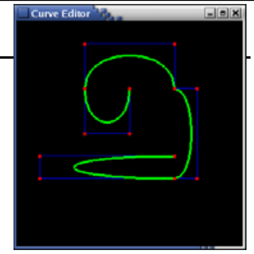
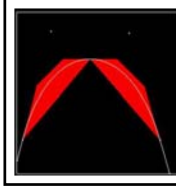
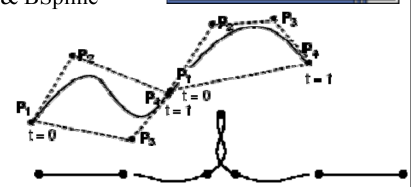
Subdivision Surfaces

Last Time?



- Curves & Surfaces
- Continuity Definitions
  - C^0, G^1, C^1, … C^∞
- Interpolation vs. Approximation Splines
- Cubic Bezier & BSpline





Today

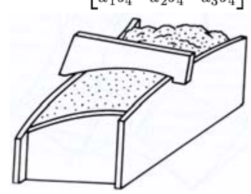
- Spline Surfaces / Patches
  - Tensor Product
  - Bilinear Patches
  - Bezier Patches
- Subdivision Surfaces

Tensor Product

- Of two vectors:

$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \otimes \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \end{bmatrix} = \begin{bmatrix} a_1b_1 & a_2b_1 & a_3b_1 \\ a_1b_2 & a_2b_2 & a_3b_2 \\ a_1b_3 & a_2b_3 & a_3b_3 \\ a_1b_4 & a_2b_4 & a_3b_4 \end{bmatrix}$$

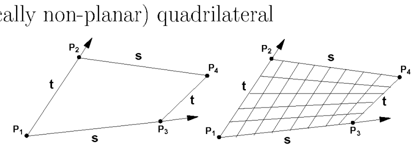
- Similarly, we can define a surface as the tensor product of two curves....



Farin, Curves and Surfaces for Computer Aided Geometric Design

Bilinear Patch

Bi-lerp a (typically non-planar) quadrilateral

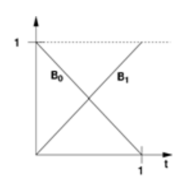


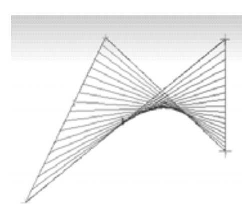
Notation: $\mathbf{L}(P_1, P_2, \alpha) \equiv (1-\alpha)P_1 + \alpha P_2$

$$Q(s,t) = \mathbf{L}(\mathbf{L}(P_1, P_2, t), L(P_3, P_4, t), s)$$

Bilinear Patch

- Smooth version of quadrilateral with non-planar vertices...





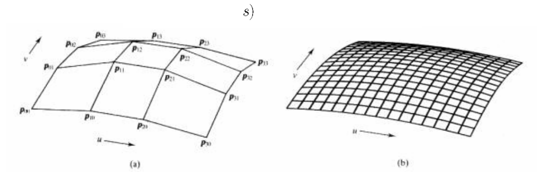
- – But will this help us model smooth surfaces?
- – Do we have control of the derivative at the edges?

Bicubic Bezier Patch

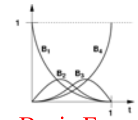
Notation: $\mathbf{CB}(P_1, P_2, P_3, P_4, \alpha)$ is Bézier curve with control points P_i evaluated at α

Define "Tensor-product" Bézier surface

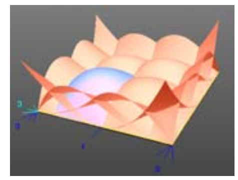
$$Q(s,t) = \mathbf{CB}(\;\mathbf{CB}(P_{00}, P_{01}, P_{02}, P_{03}, t),\; \mathbf{CB}(P_{10}, P_{11}, P_{12}, P_{13}, t),\; \mathbf{CB}(P_{20}, P_{21}, P_{22}, P_{23}, t),\; \mathbf{CB}(P_{30}, P_{31}, P_{32}, P_{33}, t),\; s)$$



