# Inverse Kinematics

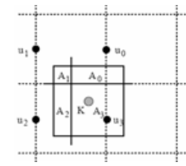
## Last Time?

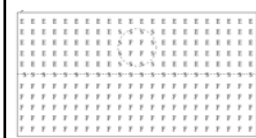
$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = -\frac{\partial p}{\partial x} + g_x + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

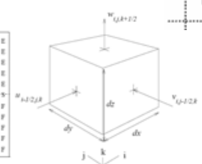
$$\frac{\partial v}{\partial t} + \frac{\partial vu}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} = -\frac{\partial p}{\partial y} + g_y + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

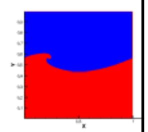
$$\frac{\partial w}{\partial t} + \frac{\partial wu}{\partial x} + \frac{\partial wv}{\partial y} + \frac{\partial w^2}{\partial z} = -\frac{\partial p}{\partial z} + g_z + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

- Navier-Stokes Equations
- Conservation of Momentum & Mass
- Incompressible Flow







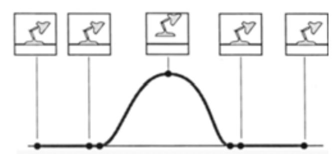


## Today

- How do we animate?
  - Keyframing
  - Procedural Animation
  - Physically-Based Animation
  - Forward and Inverse Kinematics
  - Motion Capture
- Rigid Body Dynamics
- Finite Element Method

## Keyframing

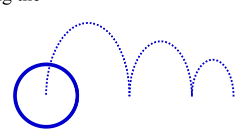
- Use spline curves to automate the in betweening
  - Good control
  - Less tedious than drawing *every* frame
- Creating a good animation still requires considerable skill and talent



ACM © 1987 "Principles of traditional animation applied to 3D computer animation"

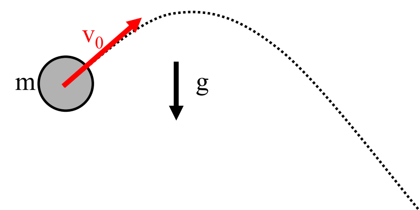
## Procedural Animation

- Describes the motion algorithmically, as a function of small number of parameters
- Example: a clock with second, minute and hour hands
  - express the clock motions in terms of a "seconds" variable
  - the clock is animated by varying the seconds parameter
- Example: A bouncing ball
  - Abs(sin($\omega$t+$\theta_0$))*$e^{-kt}$



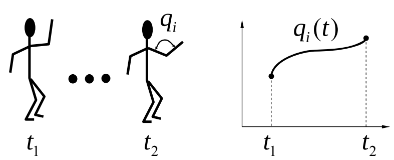
## Physically-Based Animation

- Assign physical properties to objects (masses, forces, inertial properties)
- Simulate physics by solving equations
- Realistic but difficult to control



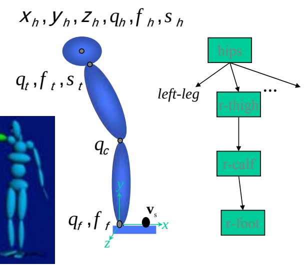
## Articulated Models

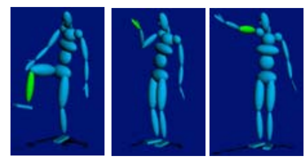
- Articulated models:
  - rigid parts
  - connected by joints
- They can be animated by specifying the joint angles as functions of time.




## Skeleton Hierarchy

- Each bone transformation described relative to the parent in the hierarchy:

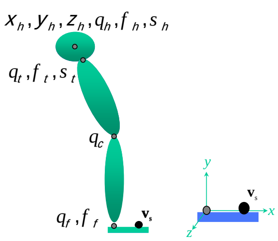


1 DOF: knee 2 DOF: wrist 3 DOF: arm

## Forward Kinematics

- Given skeleton parameters p, and the position of the effecter in local coordinates $V_s$, what is the position of the effector in the world coordinates $V_w$?

