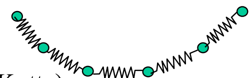
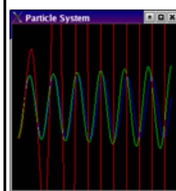
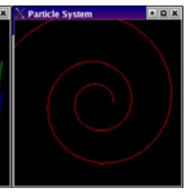
Voxels & Collisions

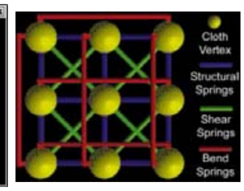
Last Time?

- Spring-Mass Systems
- Numerical Integration (Euler, Midpoint, Runge-Kutta)
- Modeling string, hair, & cloth









Today

- More on Cloth!
  - Taylor Series Analysis
  - Stiffness
  - Implicit Integration
- Implicit Surfaces
- Voxels
- Collisions
- Untangling Cloth

Analysis using Taylor Series

- Expand exact solution $\mathbf{X}(t)$

$$\mathbf{X}(t_0+h) = \mathbf{X}(t_0) + h\left(\tfrac{d}{dt}\mathbf{X}(t)\right)\Big|_{t_0} + \tfrac{h^2}{2!}\left(\tfrac{d^2}{dt^2}\mathbf{X}(t)\right)\Big|_{t_0} + \tfrac{h^3}{3!}(\cdots) + \cdots$$

- Euler's method:

$$\mathbf{X}(t_0+h) = \mathbf{X}_0 + h\,f(\mathbf{X}_0, t_0) \quad \ldots + O(h^2)\,\text{error}$$

$$h \to h/2 \;\Rightarrow\; error \to error/4 \text{ per step} \times \text{twice as many steps}$$
$$\to error/2$$

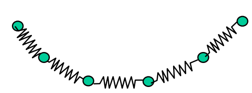
- First-order method: Accuracy varies with h
  - To get 100x better accuracy need 100x more steps

The Stiffness Issue

- What relative stiffness do we want for the different springs in the network?
- Cloth is barely elastic, shouldn't stretch so much!
- Inverse relationship between stiffness & Δt
- We really want a constraints (not springs)
- Many numerical solutions
  - reduce Δt
  - use constraints
  - implicit integration
  - …

How would you simulate a string?

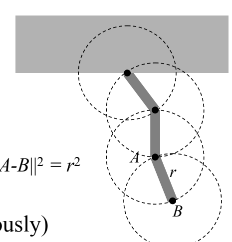
- Springs link the particles. Problems?
  - Stretch, actual length will be greater than rest length
  - Numerical oscillation



- Rigid, fixed-length bars link the particles
  - Dynamics &
  - Constraints

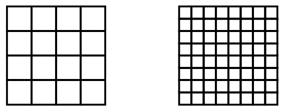
  (must be solved simultaneously)

$||A-B||^2 = r^2$



The Discretization Problem

- What happens if we discretize our cloth more finely, or with a different mesh structure?



- Do we get the same behavior?
  - Usually not! It takes a lot of effort to design a scheme that does not depend on the discretization.
- Using (explicit) Euler, how many timesteps before a force propagates across the mesh?

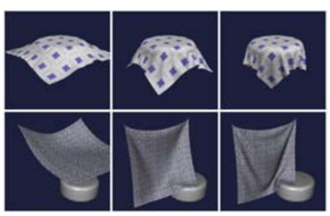
Explicit vs. Implicit Integration

- With an explicit/forward integration scheme:
  $$\mathbf{y}_{k+1} = \mathbf{y}_k + h\, \mathbf{g}(y_k)$$
  we must use a very small timestep to simulate *stable, stiff* cloth.
- Alternatively we can use an implicit/backwards scheme:
  $$\mathbf{y}_{k+1} = \mathbf{y}_k + h\, \mathbf{g}(y_{k+1})$$
  $$\mathbf{y}_k = \mathbf{y}_{k+1} - h\, \mathbf{g}(y_{k+1})$$
  Solving one step is much more expensive (Newton's Method, Conjugate Gradients, …) but overall faster than the thousands of explicit timesteps required for very stiff springs.

Questions?

