Sampling, Aliasing, & Mipmaps
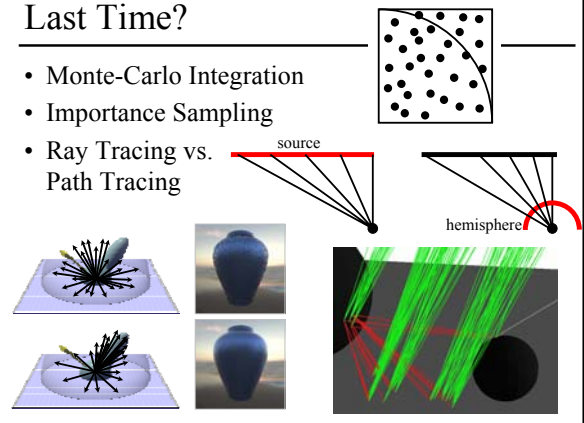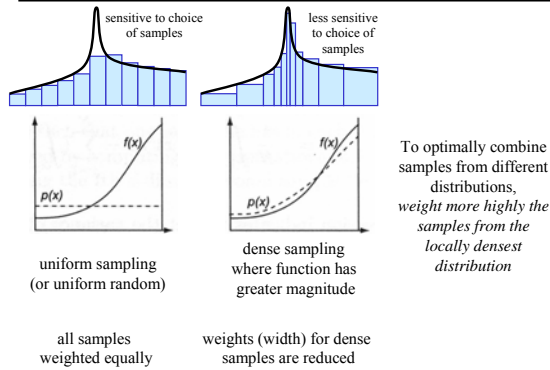

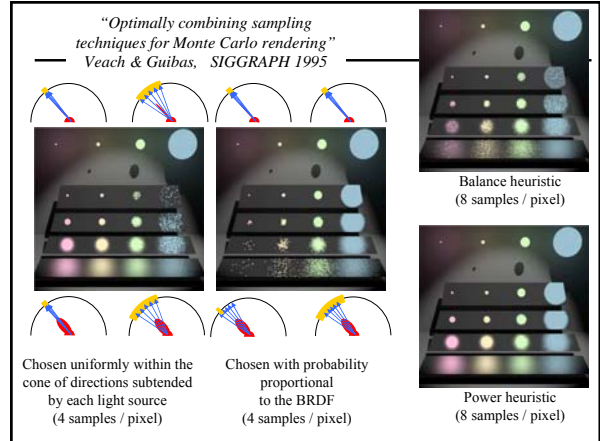## Last Time?

- Monte-Carlo Integration
- Importance Sampling
- Ray Tracing vs. Path Tracing

source

hemisphere

Sampling

sensitive to choice of samples

less sensitive to choice of samples

$f(x)$

$p(x)$

$f(x)$

$p(x)$

uniform sampling (or uniform random)

dense sampling where function has greater magnitude

all samples weighted equally

weights (width) for dense samples are reduced

To optimally combine samples from different distributions, *weight more highly the samples from the locally densest distribution*

"Optimally combining sampling techniques for Monte Carlo rendering"
Veach & Guibas, SIGGRAPH 1995

Chosen uniformly within the cone of directions subtended by each light source (4 samples / pixel)

Chosen with probability proportional to the BRDF (4 samples / pixel)

Balance heuristic (8 samples / pixel)

Power heuristic (8 samples / pixel)

## Today

- What is a Pixel?
- Examples of Aliasing
- Sampling & Reconstruction
- Filters in Computer Graphics
- Anti-Aliasing for Texture Maps

---

What is a Pixel?

