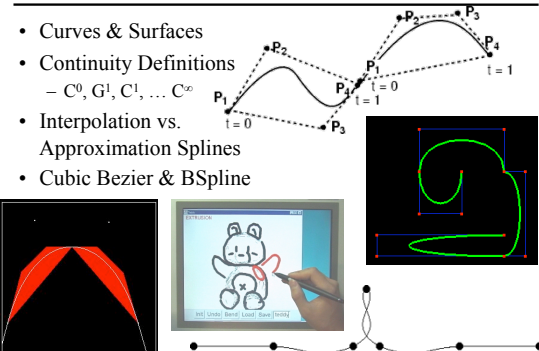


Subdivision Surfaces

Last Time?

- Curves & Surfaces
- Continuity Definitions
 - $C^0, G^1, C^1, \dots, C^\infty$
- Interpolation vs. Approximation Splines
- Cubic Bezier & BSpline



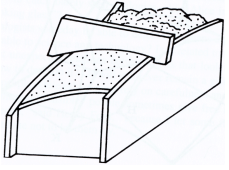
Today

- Spline Surfaces / Patches
 - Tensor Product
 - Bilinear Patches
 - Bezier Patches
 - Trimming Curves
- Subdivision Surface “Zoo”
- Seams In Subdivision
- Misc. Mesh/Surface Vocabulary
- “Subdivision Surfaces in Character Animation”

Tensor Product

- Of two vectors:

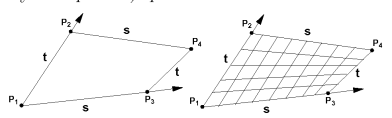
$$[a_1 \ a_2 \ a_3] \otimes [b_1 \ b_2 \ b_3 \ b_4] = \begin{bmatrix} a_1 b_1 & a_2 b_1 & a_3 b_1 \\ a_1 b_2 & a_2 b_2 & a_3 b_2 \\ a_1 b_3 & a_2 b_3 & a_3 b_3 \\ a_1 b_4 & a_2 b_4 & a_3 b_4 \end{bmatrix}$$
- Similarly, we can define a surface as the tensor product of two curves....



Farin, Curves and Surfaces for Computer Aided Geometric Design

Bilinear Patch

Bi-lerp a (typically non-planar) quadrilateral

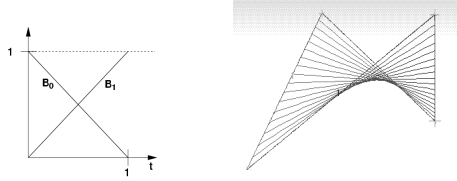


Notation: $L(P_1, P_2, \alpha) \equiv (1 - \alpha)P_1 + \alpha P_2$

$$Q(s, t) = L(L(P_1, P_2, t), L(P_3, P_4, t), s)$$

Bilinear Patch

- Smooth version of quadrilateral with non-planar vertices...



- But will this help us model smooth surfaces?
- Do we have control of the derivative at the edges?

Today

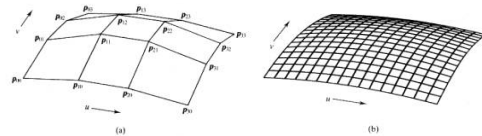
- Spline Surfaces / Patches
 - Tensor Product
 - Bilinear Patches
 - **Bezier Patches**
 - **Trimming Curves**
- Subdivision Surface “Zoo”
- Seams In Subdivision
- Misc. Mesh/Surface Vocabulary
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Bicubic Bezier Patch

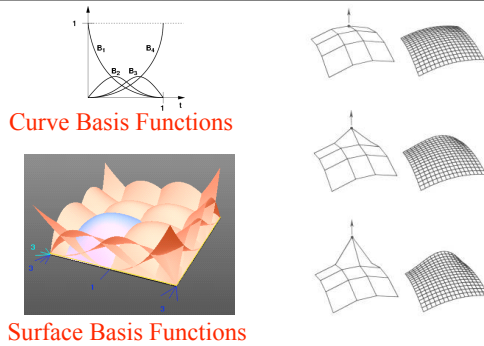
Notation: $\text{CB}(P_1, P_2, P_3, P_4, \alpha)$ is Bézier curve with control points P_i evaluated at α

Define “Tensor-product” Bézier surface

$$Q(s, t) = \text{CB}(\text{CB}(P_{00}, P_{01}, P_{02}, P_{03}, t), \text{CB}(P_{10}, P_{11}, P_{12}, P_{13}, t), \text{CB}(P_{20}, P_{21}, P_{22}, P_{23}, t), \text{CB}(P_{30}, P_{31}, P_{32}, P_{33}, t), s)$$