David Baraff & Andrew Witkin
Large Steps in Cloth Simulation
SIGGRAPH 1998

- Dynamic motion driven by animation



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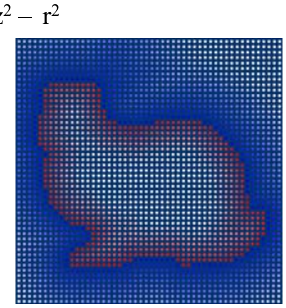
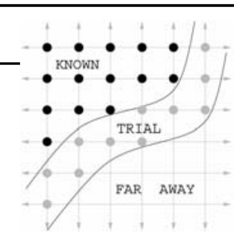
Implicit Surfaces

- For a sphere:
  $H(x,y,z) = x^2 + y^2 + z^2 - r^2$
- If $H(x,y,z) = 0$, on surface
- If $H(x,y,z) > 0$, outside surface
- If $H(x,y,z) < 0$, inside surface

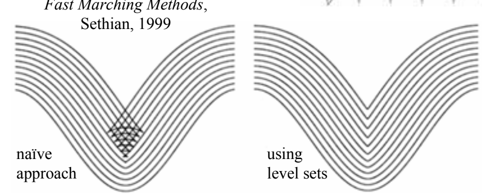


Level Sets

- Efficient method for computing signed distance field

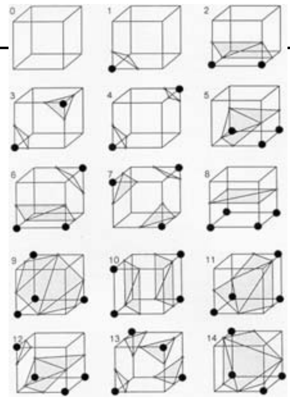


Level Set Methods and Fast Marching Methods, Sethian, 1999



Marching Cubes

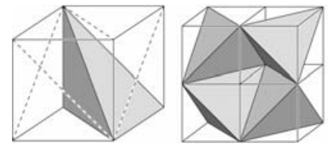
- Polygonization: extract triangle mesh from signed distance field



"Marching Cubes: A High Resolution 3D Surface Construction Algorithm", Lorensen and Cline, SIGGRAPH '87.

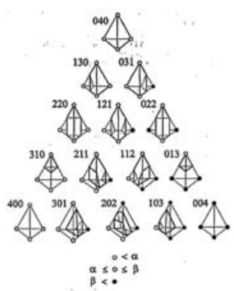
"Marching Tetrahedra"

Jules Bloomenthal
"An implicit surface polygonizer"
Graphics Gems IV



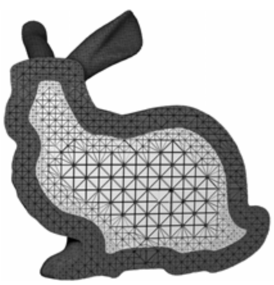
"When the Blobs Go Marching Two by Two",
Jeff Lander, Gamasutra

"Marching Tetrahedra"



"Interval volume tetrahedrization"
Visualization '97
Nielson & Sung

Similarly, we can create volumetric models:



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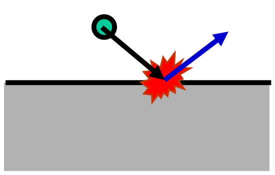
Collisions

- Detection
- Response
- Overshooting problem (when we enter the solid)



Detecting Collisions

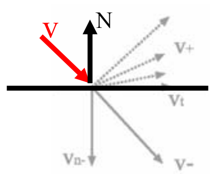
- Easy with implicit equations of surfaces
- H(x,y,z)=0 at surface
- H(x,y,z)<0 inside surface
- So just compute H and you know that you're inside if it's negative

- More complex with other surface definitions

Collision Response

- tangential velocity v_t unchanged
- normal velocity v_n reflects:

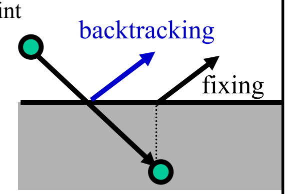
$$v = v_t + v_n$$
$$v \leftarrow v_t - \varepsilon v_n$$



- coefficient of restitution (1 for elastic, 0 for plastic)
- change of velocity = *-(1+ε)v*
- change of momentum *Impulse* = *-m(1+ε)v*

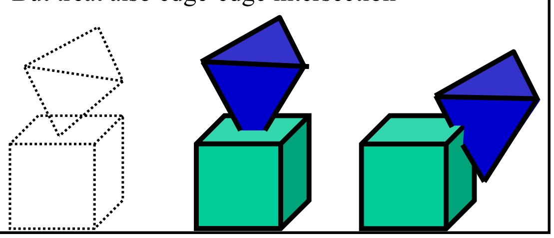
Collisions - Overshooting

- Usually, we detect collision when it's too late: we're already inside
- Solutions: back up
  - Compute intersection point
  - Compute response there
  - Advance for remaining fractional time step
- Other solution: Quick and dirty fixup
  - Just project back to object closest point



