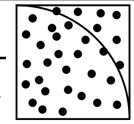
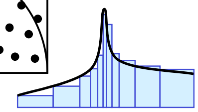
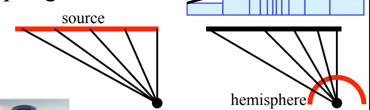
Sampling, Aliasing, & Mipmaps

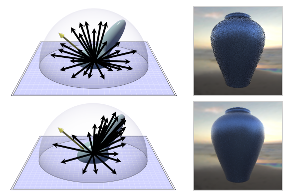
Last Time?

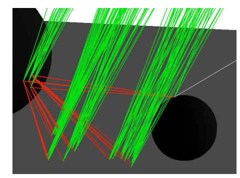
- Monte-Carlo Integration
- Importance Sampling
- Ray Tracing vs. Path Tracing









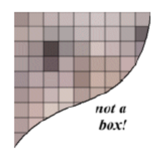


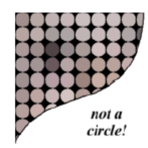
Today

- What is a Pixel?
- Examples of Aliasing
- Sampling & Reconstruction
- Filters in Computer Graphics
- Anti-Aliasing for Texture Maps

What is a Pixel?

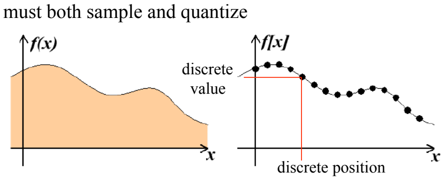
- A pixel is not:
  - a box
  - a disk
  - a teeny tiny little light
- A pixel "looks different" on different display devices
- A pixel is a sample
  - it has no dimension
  - it occupies no area
  - it cannot be seen
  - it has a coordinate
  - it has a value





More on Samples

- Most things in the real world are *continuous*, yet everything in a computer is *discrete*
- The process of mapping a continuous function to a discrete one is called *sampling*
- The process of mapping a continuous variable to a discrete one is called *quantization*
- To represent or render an image using a computer, we must both sample and quantize



An Image is a 2D Function

- An *ideal image* is a continuous function I(x,y) of intensities.
- It can be plotted as a height field.
- In general an image cannot be represented as a continuous, analytic function.
- Instead we represent images as tabulated functions.
- How do we fill this table?

Sampling Grid

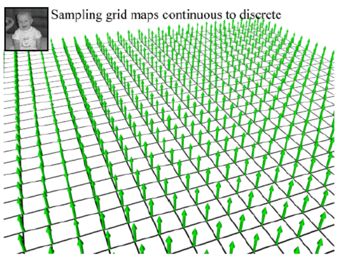
- We can generate the table values by multiplying the continuous image function by a sampling grid of Kronecker delta functions.

The definiton of the 2-D Kronecker delta is:

$$\delta(x,y) = \begin{cases} 1, & (x,y) = (0,0) \\ 0, & \text{otherwise} \end{cases}$$

And a 2-D sampling grid:

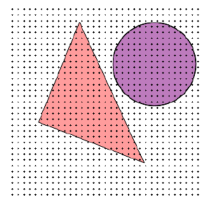
$$\sum_{j=0}^{h-1} \sum_{i=0}^{w-1} \delta(u-i, v-j)$$

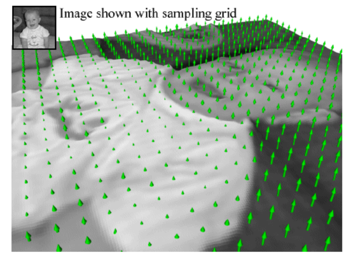


Sampling an Image

- The result is a set of point samples, or pixels.

The same analysis can be applied to geometric objects:





Questions?

