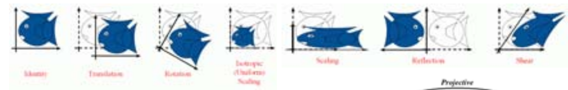
Adjacency Data Structures

material from Justin Legakis

Last Time?

- Simple Transformations



- Classes of Transformations
- Representation
  - homogeneous coordinates
- Composition
  - not commutative

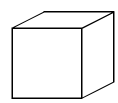
$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

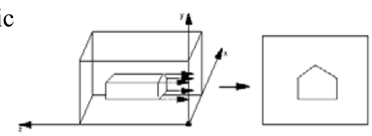
Today

- Orthographic & Perspective Projections
- OpenGL Basics
- Averaging Vertex Colors & Normals
- Surface Definitions
- Simple Data Structures
- Fixed Storage Data Structures
- Fixed Computation Data Structures

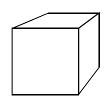
Orthographic vs. Perspective

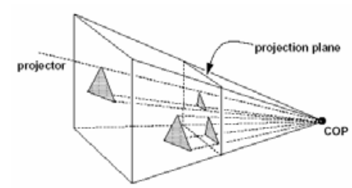
- Orthographic





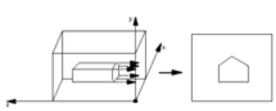
- Perspective





Simple Orthographic Projection

- Project all points along the *z* axis to the *z* = 0 plane

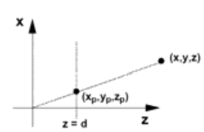


$$\begin{pmatrix} x \\ y \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Simple Perspective Projection

- Project all points along the *z* axis to the *z* = *d* plane, eyepoint at the origin:

$$x_p = \frac{d \cdot x}{z} = \frac{x}{z/d}$$
$$y_p = \frac{d \cdot y}{z} = \frac{y}{z/d}$$
$$z_p = d$$



homogenize

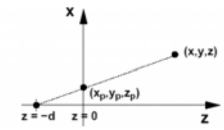
$$\begin{pmatrix} x * d / z \\ y * d / z \\ d \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ z / d \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Alternate Perspective Projection

- Project all points along the z axis to the $z = 0$ plane, eyepoint at the $(0,0,-d)$:

$$x_p = \frac{d \cdot x}{z + d} = \frac{x}{(z/d) + 1}$$

$$y_p = \frac{d \cdot y}{z + d} = \frac{y}{(z/d) + 1}$$



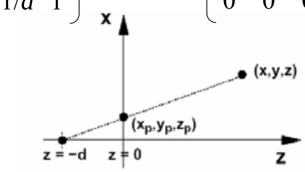
homogenize

$$\begin{pmatrix} x * d / (z + d) \\ y * d / (z + d) \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \\ (z + d)/d \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

In the limit, as $d \to \infty$

this perspective projection matrix... ...is simply an orthographic projection

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



Questions?

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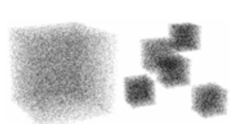
OpenGL Basics: GL_POINTS

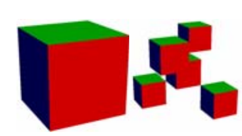
```
glDisable(GL_LIGHTING);
glBegin(GL_POINTS);
glColor3f(0.0,0.0,0.0);
glVertex3f(…);
glEnd();
```



- **lighting should be *disabled*...**

OpenGL Basics: GL_QUADS

```
glEnable(GL_LIGHTING);
glBegin(GL_QUADS);
glNormal3f(…);
glColor3f(1.0,0.0,0.0);
glVertex3f(…);
glVertex3f(…);
glVertex3f(…);
glVertex3f(…);
glEnd();
```

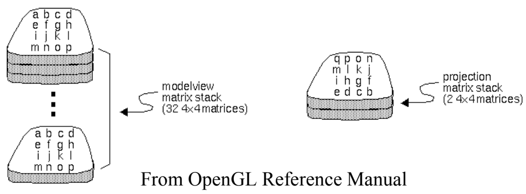


- **lighting should be *enabled*...**
- **an appropriate normal should be specified**

OpenGL Basics: Transformations

- Useful commands:

```
glMatrixMode(GL_MODELVIEW);
glPushMatrix();
glPopMatrix();
glMultMatrixf(…);
```



From OpenGL Reference Manual

Questions?

