CSCI-4530/6530
Advanced Computer Graphics

http://www.cs.rpi.edu/~cutler/classes/advancedgraphics/S09/

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MRC 309A

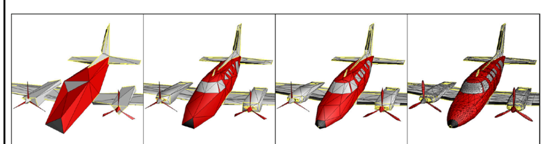
Luxo Jr.

Pixar Animation Studios, 1986

Topics for the Semester

- Meshes
  - representation
  - simplification
  - subdivision surfaces
  - generation
  - volumetric modeling
- Simulation
  - particle systems
  - rigid body, deformation, cloth, wind/water flows
  - collision detection
  - weathering
- Rendering
  - ray tracing
  - appearance models
  - shadows
  - local vs. global illumination
  - radiosity, photon mapping, subsurface scattering, etc.
- procedural modeling
- texture synthesis
- hardware & more …

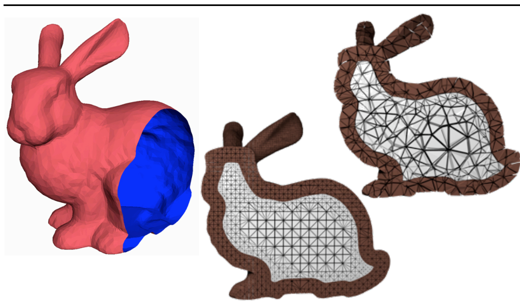
Mesh Simplification



(a) Base mesh M^0 (150 faces) (b) Mesh M^{175} (500 faces) (c) Mesh M^{425} (1,000 faces) (d) Original $\hat{M}=M^n$ (13,546 faces)

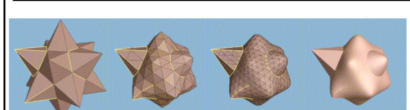
Hoppe “Progressive Meshes” SIGGRAPH 1996

Mesh Generation & Volumetric Modeling

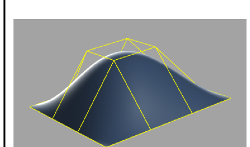


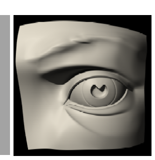
Cutler et al., “Simplification and Improvement of Tetrahedral Models for Simulation” 2004

Modeling – Subdivision Surfaces



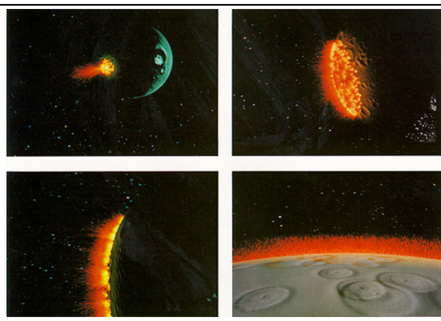
Hoppe et al., “Piecewise Smooth Surface Reconstruction” 1994





Geri’s Game
Pixar 1997

Particle Systems



Star Trek: The Wrath of Khan 1982

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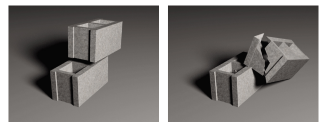
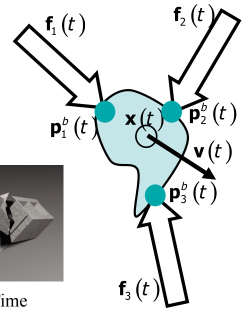
Physical Simulation

- Rigid Body Dynamics
- Collision Detection
- Fracture
- Deformation



Müller et al., "Stable Real-Time Deformations" 2002



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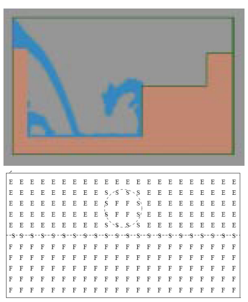
Fluid Dynamics



