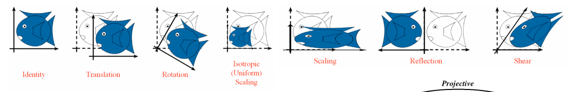
Adjacency Data Structures

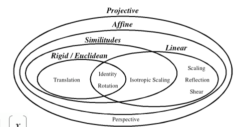
material from Justin Legakis

Last Time?

- Simple Transformations



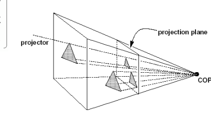
- Classes of Transformations



- Representation
  - homogeneous coordinates
- Composition
  - not commutative

$$\begin{bmatrix}x'\\y'\\z'\\1\end{bmatrix}=\begin{bmatrix}a&b&c&d\\e&f&g&h\\i&j&k&l\\0&0&0&1\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

- Orthographic & Perspective Projections



Today

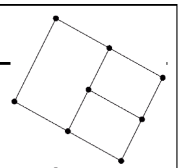
- Surface Definitions
- Simple Data Structures
- Fixed Storage Data Structures
- Fixed Computation Data Structures
- Mesh Simplification

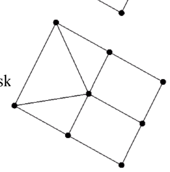
Today

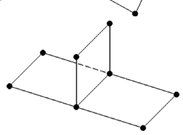
- Surface Definitions
  - Well-Formed Surfaces
  - Orientable Surfaces
  - Computational Complexity
- Simple Data Structures
- Fixed Storage Data Structures
- Fixed Computation Data Structures
- Mesh Simplification

Well-Formed Surfaces

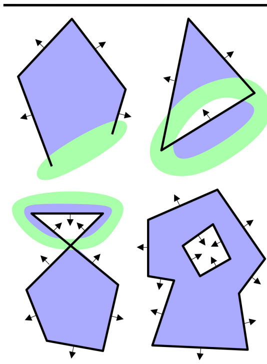
- Components Intersect "Properly"
  - Faces are: disjoint, share single Vertex, or share 2 Vertices and the Edge joining them
  - Every edge is incident to exactly 2 vertices
  - Every edge is incident to exactly 2 faces
- Local Topology is "Proper"
  - Neighborhood of a vertex is homeomorphic to a disk (permits stretching and bending, but not tearing)
  - Called a 2-manifold
  - Boundaries: half-disk, "manifold with boundaries"
- Global Topology is "Proper"
  - Connected
  - Closed
  - Bounded

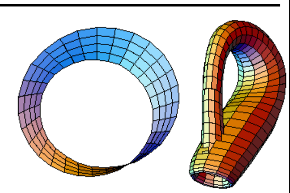




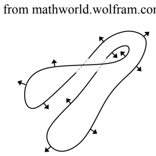


Orientable Surfaces?



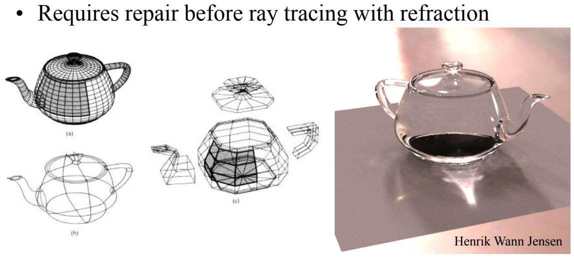


from mathworld.wolfram.com



Closed Surfaces and Refraction

- Original Teapot model is not "watertight":
  intersecting surfaces at spout & handle, no bottom,
  a hole at the spout tip, a gap between lid & base
- Requires repair before ray tracing with refraction



Computational Complexity

- Adjacent Element Access Time
  - linear, constant time average case, or constant time?
  - requires loops/recursion/if ?
- Memory
  - variable size arrays or constant size?
- Maintenance
  - ease of editing
  - ensuring consistency

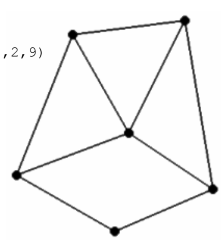
Questions?