- A pixel is not:
  - a box
  - a disk
  - a teeny tiny little light
- A pixel "looks different" on different display devices
- A pixel is a sample
  - it has no dimension
  - it occupies no area
  - it cannot be seen
  - it has a coordinate
  - it has a value

not a box!

not a circle!
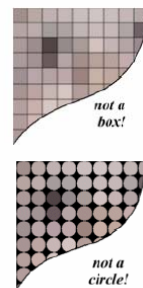
More on Samples

- Most things in the real world are *continuous*, yet everything in a computer is *discrete*
- The process of mapping a continuous function to a discrete one is called *sampling*
- The process of mapping a continuous variable to a discrete one is called *quantization*
- To represent or render an image using a computer, we must both sample and quantize
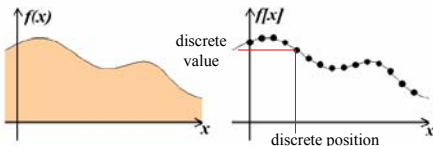
An Image is a 2D Function

- An *ideal image* is a continuous function $I(x,y)$ of intensities.
- It can be plotted as a height field.
- In general an image cannot be represented as a continuous, analytic function.
- Instead we represent images as tabulated functions.
- How do we fill this table?

An image seen as a continuous 2D function

Sampling Grid
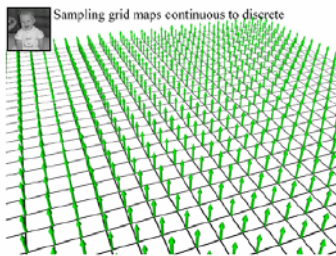
- We can generate the table values by multiplying the continuous image function by a sampling grid of Kronecker delta functions.

The definiton of the 2-D Kronecker delta is:

$$\delta(x,y) = \begin{cases} 1, & (x,y)=(0,0) \\ 0, & \text{otherwise} \end{cases}$$
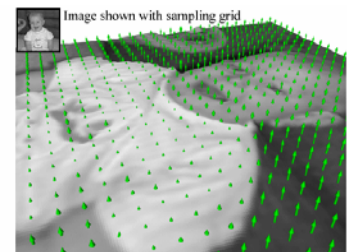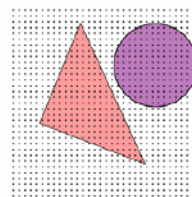
And a 2-D sampling grid:

$$\sum_{j=0}^{h-1}\sum_{i=0}^{w-1}\delta(u-i,v-j)$$

Sampling grid maps continuous to discrete

Sampling an Image

- The result is a set of point samples, or pixels.

The same analysis can be applied to geometric objects:
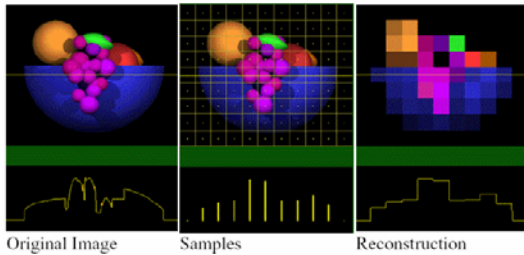
Image shown with sampling grid

Questions?

Today

- What is a Pixel?
- Examples of Aliasing
- Sampling & Reconstruction
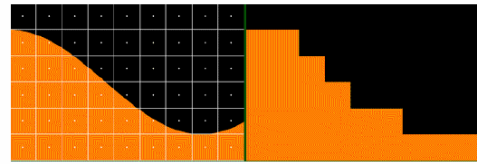- Filters in Computer Graphics
- Anti-Aliasing for Texture Maps

Examples of Aliasing

Original Image   Samples   Reconstruction

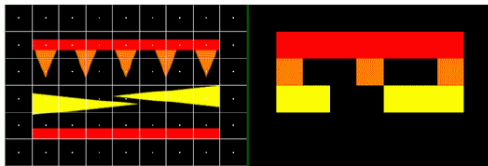- Aliasing occurs because of *sampling* and *reconstruction*

Examples of Aliasing

Jagged boundaries

Examples of Aliasing

Improperly rendered detail

## Examples of Aliasing

Texture Errors

point sampling
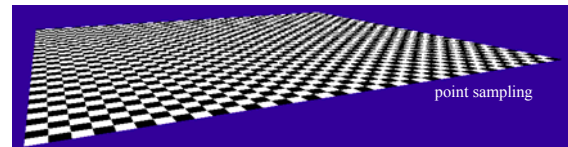
Questions?

