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# Ray Tracing

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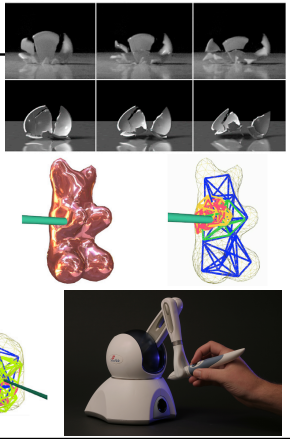
# Quiz Discussion

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- ## Announcements: Final Projects
- *Everyone* should post one or more ideas for a final project on the discussion forum (it was your “assignment” over Spring Break)
  - Connect with potential teammates (teams of 2 strongly recommended)
  - Start reading background papers
  - Proposal & summary of background research will be due in a couple weeks

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## Last Time?

- Rigid Body
- Finite Element Method
  - Stress/Strain
- Fracture
- Deformation
  - Level of Detail
  - Haptics

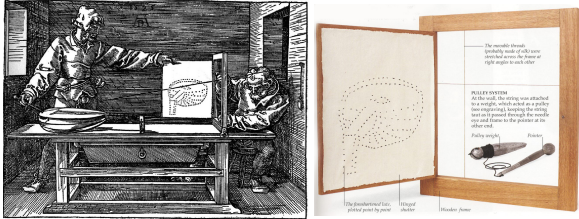


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- ## Today
- Ray Casting
    - Ray-Plane Intersection
    - Ray-Sphere Intersection
    - Point in Polygon
  - Ray Tracing
  - Recursive Ray Tracing
  - Distribution Ray Tracing

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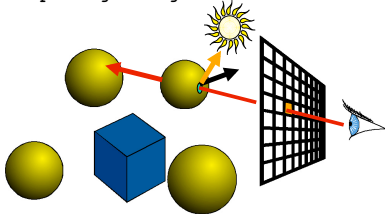
## Durer's Ray Casting Machine

- Albrecht Durer, 16<sup>th</sup> century



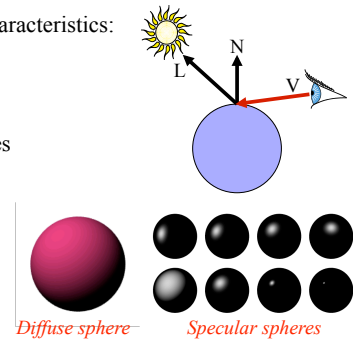
## Ray Casting

- For every pixel
- Construct a ray from the eye
  - Find the intersection and normal is the central part of ray casting
  - For every object in the scene Find **intersection** with the ray
  - Keep if closest
  - Shade depending on light and **normal** vector



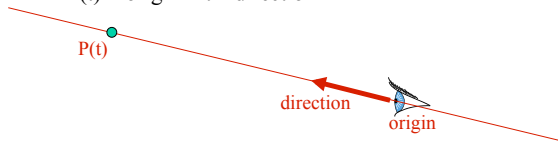
## A Note on Local Shading

- Surface/Scene Characteristics:
  - surface normal
  - direction to light
  - viewpoint
- Material Properties
  - Diffuse (matte)
  - Specular (shiny)
  - ...
- More later!



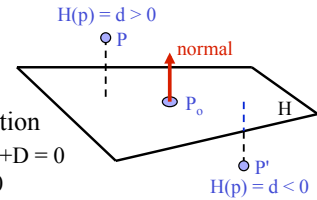
## Ray Representation?

- Two vectors:
  - Origin
  - Direction (normalized is better)
- Parametric line (*explicit* representation)
  - $P(t) = \text{origin} + t * \text{direction}$



## 3D Plane Representation?

- Plane defined by
  - $P_o = (x,y,z)$
  - $n = (A,B,C)$
- Implicit* plane equation
  - $H(P) = Ax+By+Cz+D = 0$
  - $= n \cdot P + D = 0$
- Point-Plane distance?
  - If n is normalized, distance to plane,  $d = H(P)$
  - d is the *signed distance*!