Today

- Surface Definitions
- Simple Data Structures
  - List of Polygons
  - List of Edges
  - List of Unique Vertices & Indexed Faces:
  - Simple Adjacency Data Structure
- Fixed Storage Data Structures
- Fixed Computation Data Structures
- Mesh Simplification

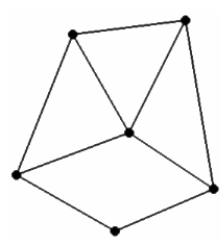
List of Polygons:

```
(3,-2,5), (3,6,2), (-6,2,4)
(2,2,4), (0,-1,-2), (9,4,0), (4,2,9)
(1,2,-2), (8,8,7), (-4,-5,1)
(-8,2,7), (-2,3,9), (1,2,-7)
```



List of Edges:

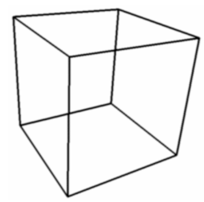
```
(3,6,2), (-6,2,4)
(2,2,4), (0,-1,-2)
(9,4,0), (4,2,9)
(8,8,7), (-4,-5,1)
(-8,2,7), (1,2,-7)
(3,0,-3), (-7,4,-3)
(9,4,0), (4,2,9)
(3,6,2), (-6,2,4)
(-3,0,-4), (7,-3,-4)
```



List of Unique Vertices & Indexed Faces:

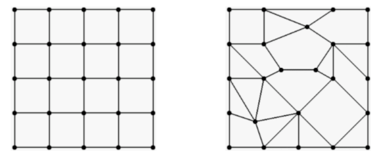
Vertices:
```
(-1, -1, -1)
(-1, -1, 1)
(-1, 1, -1)
(-1, 1, 1)
(1, -1, -1)
(1, -1, 1)
(1, 1, -1)
(1, 1, 1)
```

Faces:
```
1 2 4 3
5 7 8 6
1 5 6 2
3 4 8 7
1 3 7 5
2 6 8 4
```



Problems with Simple Data Structures

- No Adjacency Information
- Linear-time Searches



Structured *Unstructured*

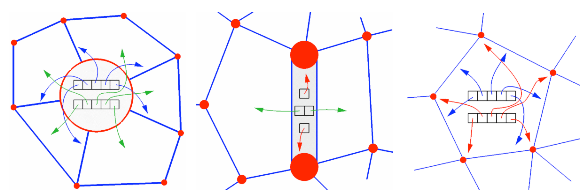
- Adjacency is implicit for structured meshes, but what do we do for unstructured meshes?

Mesh Data

- So, in addition to:
  - Geometric Information (position)
  - Attribute Information (color, texture, temperature, population density, etc.)
- Let's store:
  - Topological Information (adjacency, connectivity)

Simple Adjacency

- Each element (vertex, edge, and face) has a list of pointers to all incident elements
- Queries depend only on local complexity of mesh
- Data structures do not have fixed size
- Slow! Big! Too much work to maintain!



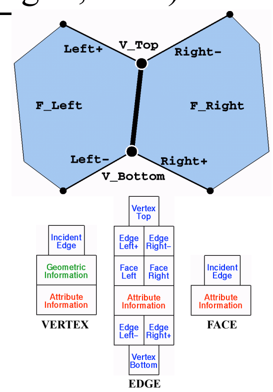
Questions?

Today

- Surface Definitions
- Simple Data Structures
- Fixed Storage Data Structures
  - Winged Edge (Baumgart, 1975)
- Fixed Computation Data Structures
- Mesh Simplification

Winged Edge (Baumgart, 1975)

- Each edge stores pointers to 4 Adjacent Edges
- Vertices and Faces have a single pointer to one incident Edge
- Data Structure Size?
  Fixed
- How do we gather all faces surrounding one vertex?
  Messy, because there is no consistent way to order pointers

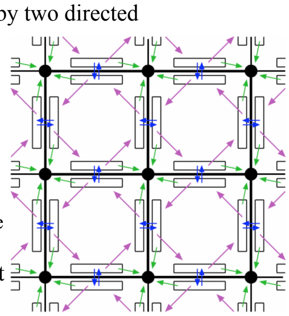


Today

- Surface Definitions
- Simple Data Structures
- Fixed Storage Data Structures
- Fixed Computation Data Structures
  - HalfEdge (Eastman, 1982)
  - SplitEdge
  - Corner
  - QuadEdge (Guibas and Stolfi, 1985)
  - FacetEdge (Dobkin and Laszlo, 1987)

HalfEdge (Eastman, 1982)

- Every edge is represented by two directed HalfEdge structures
- Each HalfEdge stores:
  - `vertex` at end of directed edge
  - `symmetric` half edge
  - `face` to left of edge
  - `next` points to the HalfEdge counter-clockwise around face on left
- Orientation is essential, but can be done consistently!



