Electromagnetics Simulator

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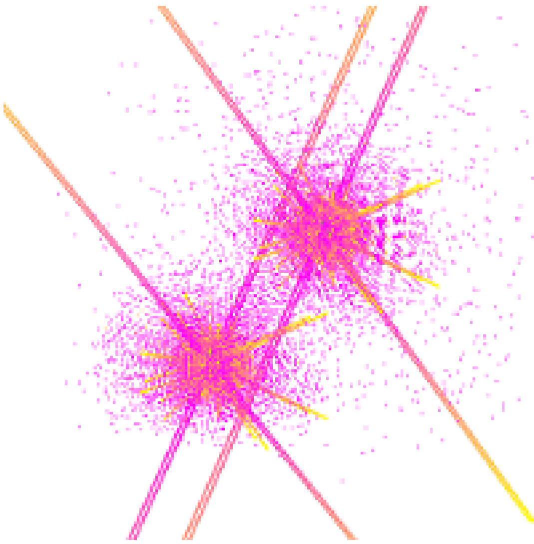

Figure 1: Two magnetic dipoles

1 Summary

The purpose of this project is to simulate electromagnetic fields in free space. While software exists to simulate these fields (such as COMSOL, a finite element software package), it can be expensive (both financially and temporally) to use. In addition, there is little support for creating quick-and-dirty scenes in the experience of the author.

One of the most useful components of the simulation is visualization of the electric and magnetic fields and the resulting forces, similar to the visualization of velocities and forces in the fluid and cloth simulations from earlier in the semester.

The majority of the physical computations in the simulations will use Maxwell's equations, largely gathered from [2]. [3] was used heavily for some of the approximations for finite wires, as was [1].

The inclusion of gravity in the physical model is a possibility, but will probably be an afterthought, as gravitational forces are many orders of magnitude weaker than electromagnetic forces.

2 Underlying Mathematics

At its core, my project is based on the Maxwell's equations:

$$\oint_{\partial V} \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{Q(V)}{\varepsilon_0}$$

$$\oint_{\partial V} \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$$

$$\oint_{\partial S} \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\frac{\partial \Phi_{B,S}}{\partial t}$$

$$\oint_{\partial S} \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \mu_0 I_S + \mu_0 \varepsilon_0 \frac{\partial \Phi_{E,S}}{\partial t}$$

As elegant as they appear in their simplicity, these integrals are extremely difficult to solve analytically in geometries without certain symmetries (loops, straight lines, points, etc.).

However, for many geometries, the equations for **B** and **E** can be easily calculated.

2.1 Point Charge

For a point charge q at displacement $\mathbf{r}$ and moving at velocity $\mathbf{v}$,

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{\mathbf{r}}$$

$$\mathbf{B} = \frac{\mu_0}{4\pi} q \frac{\mathbf{v} \times \hat{\mathbf{r}}}{r^2}$$

2.2 Conducting Sphere

For a conducting sphere of radius R, charge Q, and displacement $\mathbf{r}$,

$$\mathbf{E} = \begin{cases} \frac{Q}{4\pi\varepsilon_0 r^2}, ||\mathbf{r}|| > R \\ 0, ||\mathbf{r}|| < R \end{cases}$$

$$\mathbf{B} = 0$$

2.3 Uniform Spherical Charge

For a uniform charge distribution Q of radius R and displacement $\mathbf{r}$,

$$\mathbf{E} = \begin{cases} \frac{Q}{4\pi\varepsilon_0 r^2}, ||\mathbf{r}|| > R \\ \frac{Qr}{4\pi\varepsilon_0 R^3}, ||\mathbf{r}|| < R \end{cases}$$

$$\mathbf{B} = 0$$

2.4 Long Straight Wire

Long, straight wire axisymettric to ℓ and orthogonal to $\hat{\mathbf{r}}$

$$\mathbf{B} = \mu_0 I \ell \times \hat{\mathbf{r}}$$

$$\mathbf{E} = 0$$

2.5 Magnetic Dipole

For a magnetic dipole with moment μ and displacement $\mathbf{r}$

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{3\mathbf{r}(\mu \cdot \mathbf{r}) - \mu r^2}{r^5}$$

$$\mathbf{E} = 0$$

2.6 Solenoid

For an infinitely long ideal solenoid axisymmetric with $\hat{\mathbf{z}}$ and orthogonal to $\mathbf{r}$ with current I, n turns per length,

$$\mathbf{B} = \mu_0 n I \hat{\mathbf{z}}$$

$$\mathbf{E} = 0$$

3 Technical Approach

The implimentation of the electromagnetics simulator was a near-complete rewrite of the fluid and cloth simulators encountered earlier in the course, but borrowed heavily from the data structures. However, the entire simulation followed the steps in Algorithm 1 for animation:

The core features were the behavior of the bodies themselves and the visualizations of the electric and magnetic fields. Experience played a large part in this: does the magnetic field around that wire look like it should? What about that solenoid? Many of the simpler geometries elicit recognizable shapes.