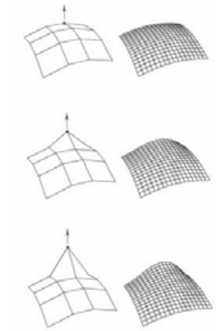
Editing Bicubic Bezier Patches



Curve Basis Functions

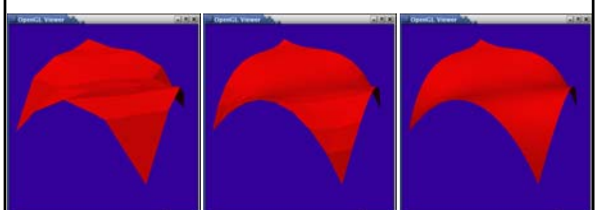


Surface Basis Functions



Bicubic Bezier Patch Tessellation

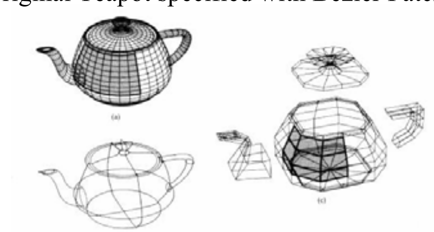
- Given 16 control points and a tessellation resolution, we can create a triangle mesh



resolution: 5x5 vertices | resolution: 11x11 vertices | resolution: 41x41 vertices

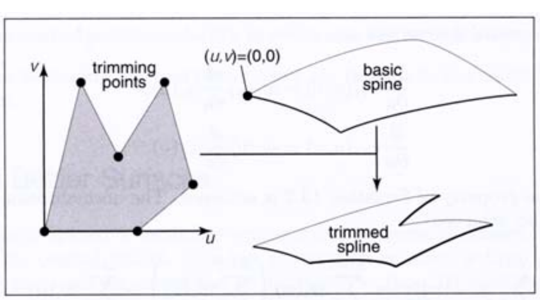
Modeling with Bicubic Bezier Patches

- Original Teapot specified with Bezier Patches



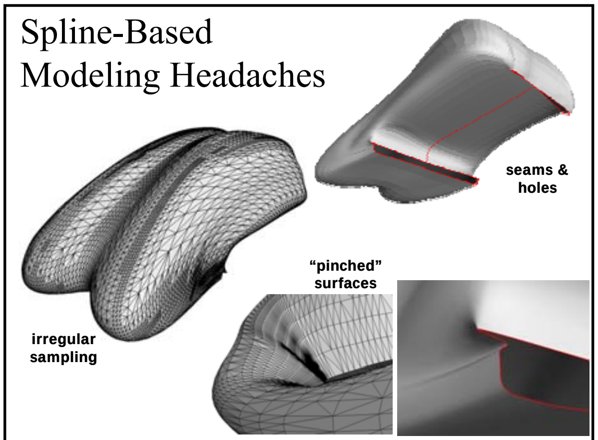
- But it's not "watertight": it has intersecting surfaces at spout & handle, no bottom, a hole at the spout tip, a gap between lid & base

Trimming Curves for Patches



Shirley, Fundamentals of Computer Graphics

Spline-Based Modeling Headaches



Questions?

- Bezier Patches?

  or

- Triangle Mesh?

Reading for Today!

- DeRose, Kass, & Truong, "Subdivision Surfaces in Character Animation", SIGGRAPH 1998

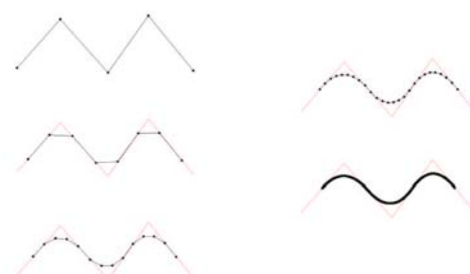
Figure 5: Geri's hand as a piecewise smooth Catmull-Clark surface. Infinitely sharp creases are used between the skin and the finger nails.

- Additional Reference: SIGGRAPH course notes Subdivision for Modeling and Animation

Subdivision Surfaces

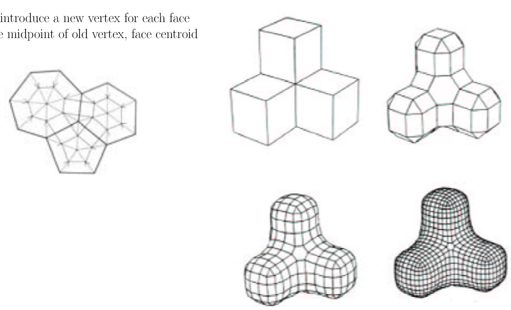
- Subdivision Zoo
  - Doo Sabin (anything!)
  - Loop (triangles only)
  - Catmull Clark (turns everything into quads)
  - … many others!
- Subdivision for Texture Coordinates

Chaikin's Algorithm

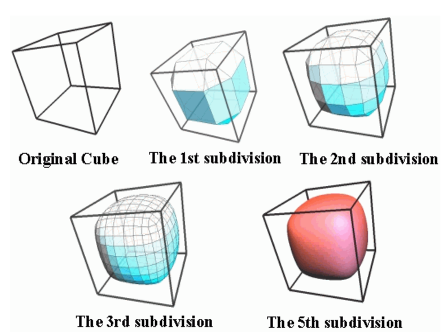


Doo-Sabin Subdivision

Idea: introduce a new vertex for each face
At the midpoint of old vertex, face centroid