HalfEdge (Eastman, 1982)

- Starting at a half edge, how do we find:

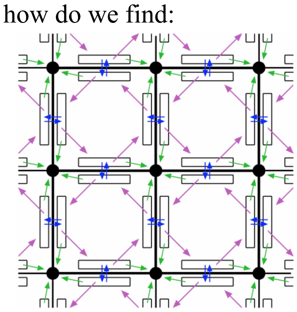
  the other vertex of the edge?

  the other face of the edge?

  the clockwise edge around the face at the left?

  all the edges surrounding the face at the left?

  all the faces surrounding the vertex?



HalfEdge (Eastman, 1982)

- Loop around a Face:

```
HalfEdgeMesh::FaceLoop(HalfEdge *HE) {
   HalfEdge *loop = HE;
   do {
      loop = loop->Next;
   } while (loop != HE);
}
```

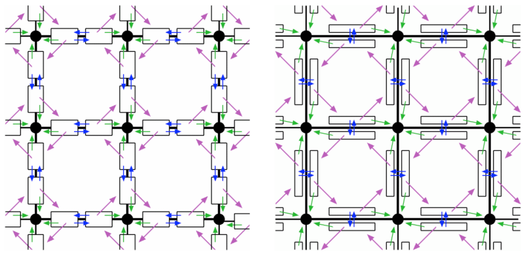
- Loop around a Vertex:

```
HalfEdgeMesh::VertexLoop(HalfEdge *HE) {
   HalfEdge *loop = HE;
   do {
      loop = loop->Next->Sym;
   } while (loop != HE);
}
```

HalfEdge (Eastman, 1982)

- Data Structure Size?
  Fixed
- Data:
  - geometric information stored at Vertices
  - attribute information in Vertices, HalfEdges, and/or Faces
  - topological information in HalfEdges only!
- Orientable surfaces only (no Mobius Strips!)
- Local consistency everywhere implies global consistency
- Time Complexity?
  linear in the amount of information gathered

SplitEdge Data Structure:

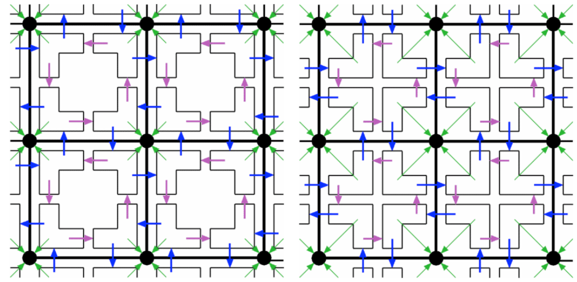


- HalfEdge and SplitEdge are dual structures!

```
SplitEdgeMesh::FaceLoop() = HalfEdgeMesh::VertexLoop()
SplitEdgeMesh::VertexLoop() = HalfEdgeMesh::FaceLoop()
```

Corner Data Structure:

- The Corner data structure is its own dual!



Questions?

Today

- Surface Definitions
- Simple Data Structures
- Fixed Storage Data Structures
- Fixed Computation Data Structures
  - HalfEdge (Eastman, 1982)
  - SplitEdge
  - Corner
  - QuadEdge (Guibas and Stolfi, 1985)
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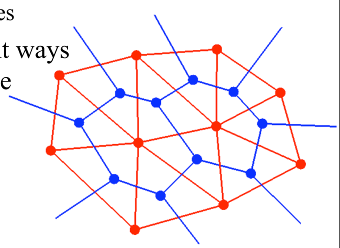
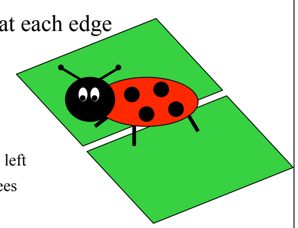
QuadEdge (Guibas and Stolfi, 1985)

- Consider the Mesh and its *Dual* simultaneously
  - Vertices and Faces switch roles, we just re-label them
  - Edges remain Edges
- Now there are eight ways to look at each edge
  - Four ways to look at primal edge
  - Four ways to look at dual edge



QuadEdge (Guibas and Stolfi, 1985)

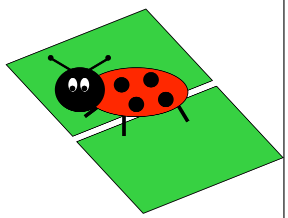
- Relations Between Edges: Edge Algebra
- Elements in Edge Algebra:
  - Each of 8 ways to look at each edge



- Operators in Edge Algebra:
  - Rot: Bug rotates 90 degrees to its left
  - Sym: Bug turns around 180 degrees
  - Flip: Bug flips up-side down
  - Onext: Bug rotates CCW about its origin (either Vertex or Face)

