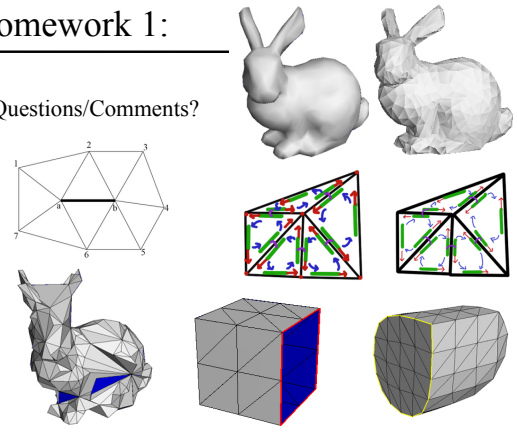
Subdivision Surfaces

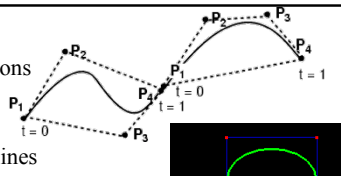
Homework 1:

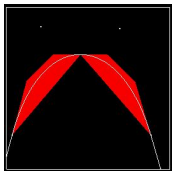
- Questions/Comments?

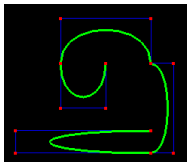


Last Time?

- Curves & Surfaces
- Continuity Definitions
  - C^0, G^1, C^1, … C^∞
- Interpolation vs. Approximation Splines
- Cubic Bezier & BSpline







Today

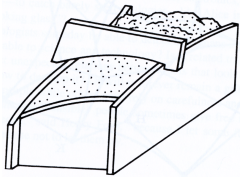
- Spline Surfaces / Patches
  - Tensor Product
  - Bilinear Patches
  - Bezier Patches
  - Trimming Curves
- Subdivision Surface "Zoo"
- Misc. Mesh/Surface Vocabulary
- "Subdivision Surfaces in Character Animation"

Tensor Product

- Of two vectors:

$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \otimes \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \end{bmatrix} = \begin{bmatrix} a_1b_1 & a_2b_1 & a_3b_1 \\ a_1b_2 & a_2b_2 & a_3b_2 \\ a_1b_3 & a_2b_3 & a_3b_3 \\ a_1b_4 & a_2b_4 & a_3b_4 \end{bmatrix}$$

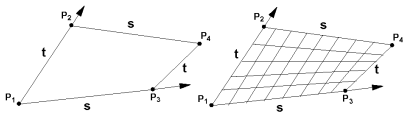
- Similarly, we can define a surface as the tensor product of two curves....



Farin, Curves and Surfaces for Computer Aided Geometric Design

Bilinear Patch

Bi-lerp a (typically non-planar) quadrilateral

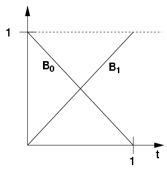


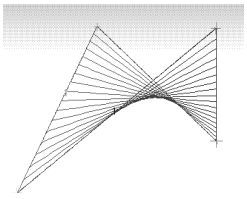
Notation: $\mathbf{L}(P_1, P_2, \alpha) \equiv (1-\alpha)P_1 + \alpha P_2$

$$Q(s,t) = \mathbf{L}(\mathbf{L}(P_1, P_2, t), L(P_3, P_4, t), s)$$

Bilinear Patch

- Smooth version of quadrilateral with non-planar vertices...



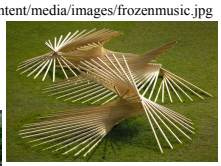


  - But will this help us model smooth surfaces?
  - Do we have control of the derivative at the edges?