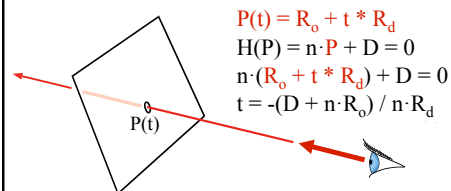


## Explicit vs. Implicit?

- Ray equation is explicit  $P(t) = R_o + t * R_d$ 
  - Parametric
  - Generates points
  - Harder to verify that a point is on the ray
- Plane equation is implicit  $H(P) = n \cdot P + D = 0$ 
  - Solution of an equation
  - Does not generate points
  - Verifies that a point is on the plane

## Ray-Plane Intersection

- Intersection means both are satisfied
- So, insert explicit equation of ray into implicit equation of plane & solve for t



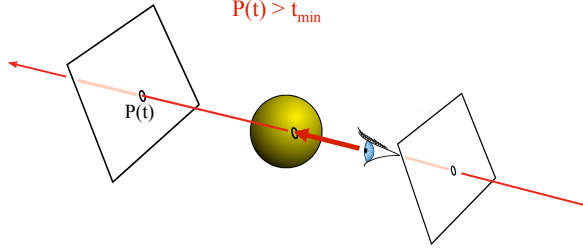
## Additional Housekeeping

- Verify that intersection is closer than previous

$$P(t) < t_{\text{current}}$$

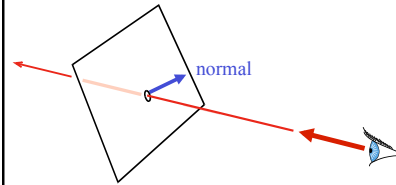
- Verify that it is not out of range (behind eye)

$$P(t) > t_{\text{min}}$$



## Normal

- For shading
  - diffuse: dot product between light and normal
- Normal is constant



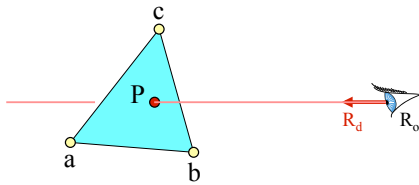
## Ray-Triangle Intersection

- Use barycentric coordinates:

$$- P(\alpha, \beta, \gamma) = \alpha a + \beta b + \gamma c$$

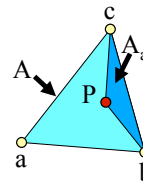
with  $\alpha + \beta + \gamma = 1$

- If  $0 < \alpha < 1$  &  $0 < \beta < 1$  &  $0 < \gamma < 1$  then the point is inside the triangle!



## How Do We Compute $\alpha, \beta, \gamma$ ?

- Ratio of opposite sub-triangle area to total area
  - $\alpha = A_a/A$     $\beta = A_b/A$     $\gamma = A_c/A$
- Use signed areas for points outside the triangle



## Using Cramer's Rule...

- Used to solve for one variable at a time in system of equations

$$\beta = \frac{\begin{vmatrix} a_x - R_{ox} & a_x - c_x & R_{dx} \\ a_y - R_{oy} & a_y - c_y & R_{dy} \\ a_z - R_{oz} & a_z - c_z & R_{dz} \end{vmatrix}}{|A|} \quad \gamma = \frac{\begin{vmatrix} a_x - b_x & a_x - R_{ox} & R_{dx} \\ a_y - b_y & a_y - R_{oy} & R_{dy} \\ a_z - b_z & a_z - R_{oz} & R_{dz} \end{vmatrix}}{|A|}$$