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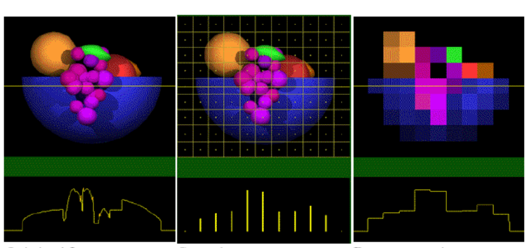
Examples of Aliasing

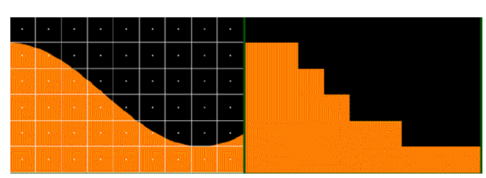


Original Image Samples Reconstruction

- Aliasing occurs because of *sampling* and *reconstruction*

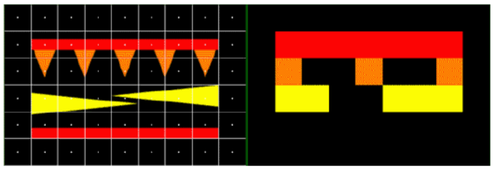
Examples of Aliasing

Jagged boundaries



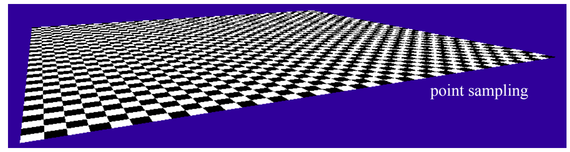
Examples of Aliasing

Improperly rendered detail



Examples of Aliasing

Texture Errors



Questions?

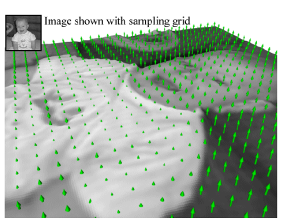
Today

- What is a Pixel?
- Examples of Aliasing
- Sampling & Reconstruction
  - Sampling Density
  - Fourier Analysis & Convolution
- Filters in Computer Graphics
- Anti-Aliasing for Texture Maps

Sampling Density

- How densely must we sample an image in order to capture its essence?

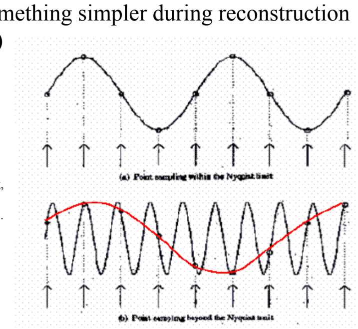
- If we under-sample the signal, we won't be able to accurately reconstruct it...



Sampling Density

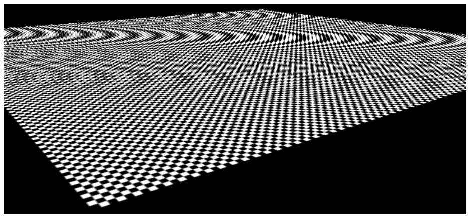
- If we insufficiently sample the signal, it may be mistaken for something simpler during reconstruction (that's aliasing!)

Image from Robert L. Cook, "Stochastic Sampling and Distributed Ray Tracing", An Introduction to Ray Tracing, Andrew Glassner, ed., Academic Press Limited, 1989.



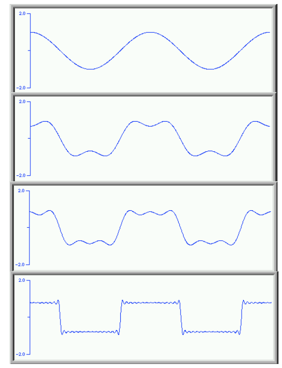
Sampling Density

- Aliasing in 2D because of insufficient sampling density



Remember Fourier Analysis?

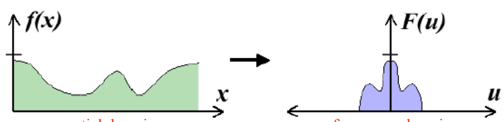
- All periodic signals can be represented as a summation of sinusoidal waves.