Ruled Surfaces in Art & Architecture

http://www.bergenwood.no/wp-content/media/images/frozenmusic.jpg

Chiras Iulia
Astri Isabella
Matiss Shteinerts



Antoni Gaudi
Children's School
Barcelona

http://www.lonelyplanetimages.com/images/399954

Today

- Spline Surfaces / Patches
  - Tensor Product
  - Bilinear Patches
  - Bezier Patches
  - Trimming Curves
- Subdivision Surface "Zoo"
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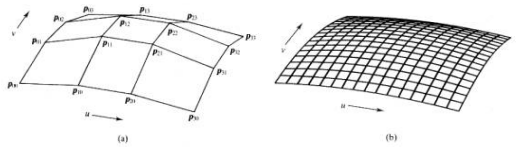
Bicubic Bezier Patch

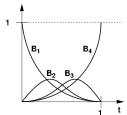
Notation: $\mathbf{CB}(P_1, P_2, P_3, P_4, \alpha)$ is Bézier curve with control points P_i evaluated at α

Define "Tensor-product" Bézier surface

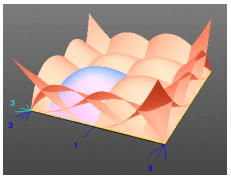
$$Q(s,t) = \mathbf{CB}(\ \mathbf{CB}(P_{00}, P_{01}, P_{02}, P_{03}, t),\ \mathbf{CB}(P_{10}, P_{11}, P_{12}, P_{13}, t),\ \mathbf{CB}(P_{20}, P_{21}, P_{22}, P_{23}, t),\ \mathbf{CB}(P_{30}, P_{31}, P_{32}, P_{33}, t),\ s)$$



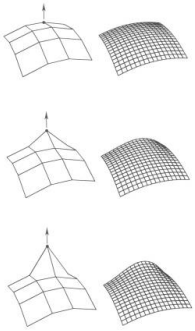
Editing Bicubic Bezier Patches



Curve Basis Functions

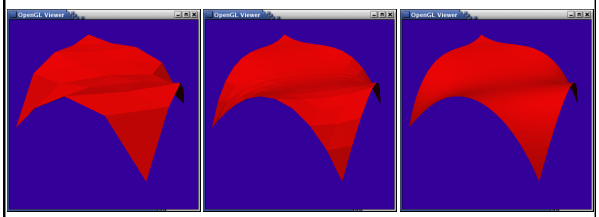


Surface Basis Functions



Bicubic Bezier Patch Tessellation

- Given 16 control points and a tessellation resolution, we can create a triangle mesh



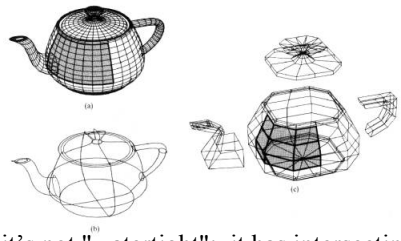
resolution: 5x5 vertices

resolution: 11x11 vertices

resolution: 41x41 vertices

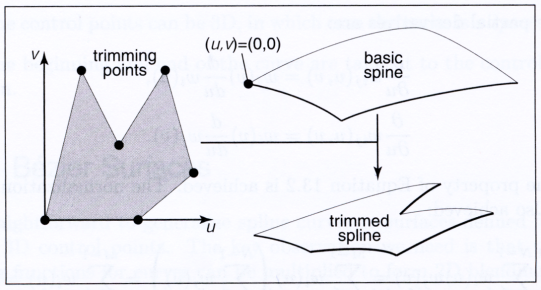
Modeling with Bicubic Bezier Patches

- Original Teapot specified with Bezier Patches



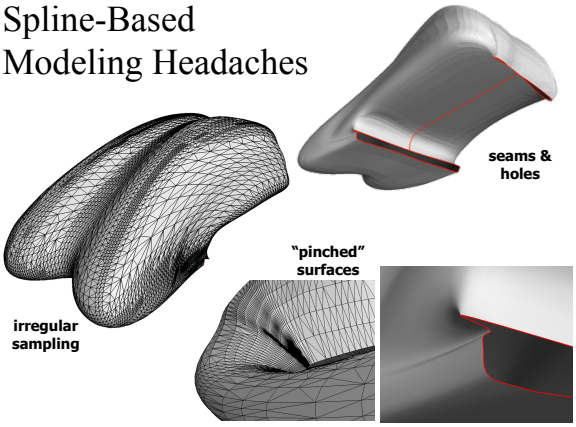
- But it's not "watertight": it has intersecting surfaces at spout & handle, no bottom, a hole at the spout tip, a gap between lid & base

Trimming Curves for Patches



Shirley, Fundamentals of Computer Graphics

Spline-Based Modeling Headaches



Questions?