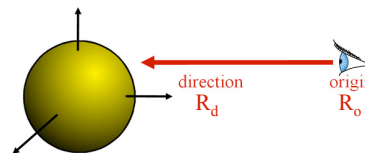
$$t = \frac{\begin{vmatrix} a_x - b_x & a_x - c_x & a_x - R_{ox} \\ a_y - b_y & a_y - c_y & a_y - R_{oy} \\ a_z - b_z & a_z - c_z & a_z - R_{oz} \end{vmatrix}}{|A|}$$

| | denotes the determinant

Can be copied mechanically into code

## Sphere Representation?

- Implicit sphere equation
  - Assume centered at origin (easy to translate)
  - $H(P) = P \cdot P - r^2 = 0$



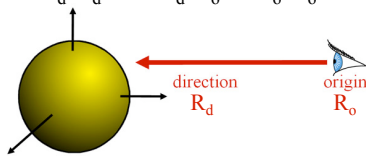
## Ray-Sphere Intersection

- Insert explicit equation of ray into implicit equation of sphere & solve for t

$$P(t) = R_o + t \cdot R_d \quad H(P) = P \cdot P - r^2 = 0$$

$$(R_o + tR_d) \cdot (R_o + tR_d) - r^2 = 0$$

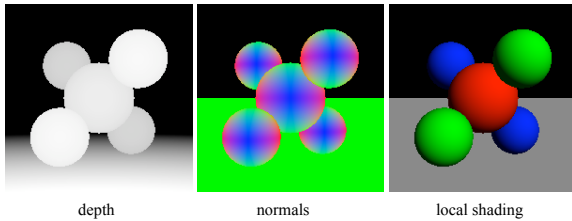
$$R_d \cdot R_d t^2 + 2R_d \cdot R_o t + R_o \cdot R_o - r^2 = 0$$



## Ray-Sphere Intersection

- Quadratic:  $at^2 + bt + c = 0$ 
  - $a = 1$  (remember,  $\|R_d\| = 1$ )
  - $b = 2R_d \cdot R_o$
  - $c = R_o \cdot R_o - r^2$
- with discriminant  $d = \sqrt{b^2 - 4ac}$
- and solutions  $t_{\pm} = \frac{-b \pm d}{2a}$
- What does it mean if there are no solutions, 1 solution, or 2 solutions?

## Questions?

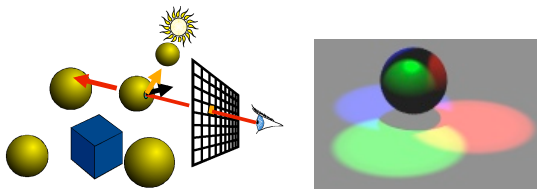


## Today

- Ray Casting
- Ray Tracing
  - Shadows
  - Reflection
  - Refraction
- Recursive Ray Tracing
- Distribution Ray Tracing

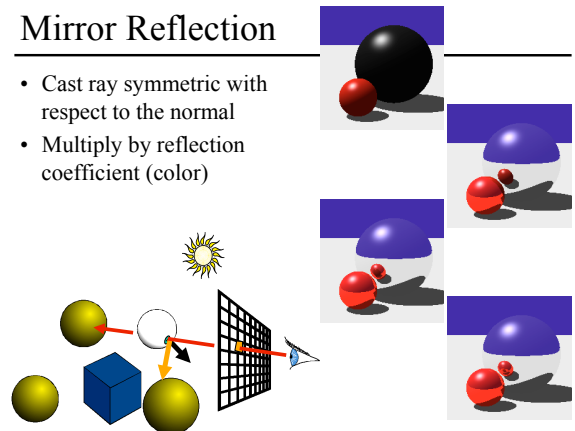
## How Can We Add Shadows?