Foster & Mataxas, 1996

"Visual Simulation of Smoke"
Fedkiw, Stam & Jensen
SIGGRAPH 2001

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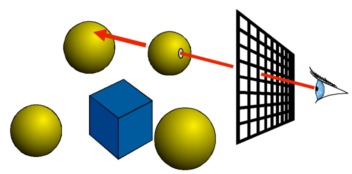
Ray Casting

- For every pixel
  construct a ray from the eye
  - For every object in the scene
    - Find intersection with the ray
    - Keep the closest



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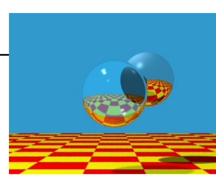
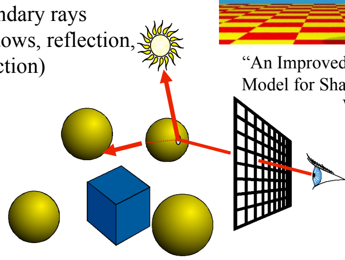
Ray Tracing

- Shade (interaction of light and material)
- Secondary rays (shadows, reflection, refraction)



"An Improved Illumination Model for Shaded Display"
Whitted 1980



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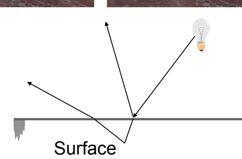
Subsurface Scattering



Jensen et al., "A Practical Model for Subsurface Light Transport" 2001

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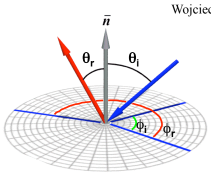
Appearance Models

Wojciech Matusik



Henrik Wann Jensen

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Syllabus & Course Website

http://www.cs.rpi.edu/~cutler/classes/advancedgraphics/S09/

- Which version should I register for?
  - CSCI 6530
    - 3 units of graduate credit
  - CSCI 4530
    - 4 units of undergraduate credit

  same lectures, assignments, quizzes, & grading criteria
- Other Questions?

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Introductions – Who are you?

- name
- year/degree
- graphics background (if any)
- research/job interests
- why you are taking this class
- something fun, interesting, or unusual about yourself

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Outline

- Course Overview
- Classes of Transformations
- Representing Transformations
- Combining Transformations
- Orthographic & Perspective Projections
- Example: Iterated Function Systems (IFS)
- OpenGL Basics

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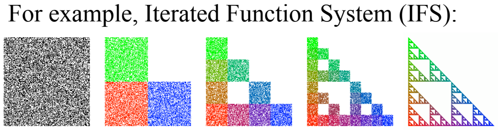
What is a Transformation?

- Maps points (x, y) in one coordinate system to points (x', y') in another coordinate system

$$x' = ax + by + c$$
$$y' = dx + ey + f$$

- For example, Iterated Function System (IFS):



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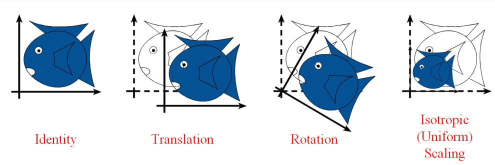
Simple Transformations



- Can be combined
- Are these operations invertible?

  Yes, except scale = 0

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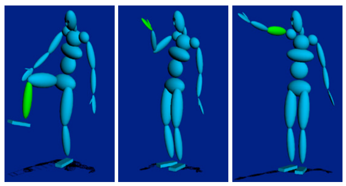
Transformations are used to:

- Position objects in a scene
- Change the shape of objects
- Create multiple copies of objects
- Projection for virtual cameras
- Describe animations



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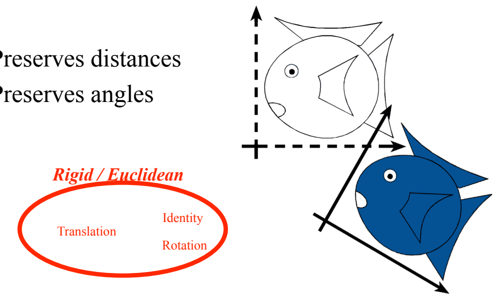
Rigid-Body / Euclidean Transforms