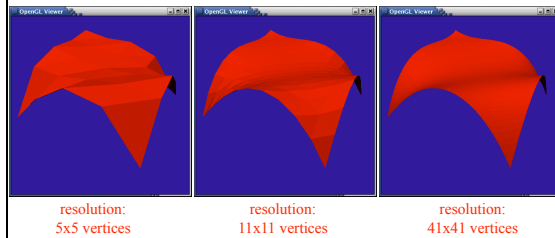


Editing Bicubic Bezier Patches



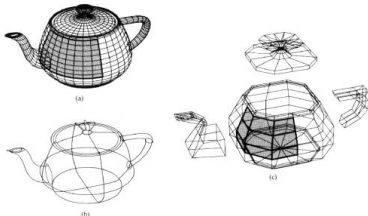
Bicubic Bezier Patch Tessellation

- Given 16 control points and a tessellation resolution, we can create a triangle mesh



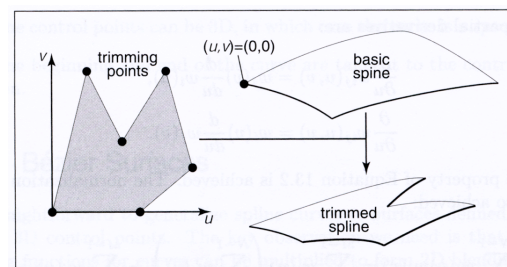
Modeling with Bicubic Bezier Patches

- Original Teapot specified with Bezier Patches



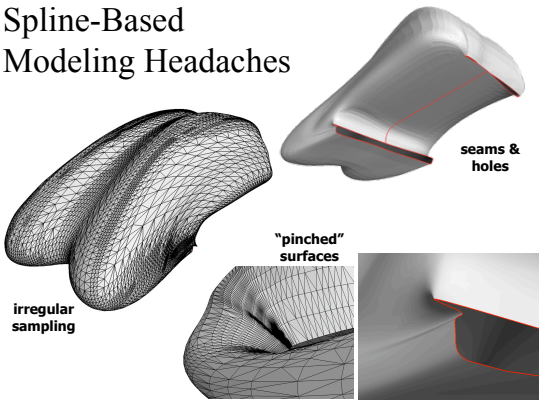
- But it's not "watertight": it has intersecting surfaces at spout & handle, no bottom, a hole at the spout tip, a gap between lid & base

Trimming Curves for Patches



Shirley, Fundamentals of Computer Graphics

Spline-Based Modeling Headaches



Questions?

- Bezier Patches?
- or
- Triangle Mesh?

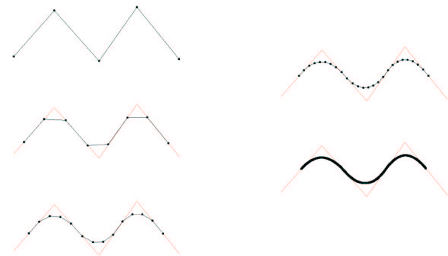


Henrik Wann Jensen

Today

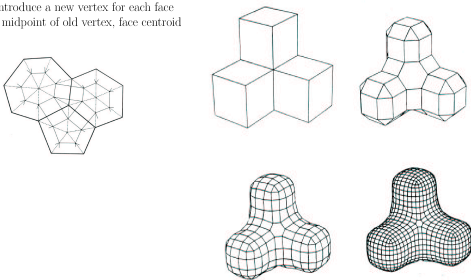
- Spline Surfaces / Patches
- Subdivision Surface "Zoo"
 - Doo Sabin (anything!)
 - Loop (triangles only)
 - Catmull Clark (turns everything into quads)
 - ... many others!
- Seams In Subdivision
- Misc. Mesh/Surface Vocabulary
- "Subdivision Surfaces in Character Animation"

Chaikin's Algorithm

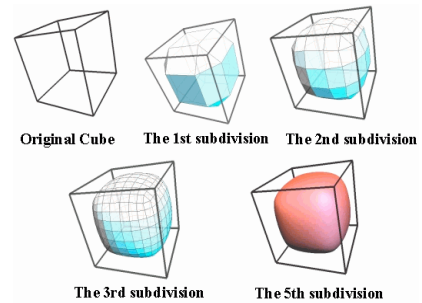


Doo-Sabin Subdivision

Idea: introduce a new vertex for each face
At the midpoint of old vertex, face centroid