Today

- What is a Pixel?
- Examples of Aliasing
- Sampling & Reconstruction
  - Sampling Density
  - Fourier Analysis & Convolution
- Filters in Computer Graphics
- Anti-Aliasing for Texture Maps
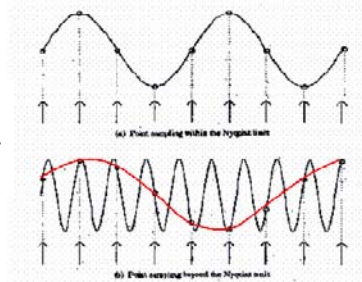
Sampling Density

- How densely must we sample an image in order to capture its essence?

- If we under-sample the signal, we won't be able to accurately reconstruct it...
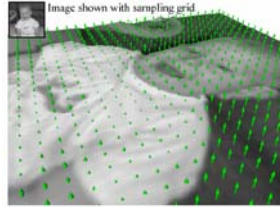
Sampling Density

- If we insufficiently sample the signal, it may be mistaken for something simpler during reconstruction (that's aliasing!)

Image from Robert L. Cook, "Stochastic Sampling and Distributed Ray Tracing", An Introduction to Ray Tracing, Andrew Glassner, ed., Academic Press Limited, 1989.
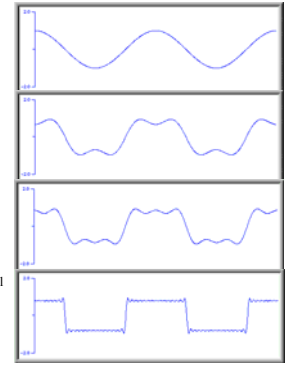
Sampling Density

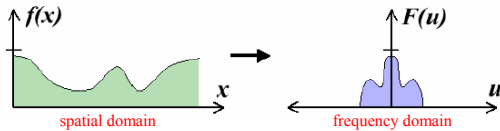- Aliasing in 2D because of insufficient sampling density
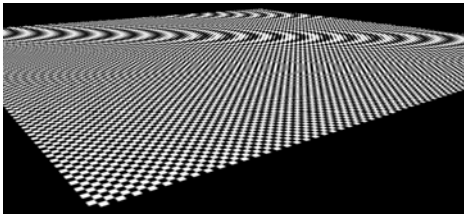
Remember Fourier Analysis?

- All periodic signals can be represented as a summation of sinusoidal waves.

Images from
http://axion.physics.ubc.ca/341-02/fourier/fourier.html

Remember Fourier Analysis?

- Every periodic signal in the *spatial domain* has a dual in the *frequency domain*.

$f(x)$

x

spatial domain

$F(u)$

u

frequency domain

- This particular signal is *band-limited*, meaning it has no frequencies above some threshold
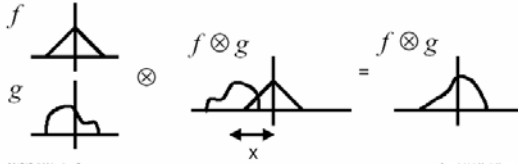
Remember Fourier Analysis?

- We can transform from one domain to the other using the Fourier Transform.

frequency domain   spatial domain

Fourier Transform
$$F(u,v) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} f(x,y)\, e^{-i2\pi(ux+vy)}\,dxdy$$

Inverse Fourier Transform
$$f(x,y) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} F(u,v)\, e^{i2\pi(ux+vy)}\,dudv$$

Remember Convolution?

Convolution describes how a system with impulse response, *h(x)*, reacts to a signal, *f(x)*.

$$f(x) * h(x) = \int_{-\infty}^{\infty} f(\lambda) h(x-\lambda) d\lambda$$

Images from Mark Meyer
http://www.gg.caltech.edu/~cs174ta/

Remember Convolution?