- Preserves distances
- Preserves angles



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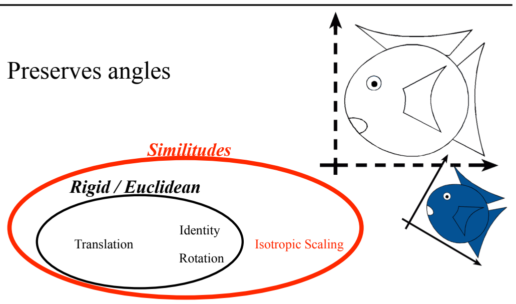
Similitudes / Similarity Transforms

- Preserves angles



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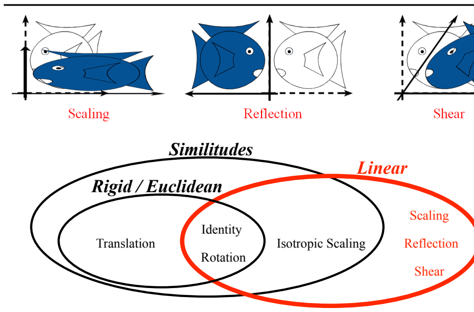
Linear Transformations



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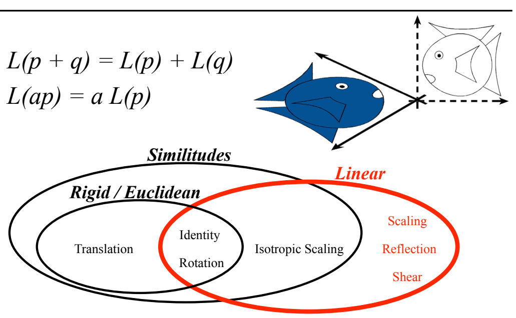
Linear Transformations

- $L(p + q) = L(p) + L(q)$
- $L(ap) = a\,L(p)$



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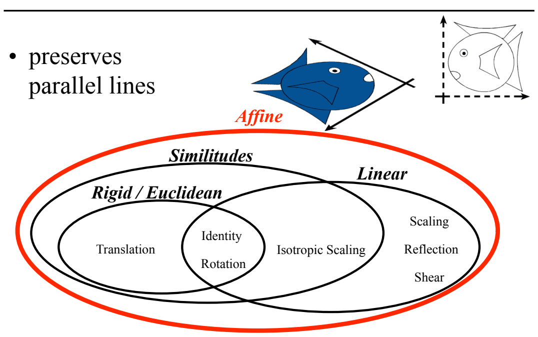
Affine Transformations

- preserves parallel lines



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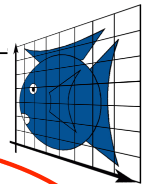
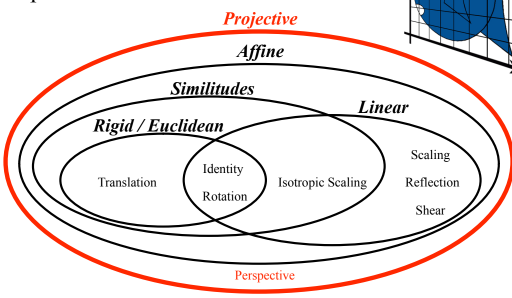
Projective Transformations

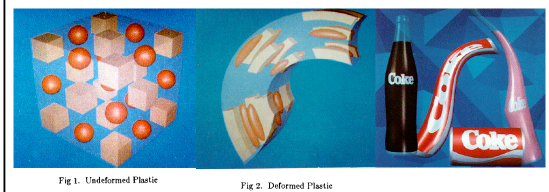
- preserves lines





General (Free-Form) Transformation

- Does not preserve lines
- Not as pervasive, computationally more involved



Sederberg and Parry, Siggraph 1986

Outline

- Course Overview
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- Combining Transformations
- Orthographic & Perspective Projections
- Example: Iterated Function Systems (IFS)
- OpenGL Basics

How are Transforms Represented?

$$x' = ax + by + c$$
$$y' = dx + ey + f$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ d & e \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} c \\ f \end{pmatrix}$$

$$p' = M p + t$$

Homogeneous Coordinates

- Add an extra dimension
  - in 2D, we use 3 x 3 matrices
  - In 3D, we use 4 x 4 matrices
- Each point has an extra value, w