4 Future Work

There is much to be done to improve the system. There is no support whatsoever for physical objects that don't phase through each other. Collision detection is non-existent. There is no gravity, although this may be benificial in that there are no forces interfering with the electromagnetic interactions.

There is also much to be done in better approximating the actual behavior of electromagnetic components. There is currently no support for real circuits, and thus the complex signals that go along with that and can in theory greatly influence the electromagnetic fields around them. For this to work properly, a very thorough simulation would probably need to be done with finite elements, which as it happens, is what I would not like to happen with it.

Overall, the project was a success, although many of the (40+?) hours were spent debugging the complex interactions between all of the data structures, which I expected. It remains a cool toy to tinker around with, as ideas for new types of geometries come to mind. One idea is a projectile railgun.

Algorithm 1 Update the positions of the particles and the values of the magnetic and electric fields for each of the cells, then draw them

Require: *particles* are the particles in the scene
Require: t is the timestep
Require: *cells* are the cells in the scene
  for all p in *particles* **do**
    if p is fixed **then**
      $p(acceleration) \Leftarrow (0,0,0)$
    else
      $position \Leftarrow p(location)$
      B at *position* $\Leftarrow$ is the magnetic field at *position* due to the contributions from all particles
      E at *position* $\Leftarrow$ is the magnetic field at *position* due to the contributions from all particles
      $bfield \Leftarrow B$ at *position*
      $efield \Leftarrow E$ at *position*
      $force \Leftarrow q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$
      if p is a magnetic dipole **then**
        $force \Leftarrow force+$ the force due to all other dipoles
      end if
    end if
    $p(acceleration) \Leftarrow force/p(mass)$
  end for
  for all p in *particles* **do**
    $p(velocity) \Leftarrow p(velocity) + p(acceleration) \times t$
    $p(position) \Leftarrow p(position) + p(velocity) \times t$
  end for
  for all *cell* in *cells* **do**
    update the bfield and magnetic field at the location of *cell*
  end for
  $draw(particles)$

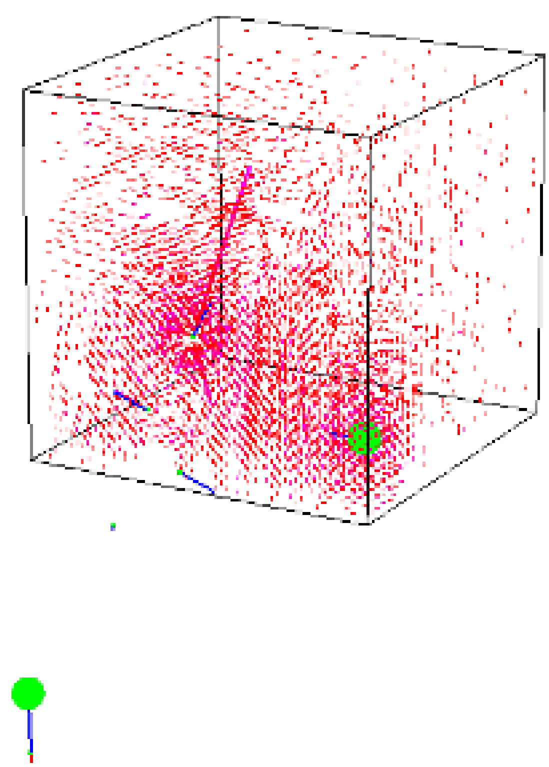

Figure 2: Moving charges, one of which has been ejected from the nest.

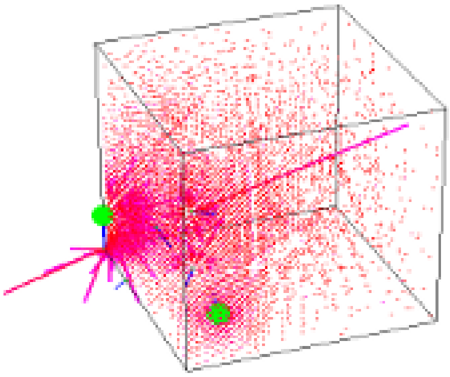

Figure 3: Moving charges

References

[1] Heinz E. Knoepfel. *Magnetic fields: A Comprehensive Theoretical Treatise for Practical Use.* John Wiley & Sons, Inc., 2000.

[2] Fawwaz T. Ulaby. *Fundamentals of Applied Electromagnetics*. Prentice Hall, 2006.

[3] Andrew Witkin. Physically based modeling: Principles and practice. In *ACM SIGGRAPH'92 Courses*, pages 1–12. ACM Press, 1992.