Images from
http://axion.physics.ubc.ca/341-02/fourier/fourier.html

Remember Fourier Analysis?

- Every periodic signal in the *spatial domain* has a dual in the *frequency domain*.



- This particular signal is *band-limited*, meaning it has no frequencies above some threshold

Remember Fourier Analysis?

- We can transform from one domain to the other using the Fourier Transform.

frequency domain — spatial domain

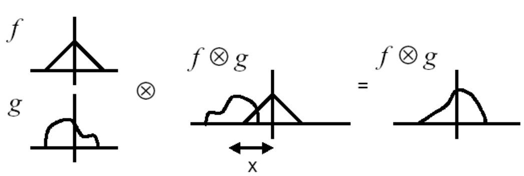
Fourier Transform
$$F(u,v)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,e^{-i2\pi(ux+vy)}dxdy$$

Inverse Fourier Transform
$$f(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)\,e^{i2\pi(ux+vy)}dudv$$

Remember Convolution?

Convolution describes how a system with impulse response, $h(x)$, reacts to a signal, $f(x)$.

$$f(x)*h(x)=\int_{-\infty}^{\infty} f(\lambda)h(x-\lambda)d\lambda$$



Images from Mark Meyer
http://www.gg.caltech.edu/~cs174ta/

Remember Convolution?

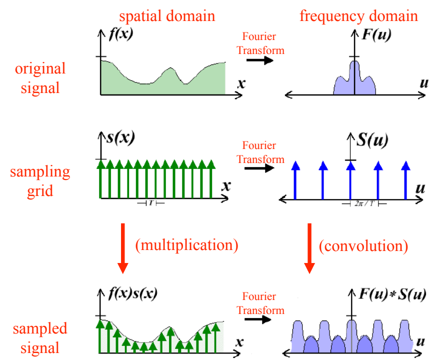
- Some operations that are difficult to compute in the spatial domain can be simplified by transforming to its dual representation in the frequency domain.
- For example, convolution in the spatial domain is the same as multiplication in the frequency domain.

$$f(x)*h(x)\rightarrow F(u)H(u)$$

- And, convolution in the frequency domain is the same as multiplication in the spatial domain

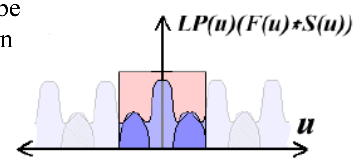
$$F(u)*H(u)\rightarrow f(x)h(x)$$

Sampling in the Frequency Domain



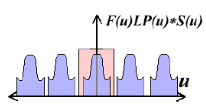
Reconstruction

- If we can extract a copy of the original signal from the frequency domain of the sampled signal, we can reconstruct the original signal!
- But there may be overlap between the copies.

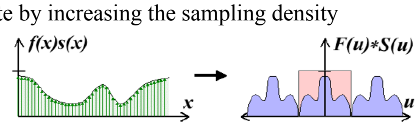


Guaranteeing Proper Reconstruction

- Separate by removing high frequencies from the original signal (low pass pre-filtering)



- Separate by increasing the sampling density



- If we can't separate the copies, we will have overlapping frequency spectrum during reconstruction → *aliasing*.

Sampling Theorem

- When sampling a signal at discrete intervals, the sampling frequency must be *greater than twice* the highest frequency of the input signal in order to be able to reconstruct the original perfectly from the sampled version (Shannon, Nyquist)

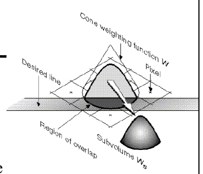
Questions?