Find the point to be shaded  
 For every light,  
 Construct ray from point to light  
 For every object  
 find intersection of ray with object  
 If no objects between point and light  
 Add contribution from light



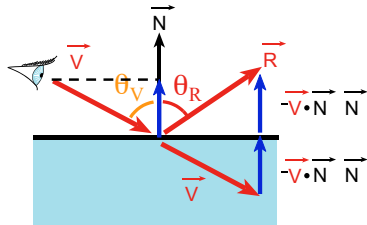
## Mirror Reflection

- Cast ray symmetric with respect to the normal
- Multiply by reflection coefficient (color)



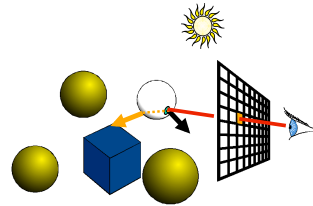
## Reflection

- Reflection angle = view angle
- $\mathbf{R} = \mathbf{V} - 2(\mathbf{V} \cdot \mathbf{N})\mathbf{N}$

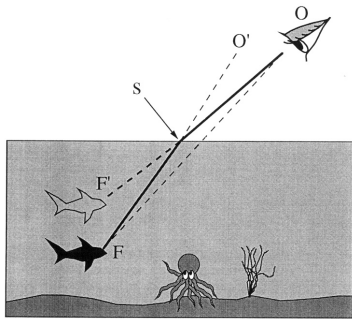


## Transparency

- Cast ray in refracted direction
- Multiply by transparency coefficient (color)

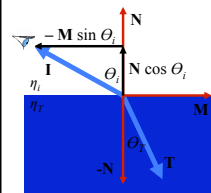


## Qualitative Refraction



From "Color and Light in Nature" by Lynch and Livingston

## Refraction



$$\begin{aligned} \mathbf{I} &= \mathbf{N} \cos \theta_i - \mathbf{M} \sin \theta_i \\ \mathbf{M} &= (\mathbf{N} \cos \theta_i - \mathbf{I}) / \sin \theta_i \\ \mathbf{T} &= -\mathbf{N} \cos \theta_r + \mathbf{M} \sin \theta_r \\ &= -\mathbf{N} \cos \theta_r + (\mathbf{N} \cos \theta_i - \mathbf{I}) \sin \theta_r / \sin \theta_i \\ &= -\mathbf{N} \cos \theta_r + (\mathbf{N} \cos \theta_i - \mathbf{I}) \eta_r \\ &= [\eta_r \cos \theta_i - \cos \theta_r] \mathbf{N} - \eta_r \mathbf{I} \\ &= [\eta_r \cos \theta_i - \sqrt{1 - \sin^2 \theta_r}] \mathbf{N} - \eta_r \mathbf{I} \\ &= [\eta_r \cos \theta_i - \sqrt{1 - \eta_r^2 \sin^2 \theta_i}] \mathbf{N} - \eta_r \mathbf{I} \\ &= [\eta_r \cos \theta_i - \sqrt{1 - \eta_r^2 (1 - \cos^2 \theta_i)}] \mathbf{N} - \eta_r \mathbf{I} \\ &= [\eta_r (\mathbf{N} \cdot \mathbf{I}) - \sqrt{1 - \eta_r^2 (1 - (\mathbf{N} \cdot \mathbf{I})^2)}] \mathbf{N} - \eta_r \mathbf{I} \end{aligned}$$

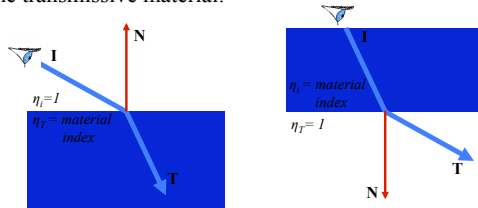
**Snell-Descartes Law:**  
 $\eta_i \sin \theta_i = \eta_r \sin \theta_r$

$$\frac{\sin \theta_r}{\sin \theta_i} = \frac{\eta_i}{\eta_r} = \eta_r$$

- Total internal reflection when the square root is imaginary
- Don't forget to normalize!

## Refraction & the Sidedness of Objects

- Make sure you know whether you're entering or leaving the transmissive material:



- What about intersecting transparent objects?

