrix} v^{l+1} \\ p_n^{l+1} \end{bmatrix} + \begin{bmatrix} -\cancelto{0}{M v^l} - p_{ext}^l \\ \psi_n^l / h + \cancelto{0}{\frac{\partial \psi_n}{\partial t}} \end{bmatrix}$$

$$0 \le \rho_n^{l+1} \perp p_n^{l+1} \ge 0$$

$$v^{l+1} = v^l + G_n p_n^{l+1} + p_{ext} \Rightarrow \begin{bmatrix} v_x^{l+1} \\ v_y^{l+1} \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} p_{1n}^{l+1} \\ p_{2n}^{l+1} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\rho_n^{l+1} = G_n^T v^{l+1} + \frac{\psi_n^l}{\cancelto{1}{h}} \ge 0 \Rightarrow$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 \\ -2 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} p_{1n}^{l+1} \\ p_{2n}^{l+1} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) + \begin{bmatrix} 1 \\ -1/2 \end{bmatrix} \ge 0$$

$$0 \le \begin{bmatrix} p_{1n}^{l+1} \\ p_{2n}^{l+1} \end{bmatrix} \perp \begin{bmatrix} p_{1n}^{l+1} \\ p_{2n}^{l+1} \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -1/2 \end{bmatrix} \ge 0$$

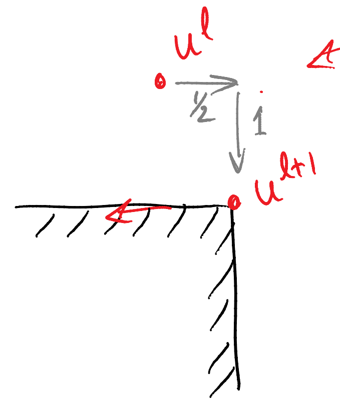
Solution is unique ... $\boxed{\begin{bmatrix} p_{1n}^{l+1} \\ p_{2n}^{l+1} \end{bmatrix} = \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}}$

$$v^{l+1} = \begin{bmatrix} v_x^{l+1} \\ v_y^{l+1} \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix} + \begin{bmatrix} p_{2n}^{l+1} \\ p_{1n}^{l+1} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} 1/2 \\ -1 \end{bmatrix} = v^{l+1}}$$

$$u^{l+1} = \begin{bmatrix} x^l \\ u^l \end{bmatrix} + \cancel{h} v^{l+1} \cancel{h} \Rightarrow \boxed{u^{l+1} = \begin{bmatrix} x^l + 1/2 \\ u^l - 1 \end{bmatrix}}$$

$$u^{l+1} = \begin{bmatrix} x^l \\ y^l \end{bmatrix} + \underbrace{\cancel{Y}}_{I_{2\times2}} v^{l+1} \underbrace{\cancel{h}}_{1} \Rightarrow \boxed{u^{l+1} = \begin{bmatrix} x^l + 1/2 \\ y^l - 1 \end{bmatrix}}$$

Particle moves to corner.



c. If you did part b. correctly, then $v_x^{l+1} > 0$.

Since $g_{app} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ & $v_x^l = 0$,

what caused v_x^{l+1} to change?

The particle was in violation of the extended vertical wall constraint.