- Some operations that are difficult to compute in the spatial domain can be simplified by transforming to its dual representation in the frequency domain.
- For example, convolution in the spatial domain is the same as multiplication in the frequency domain.

$$f(x) * h(x) \rightarrow F(u)H(u)$$

- And, convolution in the frequency domain is the same as multiplication in the spatial domain

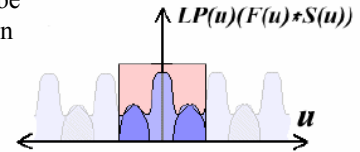$$F(u) * H(u) \rightarrow f(x)h(x)$$

Sampling in the Frequency Domain
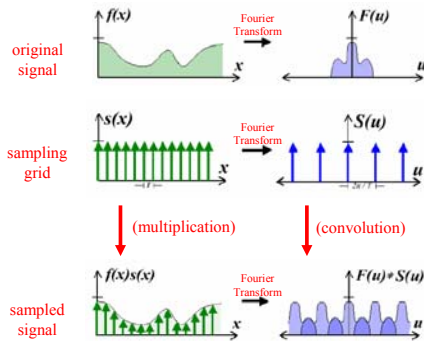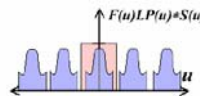

Reconstruction

- If we can extract a copy of the original signal from the frequency domain of the sampled signal, we can reconstruct the original signal!

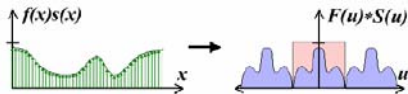- But there may be overlap between the copies.

Guaranteeing Proper Reconstruction

- Separate by removing high frequencies from the original signal (low pass pre-filtering)

- Separate by increasing the sampling density

- If we can't separate the copies, we will have overlapping frequency spectrum during reconstruction → *aliasing*.

Sampling Theorem

- When sampling a signal at discrete intervals, the sampling frequency must be *greater than twice* the highest frequency of the input signal in order to be able to reconstruct the original perfectly from the sampled version (Shannon, Nyquist)

Questions?

Today

- What is a Pixel?
- Examples of Aliasing
- Sampling & Reconstruction
- Filters in Computer Graphics
  - Pre-Filtering, Post-Filtering
  - Ideal, Gaussian, Box, Bilinear, Bicubic
- Anti-Aliasing for Texture Maps

Filters

- Weighting function or a convolution kernel
- Area of influence often bigger than "pixel"
- Sum of weights = 1
  - Each sample contributes the same total to image
  - Constant brightness as object moves across the screen.
- No negative weights/colors (optional)
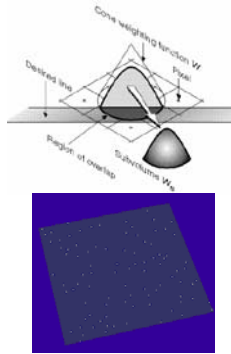
Filters

- Filters are used to
  - reconstruct a continuous signal from a sampled signal (reconstruction filters)
  - band-limit continuous signals to avoid aliasing during sampling (low-pass filters)
- Desired frequency domain properties are the same for both types of filters
- Often, the same filters are used as reconstruction and low-pass filters

Pre-Filtering

- Filter continuous primitives
- Treat a pixel as an area
- Compute weighted amount of object overlap
- What weighting function should we use?
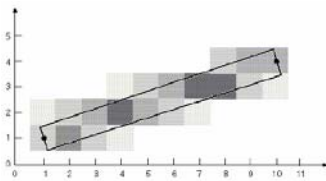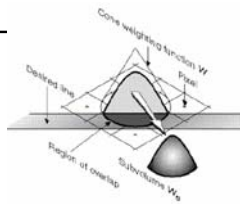
Source: Foley, VanDam, Feiner, Hughes - Computer Graphics, Second Edition, Addison Wesley