QuadEdge (Guibas and Stolfi, 1985)

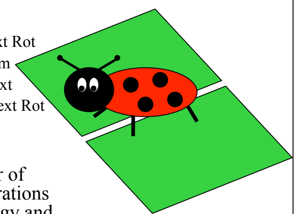
- Some Properties of Flip, Sym, Rot, and Onext:
  - e Rot^4 = e
  - e $Rot^2 \neq$ e
  - e $Flip^2$ = e
  - e Flip Rot Flip Rot = e
  - e Rot Flip Rot Flip = e
  - e Rot Onext Rot Onext = e
  - e Flip Onext Flip Onext = e
  - e $Flip^{-1}$ = e Flip
  - e Sym = e Rot^2
  - e Rot^{-1} = e Rot^3
  - e Rot^{-1} = e Flip Rot Flip
  - e $Onext^{-1}$ = e Rot Onext Rot
  - e $Onext^{-1}$ = e Flip Onext Flip



QuadEdge (Guibas and Stolfi, 1985)

- Other Useful Definitions:
  - e Lnext = e Rot^{-1} Onext Rot
  - e Rnext = e Rot Onext Rot^{-1}
  - e Dnext = e Sym Onext Sym^{-1}

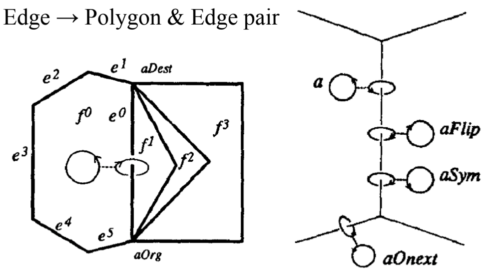
  - e Oprev = e $Onext^{-1}$ = e Rot Onext Rot
  - e Lprev = e $Lnext^{-1}$ = e Onext Sym
  - e Rprev = e $Rnext^{-1}$ = e Sym Onext
  - e Dprev = e $Dnext^{-1}$ = e Rot^{-1} Onext Rot



- All of these functions can be expressed as a constant number of Rot, Sym, Flip, and Onext operations independent of the local topology and the global size and complexity of the mesh.

FacetEdge (Dobkin and Laszlo, 1987)

- QuadEdge (2D, surface) → FacetEdge (3D, volume)
- Faces → Polyhedra / Cells
- Edge → Polygon & Edge pair



Questions?

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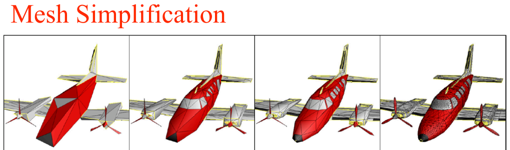


(a) Base mesh M^0 (150 faces) (b) Mesh M^{175} (500 faces) (c) Mesh M^{425} (1,000 faces) (d) Original $M=M^n$ (13,546 faces)

Hugues Hoppe "Progressive Meshes" SIGGRAPH 1996

Progressive Meshes

- Mesh Simplification
  - vertex split / edge collapse
  - geometry & discrete/scalar attributes
  - priority queue
- Level of Detail
  - geomorphs
- Progressive Transmission
- Mesh Compression
- Selective Refinement
  - view dependent

Selective Refinement

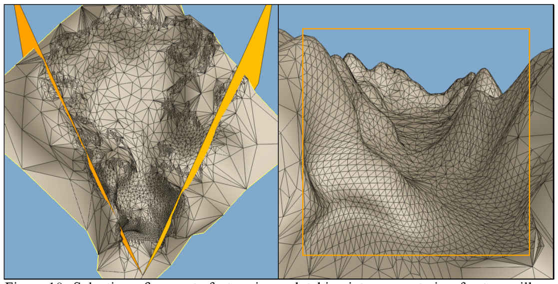


Figure 10: Selective refinement of a terrain mesh taking into account view frustum, silhouette regions, and projected screen size of faces (7,438 faces).