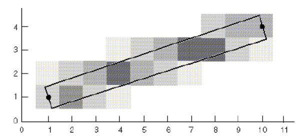
Today

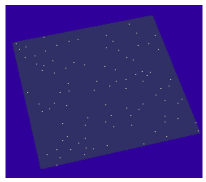
- What is a Pixel?
- Examples of Aliasing
- Sampling & Reconstruction
- Filters in Computer Graphics
  - Ideal, Gaussian, Box, Bilinear, Bicubic
- Anti-Aliasing for Texture Maps

Filters

- Weighting function (convolution kernel)
- Area of influence often bigger than "pixel"
- Sum of weights = 1
  - Each sample contributes the same total to image
  - Constant brightness as object moves across the screen.
- No negative weights/colors (optional)







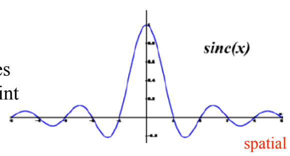
Source: Foley, VanDam, Feiner, Hughes - Computer Graphics, Second Edition,

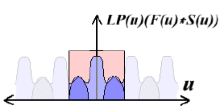
Filters

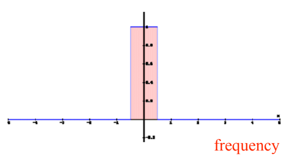
- Filters are used to
  - reconstruct a continuous signal from a sampled signal (reconstruction filters)
  - band-limit continuous signals to avoid aliasing during sampling (low-pass filters)
- Desired frequency domain properties are the same for both types of filters
- Often, the same filters are used as reconstruction and low-pass filters

The Ideal Filter

- Unfortunately it has *infinite* spatial extent
  - Every sample contributes to every interpolated point
- Expensive/impossible to compute

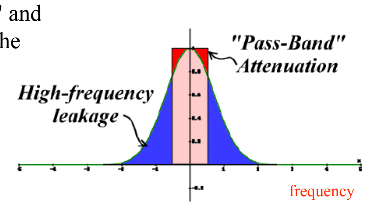






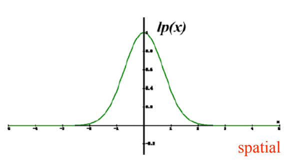
Problems with Practical Filters

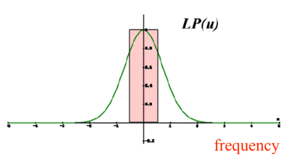
- Many visible artifacts in re-sampled images are caused by poor reconstruction filters
- Excessive pass-band attenuation results in blurry images
- Excessive high-frequency leakage causes "ringing" and can accentuate the sampling grid (anisotropy)



Gaussian Filter

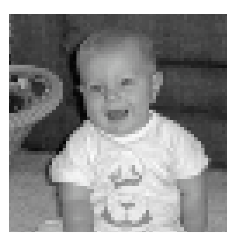
- This is what a CRT does for free!

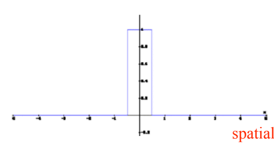


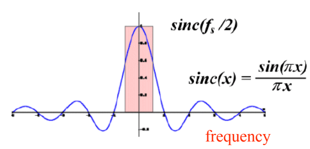


Box Filter / Nearest Neighbor

- Pretending pixels are little squares.

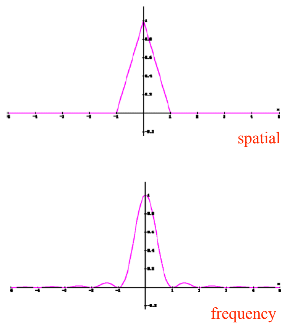






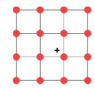
Tent Filter / Bi-Linear Interpolation

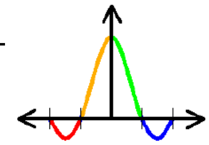
- Simple to implement
- Reasonably smooth

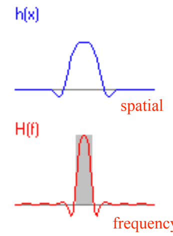


Bi-Cubic Interpolation

- Begins to approximate the ideal spatial filter, the sinc function







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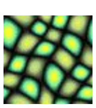
  - Magnification & Minification
  - Mipmaps
  - Anisotropic Mipmaps

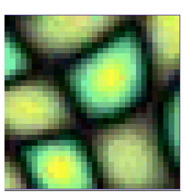
Sampling Texture Maps

- When texture mapping it is rare that the screen-space sampling density matches the sampling density of the texture.