Post-Filtering

- Filter samples
- Compute the weighted average of many samples
- Regular or jittered sampling (better)

Regular   Jittered
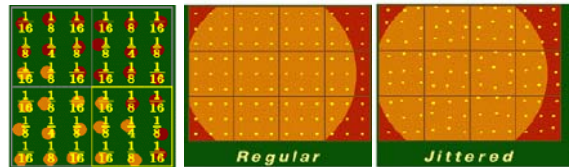
The Ideal Filter

- Unfortunately it has *infinite* spatial extent
  - Every sample contributes to every interpolated point
- Expensive/impossible to compute

sinc(x)
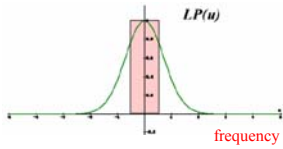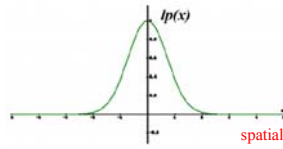
spatial

$LP(u)(F(u) \ast S(u))$

u
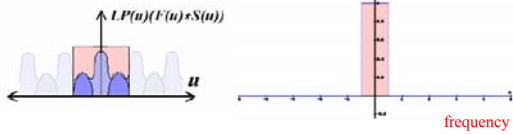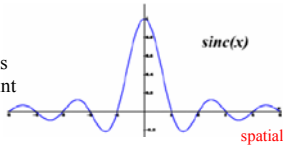
frequency

Problems with Practical Filters

- Many visible artifacts in re-sampled images are caused by poor reconstruction filters
- Excessive pass-band attenuation results in blurry images
- Excessive high-frequency leakage causes "ringing" and can accentuate the sampling grid (anisotropy)
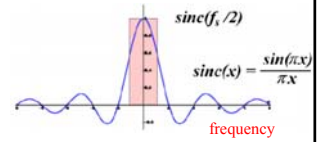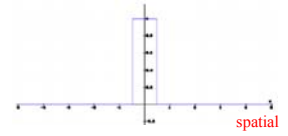
High-frequency leakage

"Pass-Band" Attenuation

frequency
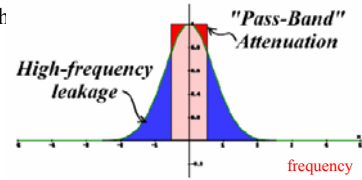
Gaussian Filter

- This is what a CRT does for free!
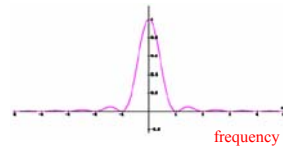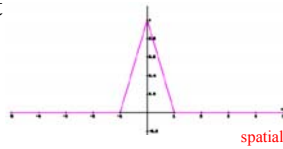
lp(x)

spatial

LP(u)

frequency

Box Filter / Nearest Neighbor

- Pretending pixels are little squares.

spatial

$sinc(f_x /2)$

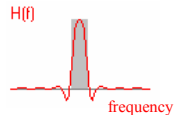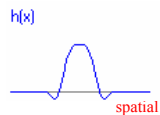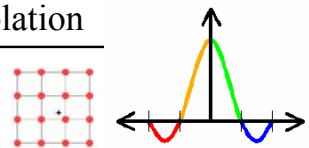$sinc(x) = \dfrac{sin(\pi x)}{\pi x}$

frequency

Tent Filter / Bi-Linear Interpolation

- Simple to implement
- Reasonably smooth

spatial

frequency

Bi-Cubic Interpolation

- Begins to approximate the ideal spatial filter, the sinc function

h(x)

spatial

H(f)

frequency

Questions?