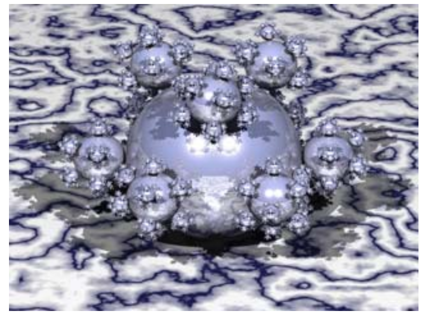


Image by Henrik Wann Jensen

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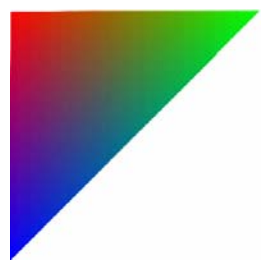
Color Interpolation

- Interpolate colors of the 3 vertices
- Linear interpolation, barycentric coordinates

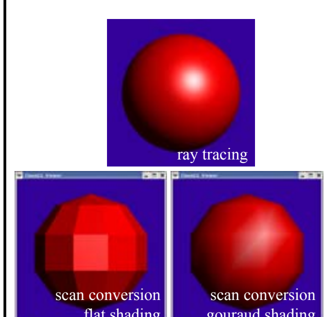


```
glBegin(GL_TRIANGLES);
glColor3f(1.0,0.0,0.0);
glVertex3f(…);
glColor3f(0.0,1.0,0.0);
glVertex3f(…);
glColor3f(0.0,0.0,1.0);
glVertex3f(…);
glEnd();
```

glShadeModel (GL_SMOOTH);

- From OpenGL Reference Manual:
  - Smooth shading, the default, causes the computed colors of vertices to be interpolated as the primitive is rasterized, typically assigning different colors to each resulting pixel fragment.
  - Flat shading selects the computed color of just one vertex and assigns it to all the pixel fragments generated by rasterizing a single primitive.
  - In either case, the computed color of a vertex is the result of lighting if lighting is enabled, or it is the current color at the time the vertex was specified if lighting is disabled.

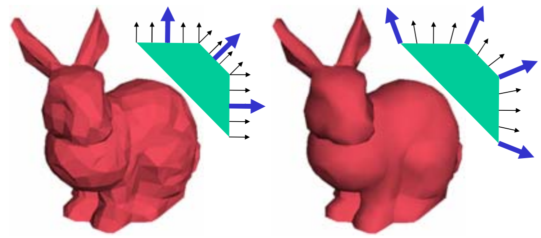
Normal Interpolation




```
glBegin(GL_TRIANGLES);
glNormal3f(…);
glVertex3f(…);
glNormal3f(…);
glVertex3f(…);
glNormal3f(…);
glVertex3f(…);
glEnd();
```

Gouraud Shading

- Instead of shading with the normal of the triangle, we’ll shade the vertices with the *average normal* and *interpolate the shaded color* across each face



- How do we compute Average Normals? Is it expensive??

Questions?

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- Orthographic & Perspective Projections
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  – Well-Formed Surfaces
  – Orientable Surfaces
  – Computational Complexity
- Simple Data Structures
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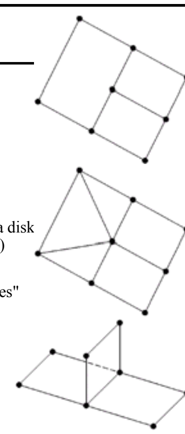
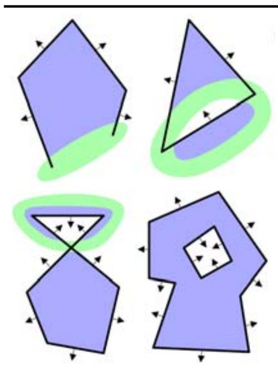
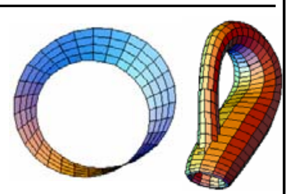
Well-Formed Surfaces



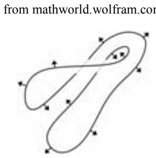
- Components Intersect "Properly"
  – Faces are: disjoint, share single Vertex, or share 2 Vertices and the Edge joining them
  – Every edge is incident to exactly 2 vertices
  – Every edge is incident to exactly 2 faces
- Local Topology is "Proper"
  – Neighborhood of a vertex is homeomorphic to a disk (permits stretching and bending, but not tearing)
  – Called a 2-manifold
  – Boundaries: half-disk, "manifold with boundaries"
- Global Topology is "Proper"
  – Connected
  – Closed
  – Bounded

Orientable Surfaces?