Collision Detection for Solids

- How to detect collision between 2 polyhedra?
- Need an inside/outside test
- Test if a vertex is inside the other polyhedron
- But treat also edge-edge intersection

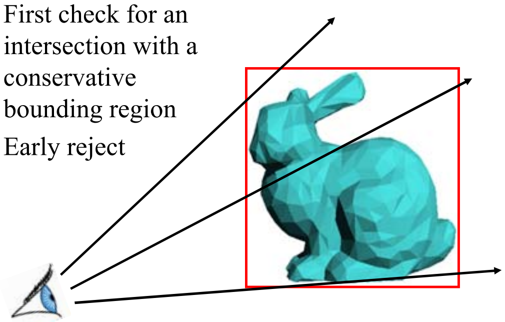


Cost of Detection?

- Test each edge with each face?
  - $O(N^2)$
- How would you detect collision between two bunnies?
  - $O(N^2)$ is too expensive!
  - Use spatial hierarchy

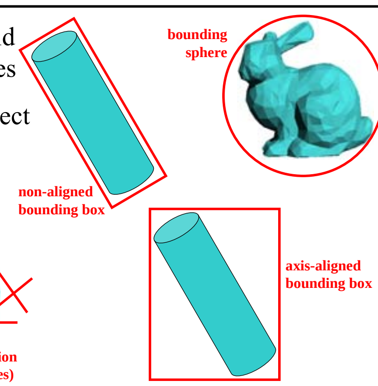
Conservative Bounding Region

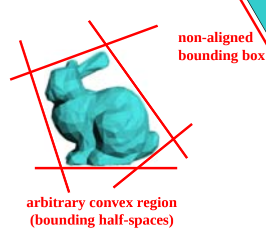
- First check for an intersection with a conservative bounding region
- Early reject



Conservative Bounding Regions

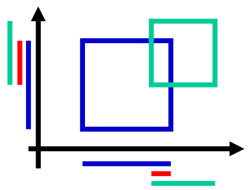
- tight → avoid false positives
- fast to intersect

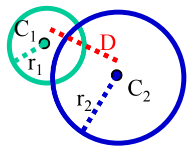




Overlap test

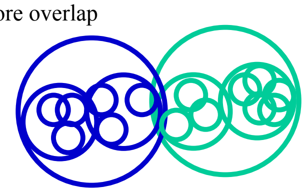
- Overlap between two axis-aligned boxes?
  - Check if the intervals along the 3 dimensions overlap
- Overlap test between two spheres?
  - $D(center_1, center_2) < r_1+r_2$





General Collision Detection

- Put a hierarchy around your objects
- Use the fast overlap test recursively
- Handle exact case at the leaves (when necessary)
- More difficult for self-collision (e.g. cloth)
  - Because there is more overlap



Reduced Deformation

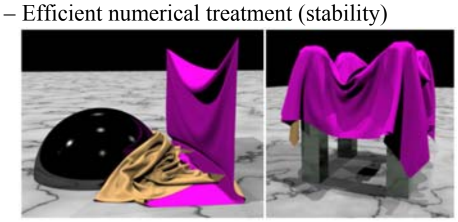
Doug L. James & Dinesh K. Pai
BD-Tree: Output-Sensitive Collision Detection for Reduced Deformable Models
SIGGRAPH 2004

- Collisions are expensive
- Deformation is expensive
- This is a lot of geometry!
- Simplify the simulation model

Cloth Collision

Robert Bridson, Ronald Fedkiw & John Anderson
Robust Treatment of Collisions, Contact and Friction for Cloth Animation
SIGGRAPH 2002

- A cloth has many points of contact
- Stays in contact
- Requires
  - Efficient collision detection
  - Efficient numerical treatment (stability)



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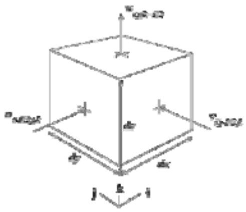
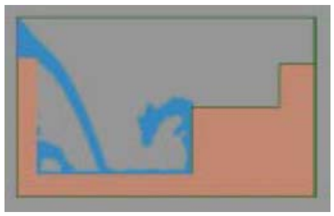
Reading for Today:

- Baraff, Witkin & Kass, *Untangling Cloth,* SIGGRAPH 2003

Reading for Friday 2/8:

- “Realistic Animation of Liquids”,
  Foster & Metaxas, 1996





- Post a comment or question on the LMS discussion by 10am on Friday 2/8

Reading for Tuesday 2/12:

- “Synthesis of Complex Dynamic Character Motion from Simple Animation”, Liu & Popović, 2002.

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