Today

- What is a Pixel?
- Examples of Aliasing
- Sampling & Reconstruction
- Filters in Computer Graphics
- Anti-Aliasing for Texture Maps
  - Magnification & Minification
  - Mipmaps
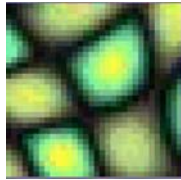 - Anisotropic Mipmaps

Sampling Texture Maps

- When texture mapping it is rare that the screen-space sampling density matches the sampling density of the texture.
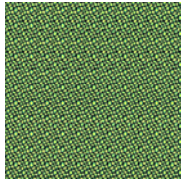

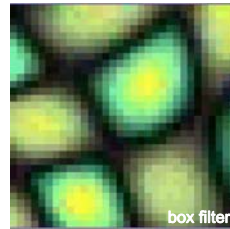64x64 pixels

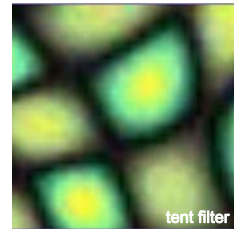Original Texture   Magnification for Display   Minification for Display

for which we must use a reconstruction filter

Linear Interpolation

- Tell OpenGL to use a tent filter instead of a box filter.
- Magnification looks better, but blurry
  - (texture is under-sampled for this resolution)

box filter   tent filter

Spatial Filtering

- Remove the high frequencies which cause artifacts in texture minification.
- Compute a spatial integration over the extent of the pixel
- This is equivalent to convolving the texture with a filter kernel centered at the sample (i.e., pixel center)!
- Expensive to do during rasterization, but an approximation it can be precomputed

projected texture in image plane
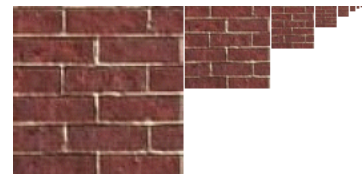

box filter in texture plane

MIP Mapping

- Construct a pyramid of images that are pre-filtered and re-sampled at 1/2, 1/4, 1/8, etc., of the original image's sampling

- During rasterization we compute the index of the decimated image that is sampled at a rate closest to the density of our desired sampling rate
- MIP stands for *multum in parvo* which means *many in a small place*

MIP Mapping Example

- Thin lines may become disconnected / disappear

Nearest Neighbor   MIP Mapped (Bi-Linear)

MIP Mapping Example

- Small details may "pop" in and out of view

Nearest Neighbor   MIP Mapped (Bi-Linear)

Examples of Aliasing

Texture Errors

point sampling

mipmaps & linear interpolation

Storing MIP Maps

- Can be stored compactly
- Illustrates the 1/3 overhead of maintaining the MIP map

10-level mip map   Memory format of a mip map

Anisotropic MIP-Mapping

- What happens when the surface is tilted?

Nearest Neighbor   MIP Mapped (Bi-Linear)

Anisotropic MIP-Mapping
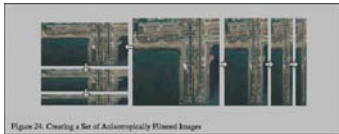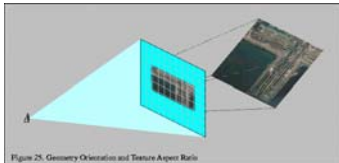

textured surface (texture map)

area pre-filtered in MIP-map

circular pixel window   image plane

- Square MIP-map area is a bad approximation

Anisotropic MIP-Mapping

- We can use different mipmaps for the 2 directions
- Additional extensions can handle non axis-aligned views

Figure 25. Geometry Orientation and Texture Aspect Ratio

Figure 24. Creating a Set of Anisotropically Filtered Images

Images from http://www.sgi.com/software/opengl/advanced98/notes/node37.html

Reading for Today:

Statistical Acceleration for Animated Global Illumination,
Meyer & Anderson,
SIGGRAPH 2006

Readings for Friday (3/30):

Global Illumination using Photon Maps,
Henrik Wann Jensen,
Rendering Techniques 1996

A Ray Tracing Solution for Diffuse Interreflection
Ward et al.
SIGGRAPH 1988