- Bezier Patches?

or

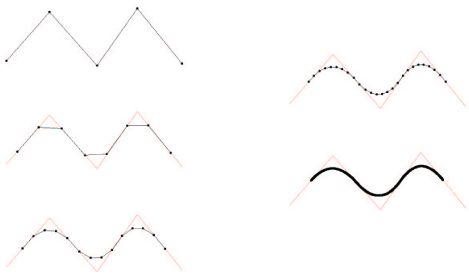
- Triangle Mesh?

Henrik Wann Jensen

Today

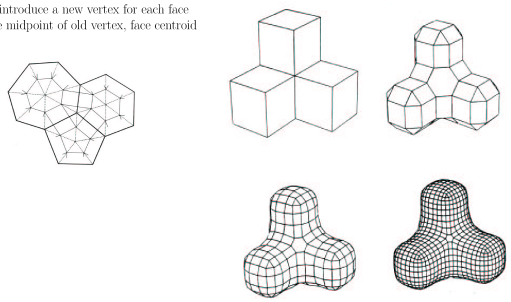
- Spline Surfaces / Patches
- Subdivision Surface "Zoo"
  - Doo Sabin (anything!)
  - Loop (triangles only)
  - Catmull Clark (turns everything into quads)
  - … many others!
- Misc. Mesh/Surface Vocabulary
- "Subdivision Surfaces in Character Animation"

Chaikin's Algorithm

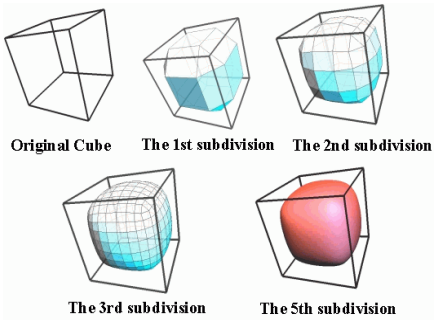


Doo-Sabin Subdivision

Idea: introduce a new vertex for each face
At the midpoint of old vertex, face centroid

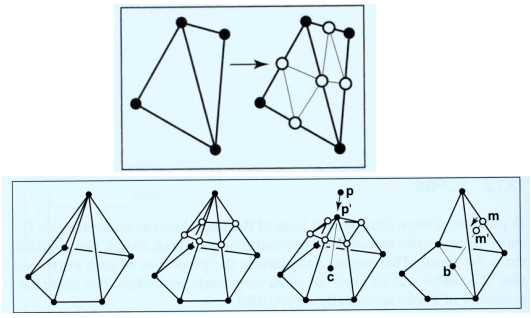


Doo-Sabin Subdivision



http://www.ke.ics.saitama-u.ac.jp/xuz/pic/doo-sabin.gif

Loop Subdivision



Shirley, Fundamentals of Computer Graphics

Loop Subdivision

Subdivision Rules. The masks for the Loop scheme are shown in Figure 4.3. For boundaries and edges tagged as *crease* edges, special rules are used. These rules produce a cubic spline curve along the boundary/crease. The curve only depends on control points on the boundary/crease.