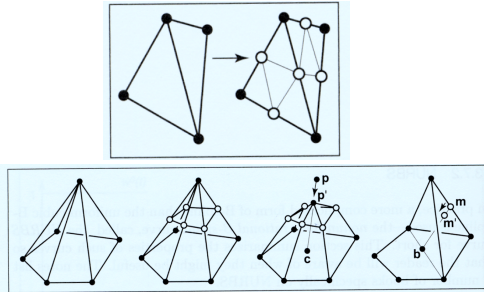


Doo-Sabin Subdivision



<http://www.ke.ics.saitama-u.ac.jp/xuz/pic/doo-sabin.gif>

Loop Subdivision



Shirley, Fundamentals of Computer Graphics

Loop Subdivision

Subdivision Rules. The masks for the Loop scheme are shown in Figure 4.3. For boundaries and edges tagged as *crease* edges, special rules are used. These rules produce a cubic spline curve along the boundary/crease. The curve only depends on control points on the boundary/crease.

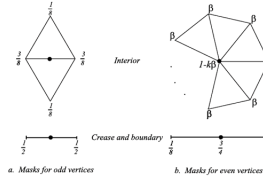


Figure 4.3: Loop subdivision: in the picture above, β can be chosen to be either $\frac{1}{4}(5/8 - (\frac{1}{4} + \frac{1}{4} \cos \frac{2\pi}{n}))^2$ (original choice of Loop [16]), or, for $n > 3$, $\beta = \frac{1}{n}$ as proposed by Warren [33]. For $n = 3$, $\beta = 3/16$ can be used.

SIGGRAPH 2000 course notes
Subdivision for Modeling and Animation (page 70)

Catmull Clark Subdivision

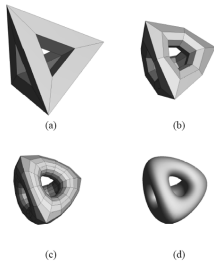


Figure 3: Recursive subdivision of a topologically complicated mesh: (a) the control mesh; (b) after one subdivision step; (c) after two subdivision steps; (d) the limit surface.

"Subdivision Surfaces in Character Animation",
DeRose, Kass & Truong, SIGGRAPH 1998

$$d_i^{n+1} = \frac{v^i + d_i^j + f_i^{n+1} + f_i^{n+1}}{4} \quad (1)$$

where subscripts are taken modulo the valence of the central vertex v^i . (The valence of a vertex is the number of edges incident to it.) Finally, a vertex point v^i is computed as

$$v^{i+1} = \frac{n-2}{n} v^i + \frac{1}{n} \sum_j d_j^i + \frac{1}{n} \sum_j f_j^{i+1} \quad (2)$$

Vertices of valence 4 are called ordinary; others are called extraordinary.

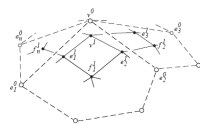
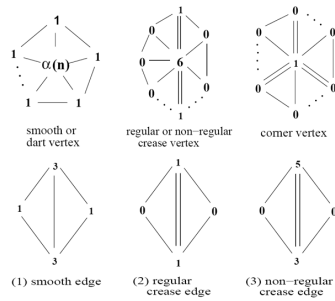


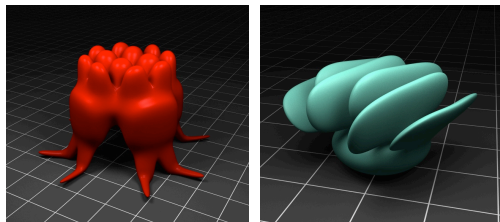
Figure 4: The situation around a vertex v^i of valence n .

Adding creases to Loop Subdivision

- Vertex & edge masks
- Limit masks
 - Position
 - Tangent



Questions?

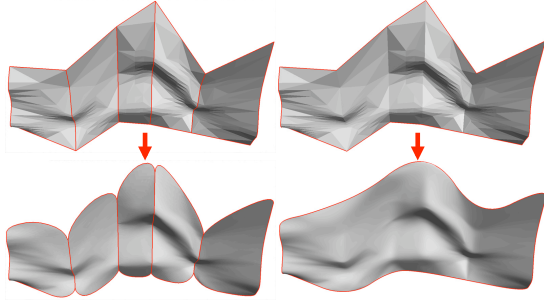


Justin Legakis

Today

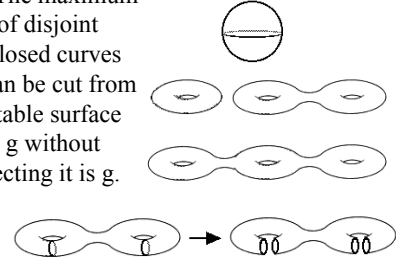
- Spline Surfaces / Patches
- Subdivision Surface "Zoo"
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Seams don't Subdivide as Expected



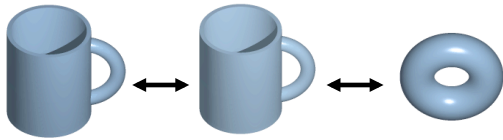
Misc. Mesh/Surface Vocabulary

- *Genus*: The maximum number of disjoint simple closed curves which can be cut from an orientable surface of genus g without disconnecting it is g .