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

$$p' = M p$$

Translation in homogeneous coordinates

$$x' = ax + by + c$$
$$y' = dx + ey + f$$

Affine formulation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ d & e \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} c \\ f \end{pmatrix}$$

$$p' = M p + t$$

Homogeneous formulation

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$p' = M p$$

Homogeneous Coordinates

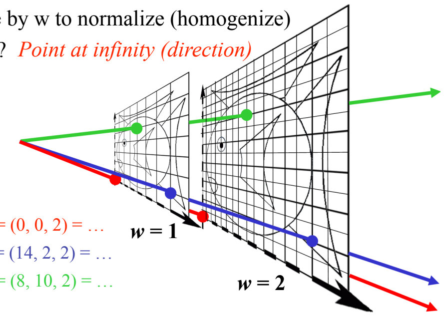
- Most of the time w = 1, and we can ignore it

$$\begin{bmatrix} x' \\ y' \\ z' \\ \color{red}{1} \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ \color{red}{0} & \color{red}{0} & \color{red}{0} & \color{red}{1} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \color{red}{1} \end{bmatrix}$$

- If we multiply a homogeneous coordinate by an *affine matrix*, w is unchanged

Homogeneous Visualization

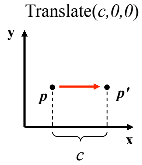
- Divide by w to normalize (homogenize)
- W = 0? *Point at infinity (direction)*



(0, 0, 1) = (0, 0, 2) = …
(7, 1, 1) = (14, 2, 2) = …
(4, 5, 1) = (8, 10, 2) = …

Translate (tx, ty, tz)

Translate(c,0,0)

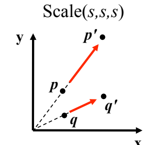


- Why bother with the extra dimension?
  Because now translations can be encoded in the matrix!

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Scale (sx, sy, sz)

Scale(s,s,s)

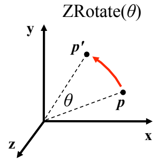


- Isotropic (uniform) scaling: $s_x = s_y = s_z$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotation

ZRotate(θ)

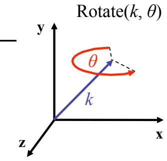


- About z axis

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotation

Rotate(k, θ)



- About (k_x, k_y, k_z), a unit vector on an arbitrary axis (Rodrigues Formula)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} k_xk_x(1-c)+c & k_zk_x(1-c)-k_zs & k_xk_z(1-c)+k_ys & 0 \\ k_yk_x(1-c)+k_zs & k_zk_x(1-c)+c & k_yk_z(1-c)-k_xs & 0 \\ k_zk_x(1-c)-k_ys & k_zk_x(1-c)-k_xs & k_zk_z(1-c)+c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

where $c = \cos\theta$ & $s = \sin\theta$

Storage

- Often, w is not stored (always 1)
- Needs careful handling of direction vs. point
  - Mathematically, the simplest is to encode directions with $w = 0$
  - In terms of storage, using a 3-component array for both direction and points is more efficient
  - Which requires to have special operation routines for points vs. directions

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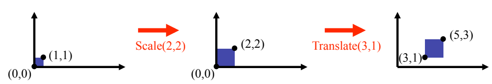
How are transforms combined?

Scale then Translate



Use matrix multiplication: p' = T (S p) = TS p

$$TS = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Caution: matrix multiplication is NOT commutative!