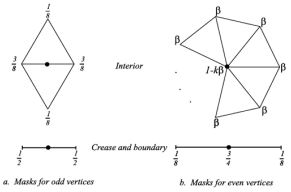


Figure 4.3: *Loop subdivision: in the picture above,* β *can be chosen to be either* $\frac{1}{n}(5/8-(\frac{3}{8}+\frac{1}{4}\cos\frac{2\pi}{n})^2)$ *(original choice of Loop [16]), or, for* $n>3$, $\beta=\frac{3}{8n}$ *as proposed by Warren [33]. For* $n=3$, $\beta=3/16$ *can be used.*

SIGGRAPH 2000 course notes
Subdivision for Modeling and Animation (page 70)

Catmull Clark Subdivision

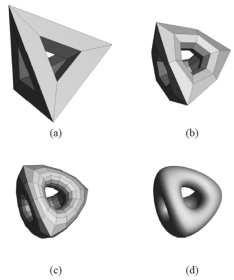


Figure 3: Recursive subdivision of a topologically complicated mesh: (a) the control mesh; (b) after one subdivision step; (c) after two subdivision steps; (d) the limit surface.

$$e_j^{l+1} = \frac{v^l + e_j^l + f_{j-1}^{l+1} + f_j^{l+1}}{4}, \quad (1)$$

where subscripts are taken modulo the valence of the central vertex v^0. (The valence of a vertex is the number of edges incident to it.) Finally, a vertex point v^l is computed as

$$v^{l+1} = \frac{n-2}{n} v^l + \frac{1}{n^2}\sum_j e_j^l + \frac{1}{n^2}\sum_j f_j^{l+1} \quad (2)$$

Vertices of valence 4 are called ordinary; others are called extraordinary.

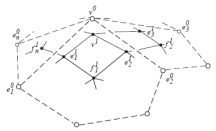
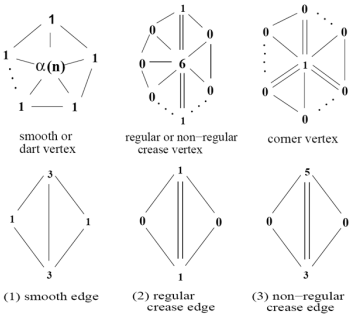


Figure 4: The situation around a vertex v^0 of valence n.

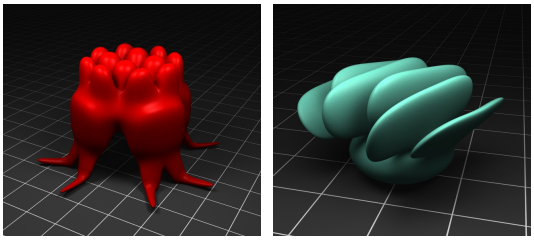
"Subdivision Surfaces in Character Animation",
DeRose, Kass & Truong, SIGGRAPH 1998

Adding creases to Loop Subdivision

- Vertex & edge masks
- Limit masks
  - Position
  - Tangent



Questions?



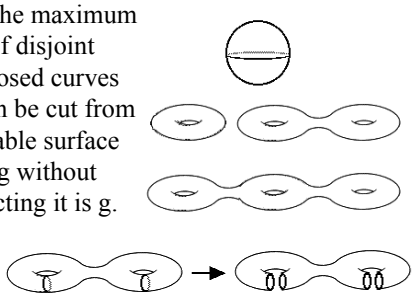
Justin Legakis

Today

- Spline Surfaces / Patches
- Subdivision Surface "Zoo"
- Misc. Mesh/Surface Vocabulary
- "Subdivision Surfaces in Character Animation"

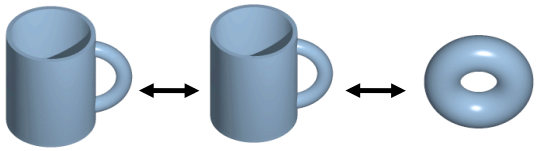
Misc. Mesh/Surface Vocabulary

- *Genus:* The maximum number of disjoint simple closed curves which can be cut from an orientable surface of genus g without disconnecting it is g.