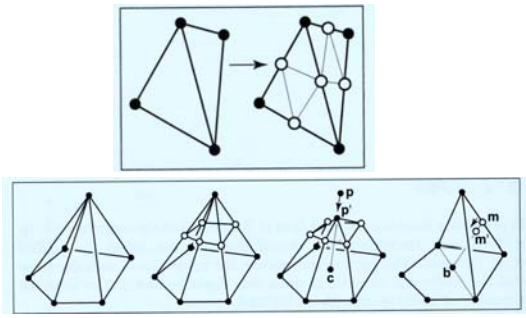


Doo-Sabin Subdivision



http://www.ke.ics.saitama-u.ac.jp/xuz/pic/doo-sabin.gif

Loop Subdivision



Shirley, Fundamentals of Computer Graphics

Catmull Clark Subdivision

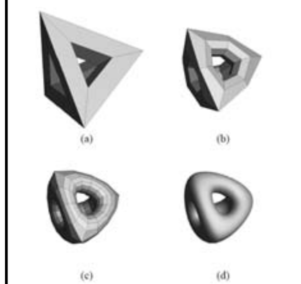


Figure 3: Recursive subdivision of a topologically complicated mesh: (a) the control mesh; (b) after one subdivision step; (c) after two subdivision steps; (d) the limit surface.

$$e_j^{i+1} = \frac{v^i + e_j^i + f_{j-1}^{i+1} + f_j^{i+1}}{4}, \quad (1)$$

where subscripts are taken modulo the valence of the central vertex v^0. (The valence of a vertex is the number of edges incident to it.) Finally, a vertex point v^i is computed as

$$v^{i+1} = \frac{n-2}{n} v^i + \frac{1}{n^2} \sum_j e_j^i + \frac{1}{n^2} \sum_j f_j^{i+1} \quad (2)$$

Vertices of valence 4 are called ordinary; others are called extraordinary.

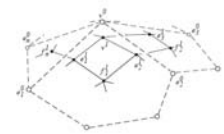


Figure 4: The situation around a vertex v^0 of valence n.

Subdivision for Texture Coordinates

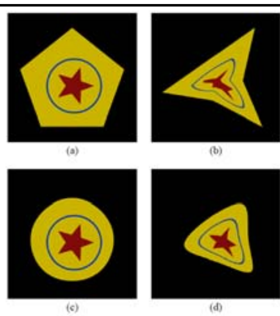
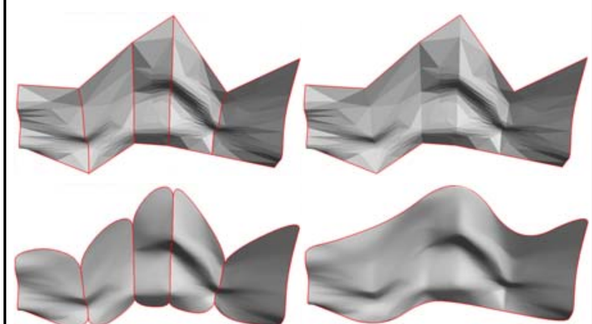
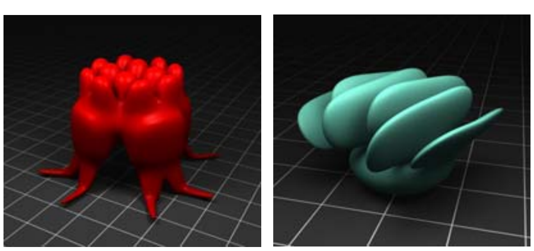


Figure 11: (a) A texture mapped regular pentagon comprised of 5 triangles; (b) the pentagonal model with its vertices moved; (c) A subdivision surface whose control mesh is the same 5 triangles in (a), and where boundary edges are marked as creases; (d) the subdivision surface with its vertices positioned as in (b).

Seams don't Subdivide as Expected



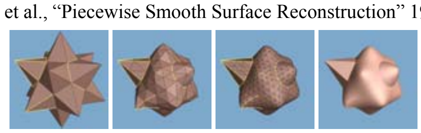
Questions?



Justin Legakis

Reading for Friday (2/2) *pick one*

Hoppe et al., "Piecewise Smooth Surface Reconstruction" 1994



"Efficient, fair interpolation using Catmull-Clark surfaces", Halstead, Kass & DeRose, SIGGRAPH 1993