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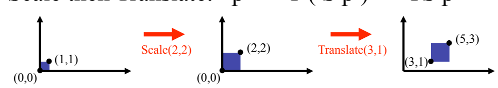
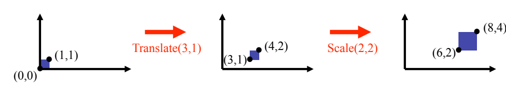
Non-commutative Composition

Scale then Translate: p' = T (S p) = TS p



Translate then Scale: p' = S (T p) = ST p



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Non-commutative Composition

Scale then Translate: p' = T (S p) = TS p

$$TS = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Translate then Scale: p' = S (T p) = ST p

$$ST = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 6 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

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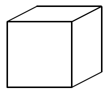
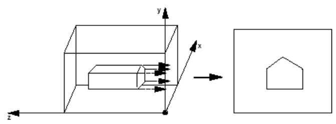
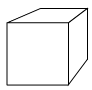
Orthographic vs. Perspective

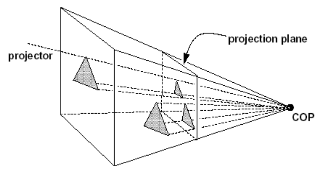
- Orthographic





- Perspective





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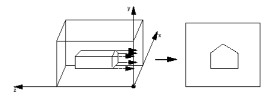
Simple Orthographic Projection

- Project all points along the z axis to the $z = 0$ plane



$$\begin{bmatrix} x \\ y \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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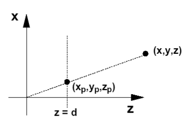
Simple Perspective Projection

- Project all points along the z axis to the $z = d$ plane, eyepoint at the origin:

$$x_p = \frac{d \cdot x}{z} = \frac{x}{z/d} \qquad y_p = \frac{d \cdot y}{z} = \frac{y}{z/d} \qquad z_p = d$$



homogenize

$$\begin{bmatrix} x * d / z \\ y * d / z \\ d \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z / d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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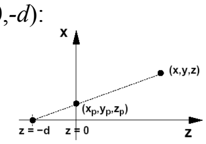
Alternate Perspective Projection

- Project all points along the z axis to the $z = 0$ plane, eyepoint at the $(0,0,-d)$:

$$x_p = \frac{d \cdot x}{z + d} = \frac{x}{(z/d) + 1} \qquad y_p = \frac{d \cdot y}{z + d} = \frac{y}{(z/d) + 1}$$



homogenize

$$\begin{bmatrix} x * d / (z + d) \\ y * d / (z + d) \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \\ (z + d)/ d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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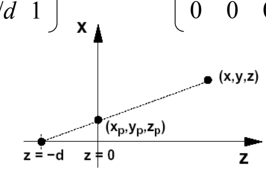
In the limit, as $d \rightarrow \infty$

this perspective projection matrix... $\rightarrow$ ...is simply an orthographic projection

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



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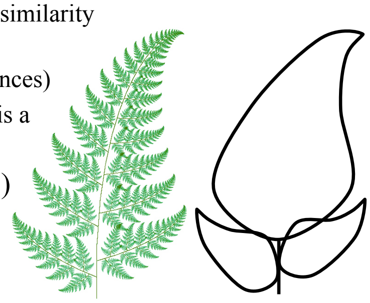
Iterated Function Systems (IFS)

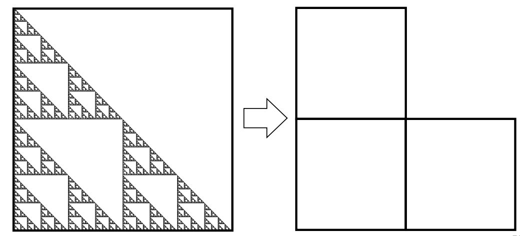
- Capture self-similarity
- Contraction (reduce distances)
- An attractor is a fixed point

$$A = \bigcup f_i(A)$$



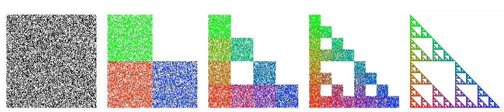
Example: Sierpinski Triangle

- Described by a set of *n* affine transformations
- In this case, $n = 3$
  - translate & scale by 0.5



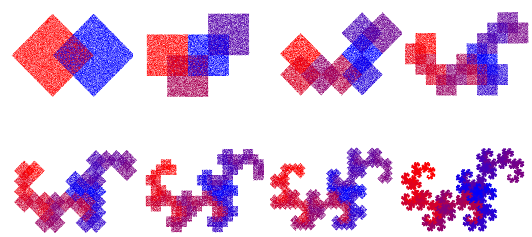
Example: Sierpinski Triangle

```
for "lots" of random input points (x_0, y_0)
   for j=0 to num_iters
      randomly pick one of the transformations
      (x_k+1, y_k+1) = f_i (x_k, y_k)
   display (x_k, y_k)
```