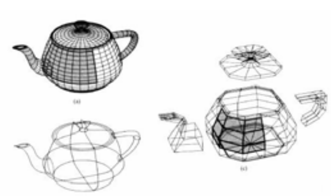


from mathworld.wolfram.com



Closed Surfaces and Refraction

- Original Teapot model is not "watertight": intersecting surfaces at spout & handle, no bottom, a hole at the spout tip, a gap between lid & base
- Requires repair before ray tracing with refraction



Henrik Wann Jensen

Computational Complexity

- Access Time
  - linear, constant time average case, or constant time?
  - requires loops/recursion/if ?
- Memory
  - variable size arrays or constant size?
- Maintenance
  - ease of editing
  - ensuring consistency

Questions?

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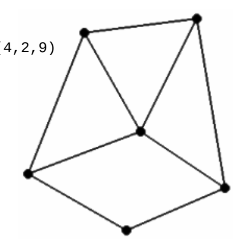
- Simple Data Structures
  - List of Polygons
  - List of Edges
  - List of Unique Vertices & Indexed Faces:
  - Simple Adjacency Data Structure
- Fixed Storage Data Structures
- Fixed Computation Data Structures

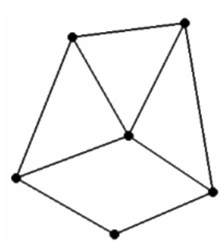
List of Polygons:

```
(3,-2,5), (3,6,2), (-6,2,4)
(2,2,4), (0,-1,-2), (9,4,0), (4,2,9)
(1,2,-2), (8,8,7), (-4,-5,1)
(-8,2,7), (-2,3,9), (1,2,-7)
```



List of Edges:

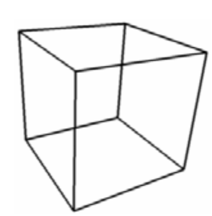
```
(3,6,2), (-6,2,4)
(2,2,4), (0,-1,-2)
(9,4,0), (4,2,9)
(8,8,7), (-4,-5,1)
(-8,2,7), (1,2,-7)
(3,0,-3), (-7,4,-3)
(9,4,0), (4,2,9)
(3,6,2), (-6,2,4)
(-3,0,-4), (7,-3,-4)
```



List of Unique Vertices & Indexed Faces:

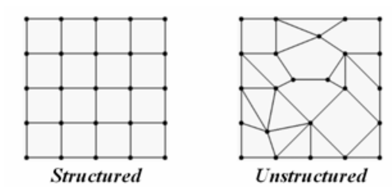
Vertices:
```
(-1, -1, -1)
(-1, -1, 1)
(-1, 1, -1)
(-1, 1, 1)
(1, -1, -1)
(1, -1, 1)
(1, 1, -1)
(1, 1, 1)
```

Faces:
```
1 2 4 3
5 7 8 6
1 5 6 2
3 4 8 7
1 3 7 5
2 6 8 4
```



Problems with Simple Data Structures

- No Adjacency Information
- Linear-time Searches



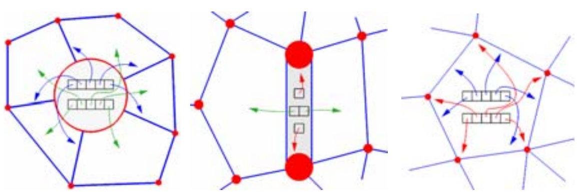
- Adjacency is implicit for structured meshes, but what do we do for unstructured meshes?

Mesh Data

- So, in addition to:
  - Geometric Information (position)
  - Attribute Information (color, texture, temperature, population density, etc.)
- Let's store:
  - Topological Information (adjacency, connectivity)

Simple Adjacency

- Each element (vertex, edge, and face) has a list of pointers to all incident elements
- Queries depend only on local complexity of mesh
- Data structures do not have fixed size
- Slow! Big! Too much work to maintain!