Misc. Mesh/Surface Vocabulary

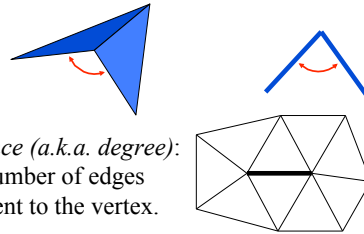
- *Homeomorphic/Topological equivalence*: a continuous stretching and bending of the object into a new shape



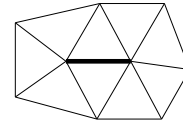
http://en.wikipedia.org/wiki/Image:Mug_and_Torus_morph.gif

Misc. Mesh/Surface Vocabulary

- *Dihedral Angle*:
 - the angle between the planes of two triangular faces
 - “looking down the edge” between two faces, the angle between the faces.

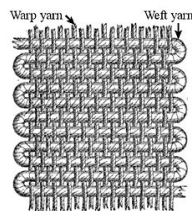


- *Valence (a.k.a. degree)*: the number of edges incident to the vertex.



Misc. Mesh/Surface Vocabulary

- *Warp & weft*: Yarns used in weaving. Because the weft does not have to be stretched in the way that the warp is, it can generally be less strong.



<http://en.wikipedia.org/wiki/Weft>

Today

- Spline Surfaces / Patches
- Subdivision Surface “Zoo”
- Seams In Subdivision
- Misc. Mesh/Surface Vocabulary
- “Subdivision Surfaces in Character Animation”

Reading for Today

- DeRose, Kass, & Truong, "Subdivision Surfaces in Character Animation", SIGGRAPH 1998



Figure 5: Gerr's hand as a piecewise smooth Catmull-Clark surface. Infinitely sharp creases are used between the skin and the fingernails.

Subdivision Surfaces in Character Animation

- Catmull Clark Subdivision Rules
- Semi-sharp vs. Infinitely-sharp creases
- Mass-Spring Cloth (*next week*)
- Hierarchical Mesh for Collision
- Texturing Subdivision Surfaces

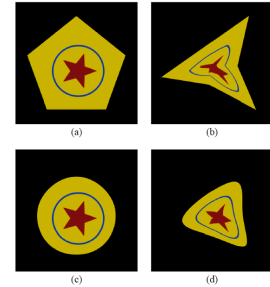
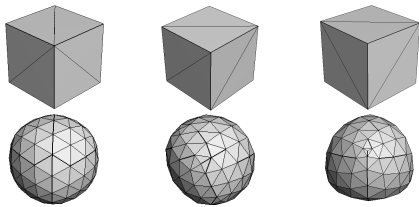


Figure 11: (a) A texture mapped regular pentagon comprised of 5 triangles; (b) the pentagonal model with its vertices moved; (c) A subdivision surface whose control mesh is the same 5 triangles in (a), and where boundary edges are marked as creases; (d) the subdivision surface with its vertices positioned as in (b).

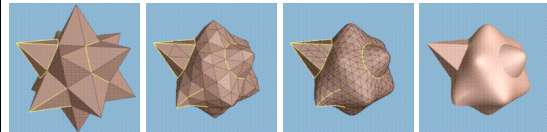
Catmull-Clark in Pixar Production

- Based on quadrilaterals
 - Like NURBS, specifically cubic bsplines
 - Implicit adjacency in subdivided microgeometry
 - Better than triangles for symmetric objects



Reading for Tuesday

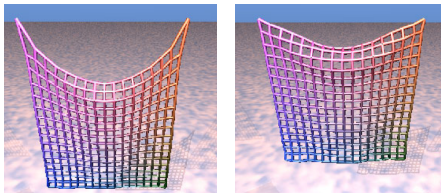
- Hoppe et al., "Piecewise Smooth Surface Reconstruction" SIGGRAPH 1994



- Post a comment or question on the LMS discussion by 10am on Tuesday

Reading for Friday

- "Deformation Constraints in a Mass-Spring Model to Describe Rigid Cloth Behavior", Provot, 1995.



- Post a comment or question on the LMS discussion by 10am on Friday

Other...

- Assigned readings & discussion

- Homework 1: Simplification & Subdivision Questions/Comments?

- Blue Sky Studio's Steve Martino
 - Wednesday, January 28, 2009
 - 3:00-4:00PM
 - Biotech Auditorium

