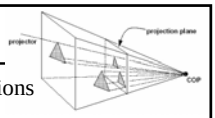
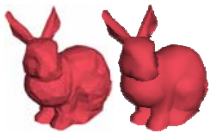
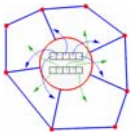
Mesh Simplification

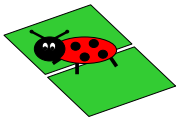
Last Time?

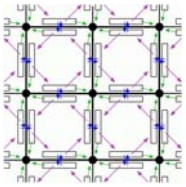


- Orthographic & Perspective Projections
- OpenGL Basics
  - Average normals
- Adjacency Data Structures
  - Geometric & topologic information
  - Dynamic allocation
  - Efficiency of access



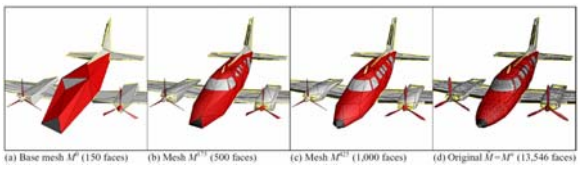






Reading for Today:

- Hugues Hoppe
  "Progressive Meshes"
  SIGGRAPH 1996



(a) Base mesh M^0 (150 faces) (b) Mesh M^{173} (500 faces) (c) Mesh M^{425} (1,000 faces) (d) Original $M = M^n$ (13,546 faces)

Progressive Meshes

- Mesh Simplification
  - vertex split / edge collapse
  - geometry & discrete/scalar attributes
  - priority queue
- Level of Detail
  - geomorphs
- Progressive Transmission
- Mesh Compression
- Selective Refinement
  - view dependent

Selective Refinement

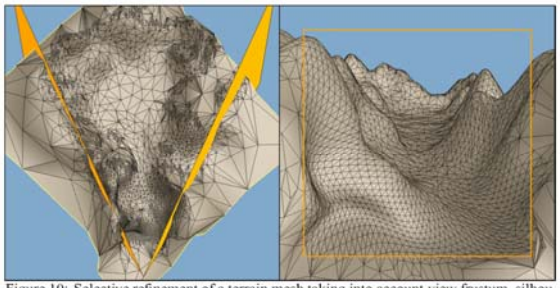
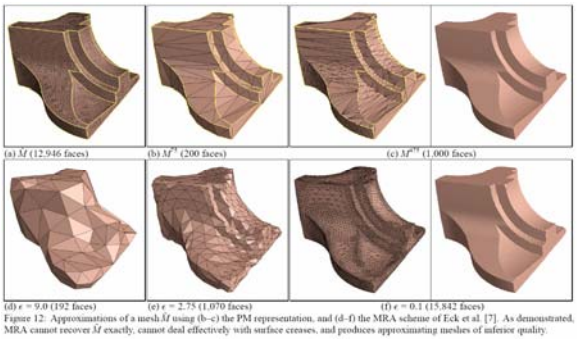


Figure 10: Selective refinement of a terrain mesh taking into account view frustum, silhouette regions, and projected screen size of faces (7,438 faces).

Preserving Discontinuity Curves



(a) M (12,946 faces) (b) M^{200} (200 faces) (c) M^{425} (1,000 faces)
(d) $\epsilon = 9.0$ (192 faces) (e) $\epsilon = 2.75$ (1,070 faces) (f) $\epsilon = 0.1$ (15,842 faces)

Figure 12: Approximations of a mesh M using (b–c) the PM representation, and (d–f) the MRA scheme of Eck et al. [7]. As demonstrated, MRA cannot recover M exactly, cannot deal effectively with surface creases, and produces approximating meshes of inferior quality.

Other Simplification Strategies

- Remove a vertex & surrounding triangles, re-triangulate the hole

- Merge Nearby Vertices
  - will likely change the topology…

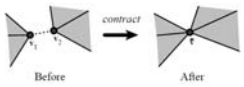


Figure 2: **Non-edge contraction.** When non-edge pairs are contracted, unconnected sections of the model are joined. The dashed line indicates the two vertices being contracted together.

from Garland & Heckbert, "Surface Simplification Using Quadric Error Metrics" SIGGRAPH 1997

When to Preserve Topology?

Figure 3: On the left is a regular grid of 100 closely spaced cubes. In the middle, an approximation built using only edge contractions demonstrates unacceptable fragmentation. On the right, the result of using more general pair contractions to achieve aggregation is an approximation much closer to the original.

from Garland & Heckbert, "Surface Simplification Using Quadric Error Metrics" SIGGRAPH 1997

Quadric Error Simplification

- Contract (merge) vertices v_i and v_j if:
  - (v_i, v_j) is an edge, or
  - $\| v_i - v_j \| < t$, where t is a threshold parameter
- Track cummulative error by summing 4x4 quadric error matrics after each operation:

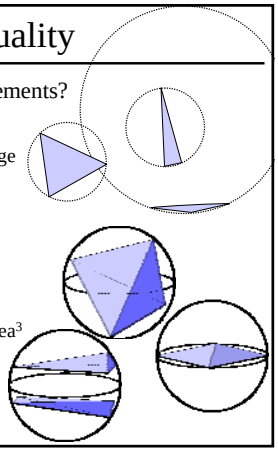
$$\begin{aligned}\Delta(\mathbf{v}) &= \sum_{\mathbf{p}\in\text{planes}(\mathbf{v})} (\mathbf{v}^T\mathbf{p})(\mathbf{p}^T\mathbf{v}) \\ &= \sum_{\mathbf{p}\in\text{planes}(\mathbf{v})} \mathbf{v}^T(\mathbf{p}\mathbf{p}^T)\mathbf{v} \\ &= \mathbf{v}^T\left(\sum_{\mathbf{p}\in\text{planes}(\mathbf{v})} \mathbf{K}_\mathbf{p}\right)\mathbf{v}\end{aligned}$$

$$\mathbf{K}_\mathbf{p} = \mathbf{p}\mathbf{p}^T = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix}$$

Garland & Heckbert, "Surface Simplification Using Quadric Error Metrics" SIGGRAPH 1997

Judging Element Quality

- How "equilateral" are the elements?
- For Triangles?
  - Ratio of shortest to longest edge
  - Ratio of area to perimeter^2
  - Smallest angle
  - Ratio of area to area of smallest circumscribed circle
- For Tetrahedra?
  - Ratio of volume^2 to surface area^3
  - Smallest *solid* angle
  - Ratio of volume to volume of smallest circumscribed sphere



Future Work

- Editing of Progressive Meshes
- Simplification of articulated or animated models
- Optimization scheme for preserving surface normals
- General simplicial complices (going to volumetric elements and beyond!)
- Data structures for efficient selective refinement

Reading for next Tuesday (1/30)

- DeRose, Kass, & Truong, "Subdivision Surfaces in Character Animation", SIGGRAPH 1998

Figure 5: Geri's hand as a piecewise smooth Catmull-Clark surface. Infinitely sharp creases are used between the skin and the finger nails.

- Additional Reference: SIGGRAPH 99 course notes Subdivision for Modeling and Animation