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  - Winged Edge (Baumgart, 1975)
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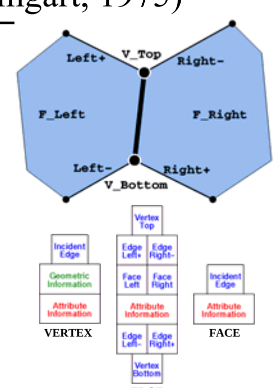
Winged Edge (Baumgart, 1975)

- Each edge stores pointers to 4 Adjacent Edges
- Vertices and Faces have a single pointer to one incident Edge
- Data Structure Size?
  Fixed
- How do we gather all faces surrounding one vertex?
  Messy, because there is no consistent way to order pointers



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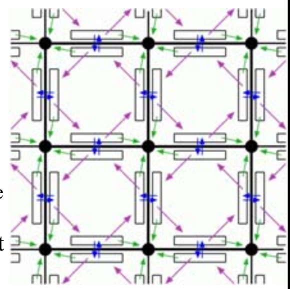
  - HalfEdge (Eastman, 1982)
  - SplitEdge
  - Corner
  - QuadEdge (Guibas and Stolfi, 1985)
  - FacetEdge (Dobkin and Laszlo, 1987)

HalfEdge (Eastman, 1982)

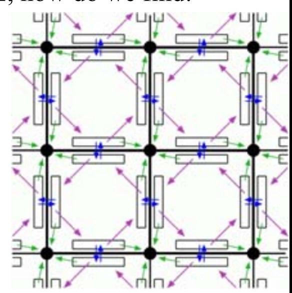
- Every edge is represented by two directed HalfEdge structures
- Each HalfEdge stores:
  - `vertex` at end of directed edge
  - `symmetric` half edge
  - `face` to left of edge
  - `next` points to the HalfEdge counter-clockwise around face on left
- Orientation is essential, but can be done consistently!



HalfEdge (Eastman, 1982)

- Starting at half edge HE, how do we find:

  the other vertex of the edge?
  the other face of the edge?
  the clockwise edge around the face at the left?
  all the edges surrounding the face at the left?
  all the faces surrounding the vertex?



HalfEdge (Eastman, 1982)

- Loop around a Face:

```
HalfEdgeMesh::FaceLoop(HalfEdge *HE) {
   HalfEdge *loop = HE;
   do {
      loop = loop->Next;
   } while (loop != HE);
}
```

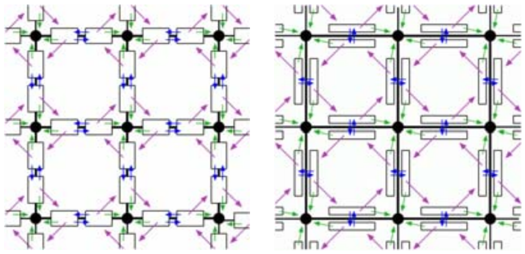
- Loop around a Vertex:

```
HalfEdgeMesh::VertexLoop(HalfEdge *HE) {
   HalfEdge *loop = HE;
   do {
      loop = loop->Next->Sym;
   } while (loop != HE);
}
```

HalfEdge (Eastman, 1982)

- Data Structure Size?
  Fixed
- Data:
  - geometric information stored at Vertices
  - attribute information in Vertices, HalfEdges, and/or Faces
  - topological information in HalfEdges only!
- Orientable surfaces only (no Mobius Strips!)
- Local consistency everywhere implies global consistency
- Time Complexity?
  linear in the amount of information gathered

SplitEdge Data Structure:

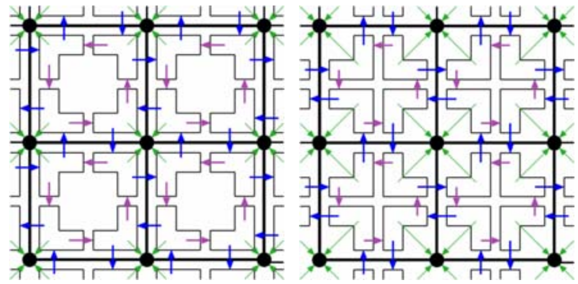


- HalfEdge and SplitEdge are dual structures!

```
SplitEdgeMesh::FaceLoop() = HalfEdgeMesh::VertexLoop()
SplitEdgeMesh::VertexLoop() = HalfEdgeMesh::FaceLoop()
```

Corner Data Structure:

- The Corner data structure is its own dual!