## Total Internal Reflection



Fig. 3.7A The optical manhole. From under water, the entire celestial hemisphere is compressed into a circle only 97.2° across. The dark boundary defining the edges of the manhole is not sharp due to surface waves. The rays are analogous to the crepuscular type seen in hazy air, Section 1.9. (Photo by D. Granger)



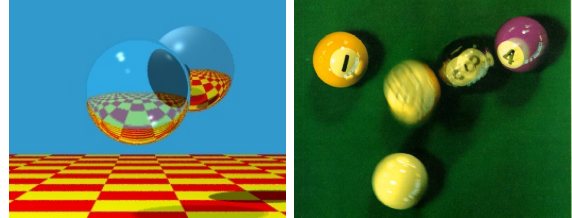
Fig. 3.7B The optical manhole. Light from the horizon (angle of incidence = 90°) is refracted downward at an angle of 48.6°. This compresses the sky into a circle with a diameter of 97.2° instead of its usual 180°.

From "Color and Light in Nature" by Lynch and Livingston

## Questions?

## Readings for Today: *(read both)*

- "An improved illumination model for shaded display" Turner Whitted, 1980.
- "Distributed Ray Tracing", Cook, Porter, & Carpenter, SIGGRAPH 1984.

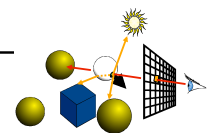


## Today

- Ray Casting
- Ray Tracing
- **Recursive Ray Tracing**
- Distribution Ray Tracing

## Ray Tracing

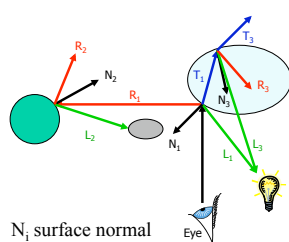
```
trace ray
↑
Intersect all objects
color = ambient term
For every light
  cast shadow ray
  color += local shading term
If mirror
  color += color_refl *
  trace reflected ray
If transparent
  color += color_trans *
  trace transmitted ray
```



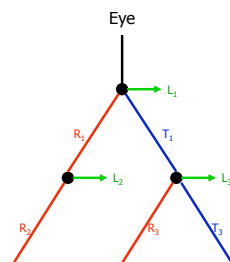
- Stopping criteria:
- **Recursion depth**
    - Stop after a number of bounces
  - **Ray contribution**
    - Stop if reflected/transmitted contribution becomes too small

- *Does it ever end?*

## The Ray Tree



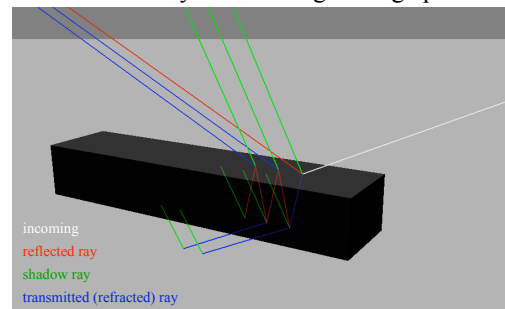
- $N_i$  surface normal  
 $R_i$  reflected ray  
 $L_i$  shadow ray  
 $T_i$  transmitted (refracted) ray



Complexity?

## Ray Debugging

- Visualize the ray tree for single image pixel



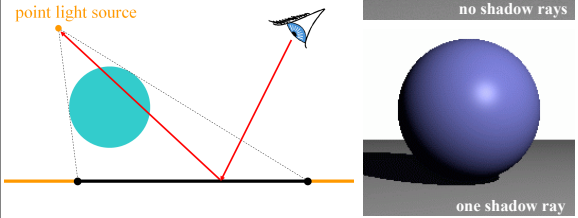
- incoming  
reflected ray  
shadow ray  
transmitted (refracted) ray