Preserving Discontinuity Curves

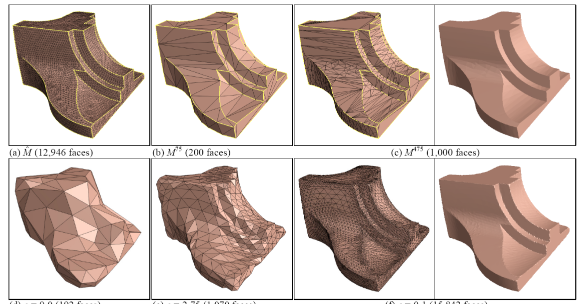
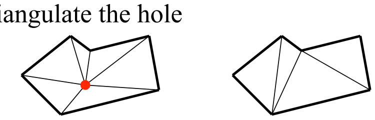


Figure 12: Approximations of a mesh $\hat{M}$ using (b–c) the PM representation, and (d–f) the MRA scheme of Eck et al. [7]. As demonstrated, MRA cannot recover $\hat{M}$ exactly, cannot deal effectively with surface creases, and produces approximating meshes of inferior quality.

Other Simplification Strategies

- Remove a vertex & surrounding triangles, re-triangulate the hole



- Merge Nearby Vertices
  - will likely change the topology…

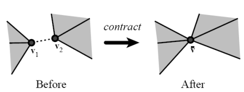


Figure 2: **Non-edge contraction.** When non-edge pairs are contracted, unconnected sections of the model are joined. The dashed line indicates the two vertices being contracted together.

from Garland & Heckbert, "Surface Simplification Using Quadric Error Metrics" SIGGRAPH 1997

When to Preserve Topology?

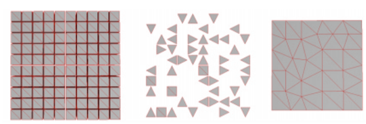


Figure 3: On the left is a regular grid of 100 closely spaced cubes. In the middle, an approximation built using only edge contractions demonstrates unacceptable fragmentation. On the right, the result of using more general pair contractions to achieve aggregation is an approximation much closer to the original.

from Garland & Heckbert, "Surface Simplification Using Quadric Error Metrics" SIGGRAPH 1997

Quadric Error Simplification

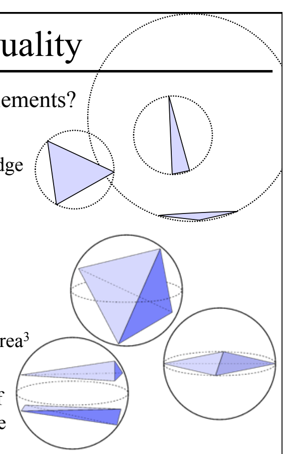
- Contract (merge) vertices v_i and v_j if:
  - (v_i, v_j) is an edge, or
  - $\| v_i - v_j \| < t$, where t is a threshold parameter
- Track cummulative error by summing 4x4 quadric error matrics after each operation:

$$
\begin{aligned}
\Delta(\mathbf{v}) &= \sum_{\mathbf{p}\in\text{planes}(\mathbf{v})} (\mathbf{v}^{\mathsf{T}}\mathbf{p})(\mathbf{p}^{\mathsf{T}}\mathbf{v}) \\
&= \sum_{\mathbf{p}\in\text{planes}(\mathbf{v})} \mathbf{v}^{\mathsf{T}}(\mathbf{p}\mathbf{p}^{\mathsf{T}})\mathbf{v} \\
&= \mathbf{v}^{\mathsf{T}}\left(\sum_{\mathbf{p}\in\text{planes}(\mathbf{v})} \mathbf{K}_{\mathbf{p}}\right)\mathbf{v}
\end{aligned}
$$

$$
\mathbf{K}_{\mathbf{p}} = \mathbf{p}\mathbf{p}^{\mathsf{T}} = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix}
$$

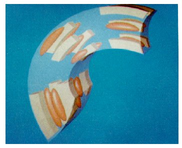
Garland & Heckbert, "Surface Simplification Using Quadric Error Metrics" SIGGRAPH 1997

Judging Element Quality

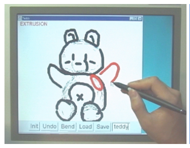


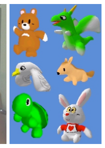
- How "equilateral" are the elements?
- For Triangles?
  - Ratio of shortest to longest edge
  - Ratio of area to perimeter^2
  - Smallest angle
  - Ratio of area to area of smallest circumscribed circle
- For Tetrahedra?
  - Ratio of volume^2 to surface area^3
  - Smallest *solid* angle
  - Ratio of volume to volume of smallest circumscribed sphere

Readings for Tuesday (1/20) *pick one*



- "Free-form deformation of solid geometric models", Sederberg & Parry, SIGGRAPH 1986
- "Teddy: A Sketching Interface for 3D Freefrom Design", Igarashi et al., SIGGRAPH 1999





- Post a comment or question on the LMS discussion by 10am on Tuesday