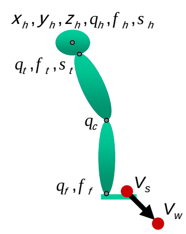



$V_w = T(x_h,y_h,z_h)R(q_h,f_h,s_h)T_hR(q_t,f_t,s_t)T_tR(q_c)T_cR(q_f,f_f)V_s$

$V_w = S(p)V_s$

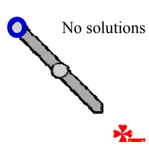
## Inverse Kinematics (IK)

- Given the position of the effecter in local coordinates $V_s$ and the *desired position* $V_w$ in world coordinates, what are the skeleton parameters p?
- Much harder requires solving the inverse of the non-linear function:

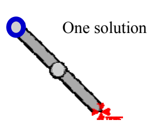


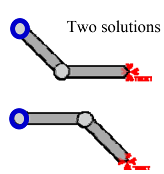

find p *such that* $S(p)V_s = V_w$

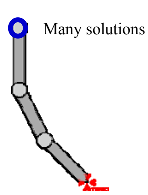
## Under-/Over- Constrained IK




- Application: Robot Motion Planning

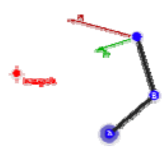


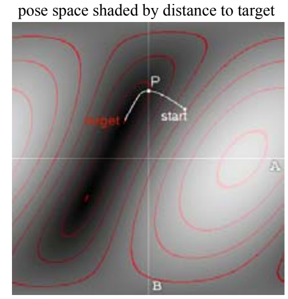




"The good-looking textured light-sourced bouncy fun smart and stretchy page" Hugo Elias, `http://freespace.virgin.net/hugo.elias/models/m_ik.htm`

## Searching Configuration Space



pose space shaded by distance to target




- Use *gradient descent* to walk from starting configuration to target
- Angle restrictions & collisions can introduce local minima

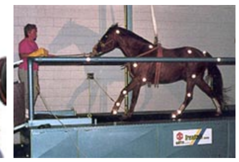
"The good-looking textured light-sourced bouncy fun smart and stretchy page" Hugo Elias, `http://freespace.virgin.net/hugo.elias/models/m_ik2.htm`

## IK Challenge

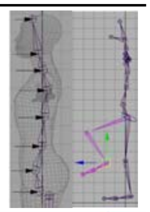
- Find a “natural” skeleton configuration for a given collection of pose constraints
- A *vector constraint function C(p) = 0* collects all pose constraints
- A *scalar objective function* $g(p)$ measures the quality of a pose, $g(p)$ is minimum for most natural poses. Example $g(p)$:
  - deviation from natural pose
  - joint stiffness
  - power consumption

## Motion Capture

- Optical markers, high-speed cameras, triangulation → 3D position
- Captures style, subtle nuances and realism
- You must observe someone do something


## How do they Animate Movies/Games?




Images from the Maya tutorial

- Keyframing mostly
- Articulated figures, inverse kinematics, motion capture
- Skinning
  - Complex deformable skin, muscle, skin motion
- Hierarchical controls
  - Smile control, eye blinking, etc.
  - Keyframes for these higher-level controls
- A huge time is spent building the 3D models, its skeleton and its controls
- Physical simulation for secondary motion
  - Hair, cloth, water

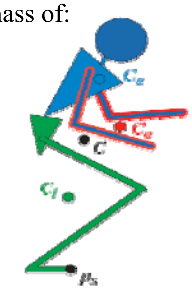
## Questions?

## Liu & Popović, “Synthesis of Complex Dynamic Character Motion from Simple Animation”, 2002.

- Rapid prototyping of realistic character motion *from rough low-quality animations*
- Obey the laws of physics & stay within space of naturally-occurring movements

## What’s a Natural Pose?

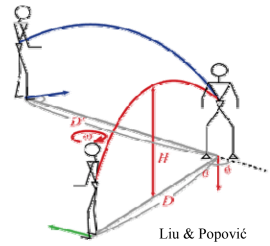
- Training database of ~50 “natural poses”
- For each, compute center of mass of:
  - Upper body
  - Arms
  - Lower body
- The relative COM of each generated pose is matched to most the most similar database example



Liu & Popović

## Linear and Angular Momentum

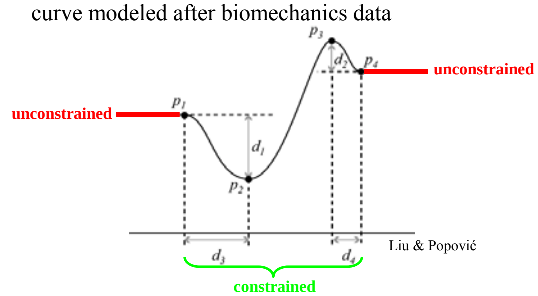
- In unconstrained animation (no contacts), both linear & angular momentum should be conserved
- The center of mass should follow a parabolic trajectory according to gravity
- The joints should move such that the angular momentum of the whole body remains constant