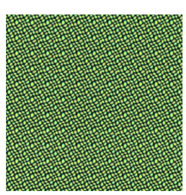


64x64 pixels

Original Texture



Magnification for Display

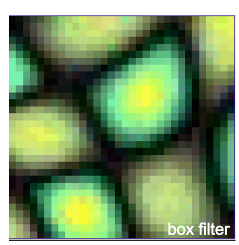


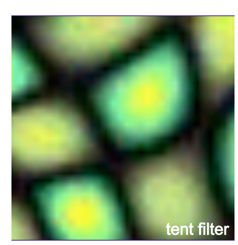
Minification for Display

for which we must use a reconstruction filter

Linear Interpolation

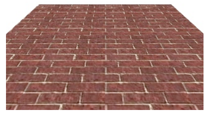
- Tell OpenGL to use a tent filter instead of a box filter.
- Magnification looks better, but blurry
  - (texture is under-sampled for this resolution)





Spatial Filtering

- Remove the high frequencies which cause artifacts in texture minification.
- Compute a spatial integration over the extent of the pixel
- This is equivalent to convolving the texture with a filter kernel centered at the sample (i.e., pixel center)!
- Expensive to do during rasterization, but an approximation it can be precomputed

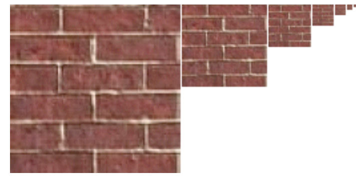


projected texture in image plane

box filter in texture plane

MIP Mapping

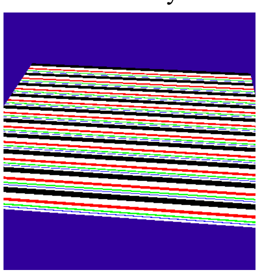
- Construct a pyramid of images that are pre-filtered and re-sampled at 1/2, 1/4, 1/8, etc., of the original image's sampling



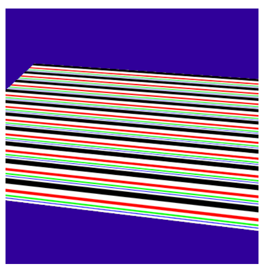
- During rasterization we compute the index of the decimated image that is sampled at a rate closest to the density of our desired sampling rate
- MIP stands for *multum in parvo* which means *many in a small place*

MIP Mapping Example

- Thin lines may become disconnected / disappear



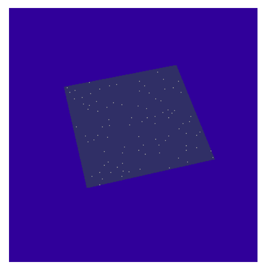
Nearest Neighbor



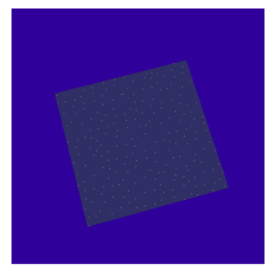
MIP Mapped (Bi-Linear)

MIP Mapping Example

- Small details may "pop" in and out of view



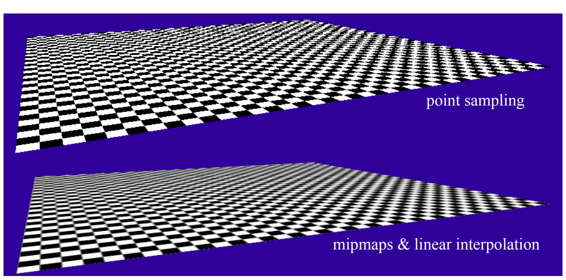
Nearest Neighbor



MIP Mapped (Bi-Linear)