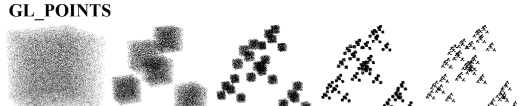
Increasing the number of iterations

Another IFS: The Dragon

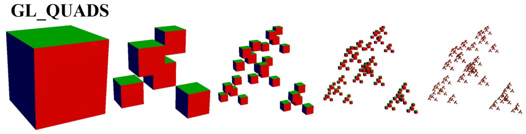


3D IFS in OpenGL

GL_POINTS

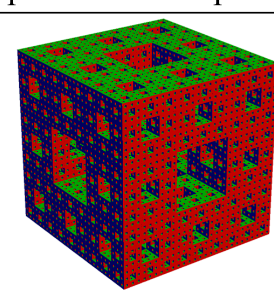


GL_QUADS



Assignment 0: OpenGL Warmup

- Get familiar with:
  - C++ environment
  - OpenGL
  - Transformations
  - simple Vector & Matrix classes
- Have Fun!



- Will not be graded
  (but you should still do it and submit it!)

Outline

- Course Overview
- Classes of Transformations
- Representing Transformations
- Combining Transformations
- Orthographic & Perspective Projections
- Example: Iterated Function Systems (IFS)
- OpenGL Basics

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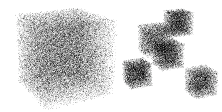
OpenGL Basics: GL_POINTS

```
glDisable(GL_LIGHTING);
glBegin(GL_POINTS);
glColor3f(0.0,0.0,0.0);
glVertex3f(…);
glEnd();
```



- **lighting should be *disabled*...**

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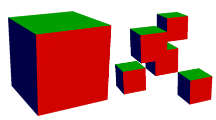
OpenGL Basics: GL_QUADS

```
glEnable(GL_LIGHTING);
glBegin(GL_QUADS);
glNormal3f(…);
glColor3f(1.0,0.0,0.0);
glVertex3f(…);
glVertex3f(…);
glVertex3f(…);
glVertex3f(…);
glEnd();
```



- **lighting should be *enabled*...**
- **an appropriate normal should be specified**

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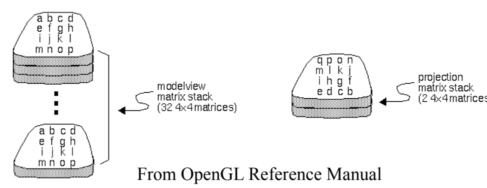
OpenGL Basics: Transformations

- Useful commands:

```
glMatrixMode(GL_MODELVIEW);
glPushMatrix();
glPopMatrix();
glMultMatrixf(…);
```



From OpenGL Reference Manual

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Questions?

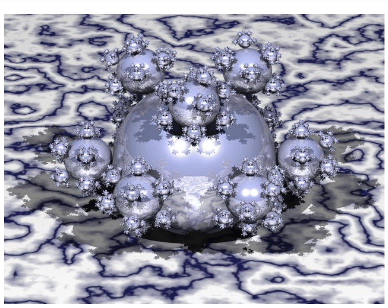

Image by Henrik Wann Jensen

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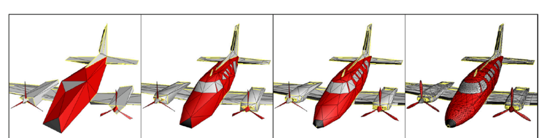
For Next Time:

- Read Hugues Hoppe "Progressive Meshes" SIGGRAPH 1996
- Post a comment or question on the course WebCT/LMS discussion by 10am on Friday 1/15



(a) Base mesh M^0 (150 faces) (b) Mesh M^{175} (500 faces) (c) Mesh M^{425} (1,000 faces) (d) Original $\hat{M} = M^n$ (13,546 faces)

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