Liu & Popović

## During Constrained Motion

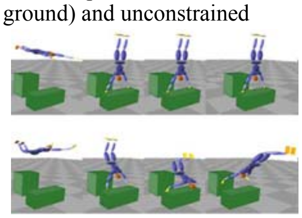
- During *constrained* motion (when in contact with the ground), the angular momentum follows a spline curve modeled after biomechanics data




Liu & Popović

## System Features

- Automatically detect point/line/plane constraints
- Divide animation into constrained portions (e.g., feet in contact with ground) and unconstrained portions (e.g., free flight)
- Linear and angular momentum constraints *without having to compute muscle forces*
- Minimize:
  - Mass displacement
  - Velocity of the degrees of freedom (DOF)
  - "Unbalance" (distance the COM projected to ground is outside of constraints)



## Questions?

## Today

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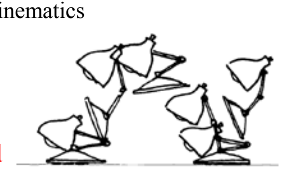
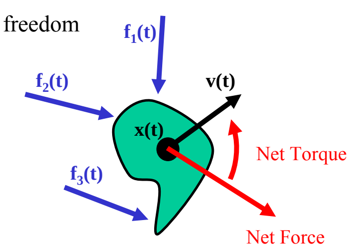
- Rigid Body Dynamics
- Finite Element Method



## Rigid Body Dynamics

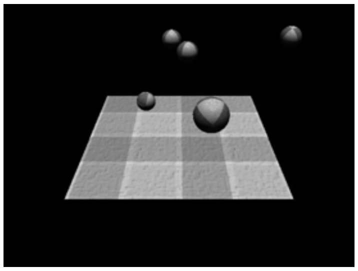
- Could use particles for all points on the object
  - But rigid body does not deform
  - Few degrees of freedom
- Use only one particle at the center of mass
- Compute Net Force & Net Torque




**Nice Reference Material: http://www.pixar.com/companyinfo/research/pbm2001/**

## Rigid Body Dynamics

- Physics
  - Velocity
  - Acceleration
  - Angular Momentum
- Collisions
- Friction



from: Darren Lewis
http://www-cs-students.stanford.edu/~dalewis/cs448a/rigidbody.html

## Simulation of Non-Rigid Objects

- We modeled string & cloth using mass-spring systems. Can we do the same?
- Yes…
- But a more physically accurate model uses *volumetric elements:*

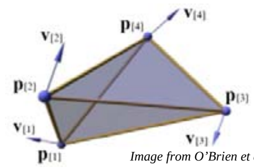
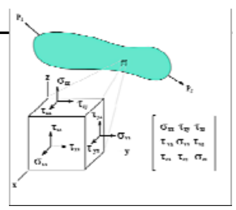



*Image from O'Brien et al. 1999*

## Strain & Stress

- Stress
  - the internal distribution of forces within a body that balance and react to the loads applied to it
  - *normal stress* & *shear stress*
- Strain
  - material deformation caused by stress.
  - measured by the change in length of a line or by the change in angle between two lines



http://en.wikipedia.org/wiki/Image:Stress_tensor.png

$$\varepsilon = \frac{\Delta_l}{l_0}$$

## Finite Element Method

- To solve the continuous problem (deformation of all points of the object)
  - Discretize the problem
  - Express the interrelationship
  - Solve a big linear system
- More principled than Mass-Spring

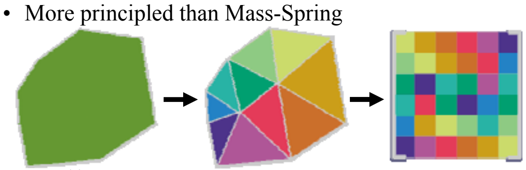
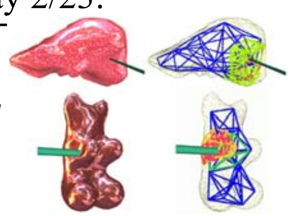
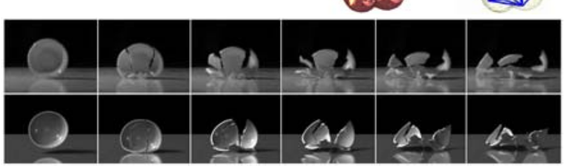



Diagram from Debunne et al. 2001

## Readings for Friday 2/23:

Gilles Debunne , Mathieu Desbrun, Marie-Paule Cani, & Alan H. Barr
*Dynamic Real-Time Deformations using Space & Time Adaptive Sampling*
SIGGRAPH 2001





James O'Brien & Jessica Hodgins "*Graphical Modeling and Animation of Brittle Fracture*" SIGGRAPH 1999