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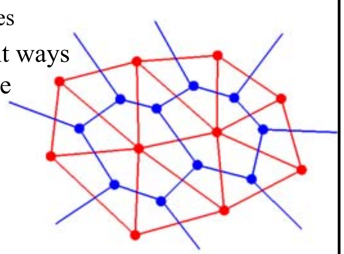
  - FacetEdge (Dobkin and Laszlo, 1987)

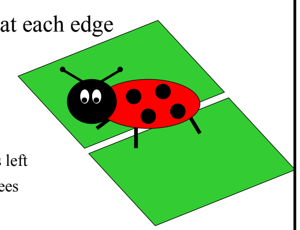
QuadEdge (Guibas and Stolfi, 1985)

- Consider the Mesh and its *Dual* simultaneously
  - Vertices and Faces switch roles, we just re-label them
  - Edges remain Edges
- Now there are eight ways to look at each edge
  - Four ways to look at primal edge
  - Four ways to look at dual edge



QuadEdge (Guibas and Stolfi, 1985)

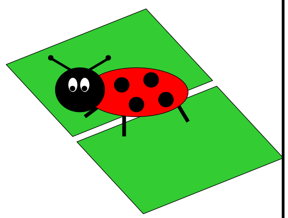
- Relations Between Edges: Edge Algebra
- Elements in Edge Algebra:
  - Each of 8 ways to look at each edge



- Operators in Edge Algebra:
  - Rot: Bug rotates 90 degrees to its left
  - Sym: Bug turns around 180 degrees
  - Flip: Bug flips up-side down
  - Onext: Bug rotates CCW about its origin (either Vertex or Face)

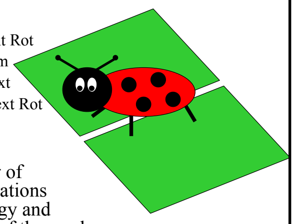
QuadEdge (Guibas and Stolfi, 1985)

- Some Properties of Flip, Sym, Rot, and Onext:
  - e Rot^4 = e
  - e $Rot^2 \neq$ e
  - e $Flip^2$ = e
  - e Flip Rot Flip Rot = e
  - e Rot Flip Rot Flip = e
  - e Rot Onext Rot Onext = e
  - e Flip Onext Flip Onext = e
  - e $Flip^{-1}$ = e Flip
  - e Sym = e Rot^2
  - e Rot^{-1} = e Rot^3
  - e Rot^{-1} = e Flip Rot Flip
  - e $Onext^{-1}$ = e Rot Onext Rot
  - e $Onext^{-1}$ = e Flip Onext Flip



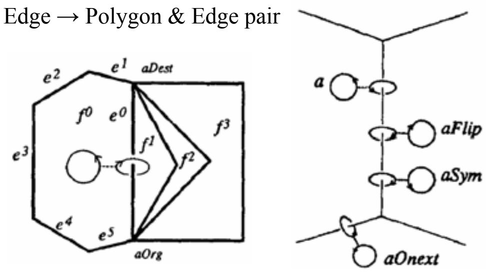
QuadEdge (Guibas and Stolfi, 1985)

- Other Useful Definitions:
  - e Lnext = e Rot^{-1} Onext Rot
  - e Rnext = e Rot Onext Rot^{-1}
  - e Dnext = e Sym Onext Sym^{-1}
  - e Oprev = e $Onext^{-1}$ = e Rot Onext Rot
  - e Lprev = e $Lnext^{-1}$ = e Onext Sym
  - e Rprev = e $Rnext^{-1}$ = e Sym Onext
  - e Dprev = e $Dnext^{-1}$ = e Rot^{-1} Onext Rot
- All of these functions can be expressed as a constant number of Rot, Sym, Flip, and Onext operations independent of the local topology and the global size and complexity of the mesh.



FacetEdge (Dobkin and Laszlo, 1987)

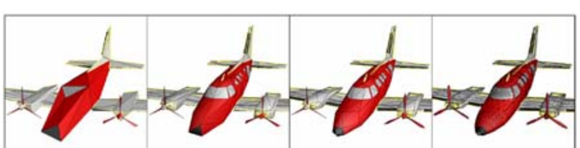
- QuadEdge (2D, surface) → FacetEdge (3D, volume)
- Faces → Polyhedra / Cells
- Edge → Polygon & Edge pair



Questions?

For Next Time:

- Read Hugues Hoppe “Progressive Meshes” SIGGRAPH 1996



(a) Base mesh M^0 (150 faces) (b) Mesh M^{175} (500 faces) (c) Mesh M^{425} (1,000 faces) (d) Original $\hat{M}=M^n$ (13,546 faces)