## Today

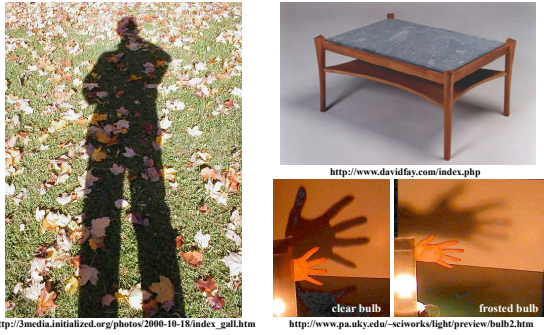
- Ray Casting
- Ray Tracing
- Recursive Ray Tracing
- **Distribution Ray Tracing**
  - Soft shadows
  - Antialiasing (getting rid of jaggies)
  - Glossy reflection
  - Motion blur
  - Depth of field (focus)

## Shadows

- one shadow ray per intersection per point light source

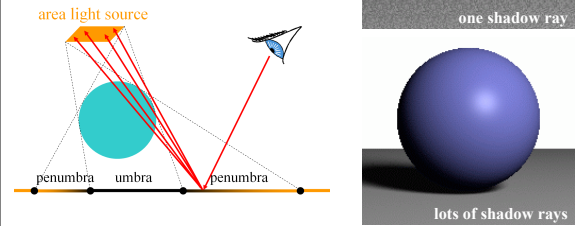


## Shadows & Light Sources



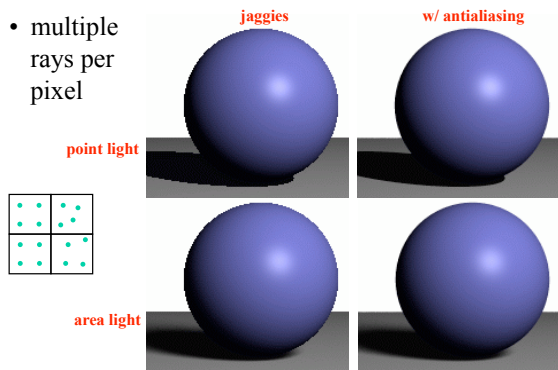
## Soft Shadows

- multiple shadow rays to sample area light source



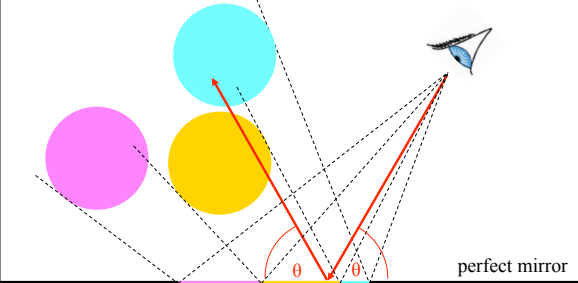
## Antialiasing – Supersampling

- multiple rays per pixel



## Reflection

- one reflection ray per intersection



### Glossy Reflection

- multiple reflection rays

Justin Legakis

polished surface

### Motion Blur

- Sample objects temporally

Rob Cook

### Depth of Field

- multiple rays per pixel

Justin Legakis

film

focal length

### Ray Tracing Algorithm Analysis

- Ray casting
- Lots of primitives
- Recursive
- Distributed Ray Tracing Effects
  - Soft shadows
  - Anti-aliasing
  - Glossy reflection
  - Motion blur
  - Depth of field

cost  $\approx$  height \* width \*  
 num primitives \*  
 intersection cost \*  
 size of recursive ray tree \*  
 num shadow rays \*  
 num supersamples \*  
 num glossy rays \*  
 num temporal samples \*  
 num focal samples \*  
 ...

can we reduce this?

these can serve double duty

### Reading for Friday 3/19:

- Goral, Torrance, Greenberg & Battaile "Modeling the Interaction of Light Between Diffuse Surfaces", SIGGRAPH '84

white paper enclosure

set of illuminating lights

camera

test cube

white diffuse surface

Post a comment or question on the LMS discussion by 10am on Friday 2/29