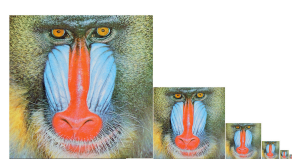
Examples of Aliasing

Texture Errors

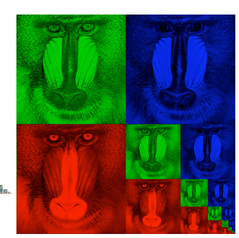


Storing MIP Maps

- Can be stored compactly
- Illustrates the 1/3 overhead of maintaining the MIP map



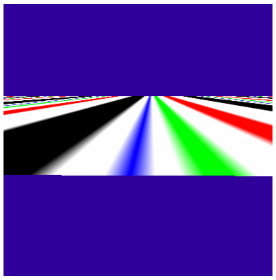
10-level mip map



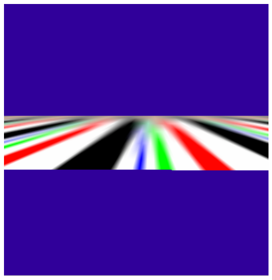
Memory format of a mip map

Anisotropic MIP-Mapping

- What happens when the surface is tilted?

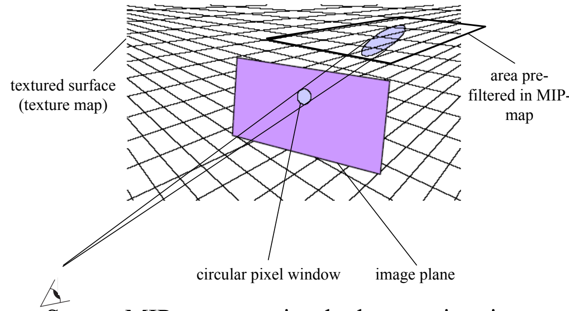


Nearest Neighbor



MIP Mapped (Bi-Linear)

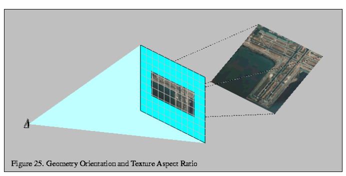
Anisotropic MIP-Mapping

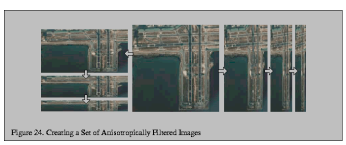


- Square MIP-map area is a bad approximation

Anisotropic MIP-Mapping

- We can use different mipmaps for the 2 directions
- Additional extensions can handle non axis-aligned views

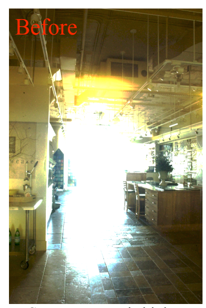




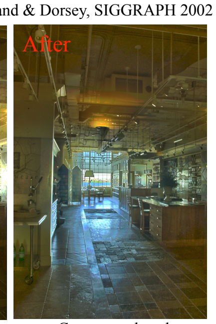
Images from http://www.sgi.com/software/opengl/advanced98/notes/node37.html

Reading for Today:

"Fast Bilateral Filtering for the Display of High-Dynamic Range Images",
Durand & Dorsey, SIGGRAPH 2002

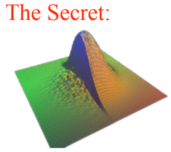


Scene contrast is high
Display contrast is low



Contrast reduced
Details preserved

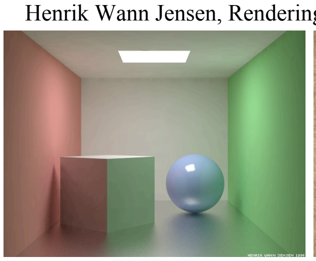
The Secret:



Bilateral filter

Reading for Tuesday (4/1):

- *Global Illumination using Photon Maps*,
  Henrik Wann Jensen, Rendering Techniques 1996



- Post a comment or question on the LMS discussion by 10am on Tuesday 1/29