Misc. Mesh/Surface Vocabulary

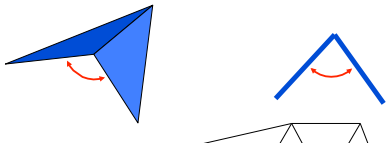
- *Homeomorphic/Topological equivalence*: a continuous stretching and bending of the object into a new shape



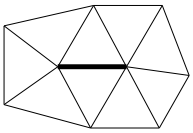
http://en.wikipedia.org/wiki/Image:Mug_and_Torus_morph.gif

Misc. Mesh/Surface Vocabulary

- *Dihedral Angle*:
  - the angle between the planes of two triangular faces
  - "looking down the edge" between two faces, the angle between the faces.

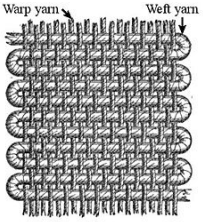


- *Valence (a.k.a. degree)*: the number of edges incident to the vertex.



Misc. Mesh/Surface Vocabulary

- *Warp & weft:* Yarns used in weaving. Because the weft does not have to be stretched in the way that the warp is, it can generally be less strong.



http://en.wikipedia.org/wiki/Weft

Today

- Spline Surfaces / Patches
- Subdivision Surface "Zoo"
- Seams In Subdivision
- Misc. Mesh/Surface Vocabulary
- "Subdivision Surfaces in Character Animation"

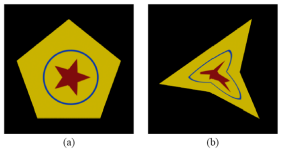
Reading for Today

- DeRose, Kass, & Truong, "Subdivision Surfaces in Character Animation", SIGGRAPH 1998

Figure 5: Geri's hand as a piecewise smooth Catmull-Clark surface. Infinitely sharp creases are used between the skin and the finger nails.

Subdivision Surfaces in Character Animation

- Catmull Clark Subdivision Rules
- Semi-sharp vs. Infinitely-sharp creases
- Mass-Spring Cloth *(next week)*
- Hierarchical Mesh for Collision
- Texturing Subdivision Surfaces

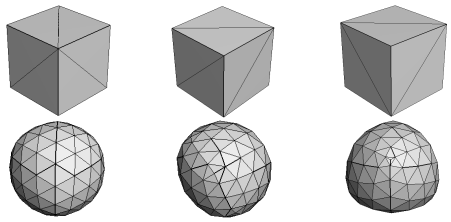


(a) (b) (c) (d)

Figure 11: (a) A texture mapped regular pentagon comprised of 5 triangles; (b) the pentagonal model with its vertices moved; (c) A subdivision surface whose control mesh is the same 5 triangles in (a), and where boundary edges are marked as creases; (d) the subdivision surface with its vertices positioned as in (b).

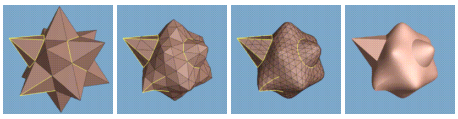
Catmull-Clark in Pixar Production

- Based on quadrilaterals
  - Like NURBS, specifically cubic bsplines
  - Implicit adjacency in subdivided microgeometry
  - Better than triangles for symmetric objects

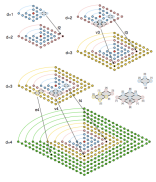


Readings for Tuesday *(pick one)*

- Hoppe et al., "Piecewise Smooth Surface Reconstruction" SIGGRAPH 1994



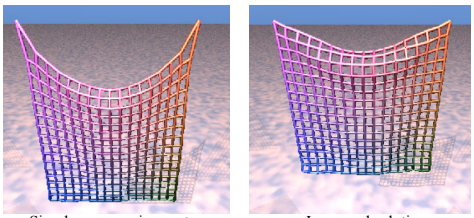
- Shiue, Jones, and Peters, "A Realtime GPU Subdivision Kernel", SIGGRAPH 2005



Post a comment or question on the LMS discussion by 10am on Tuesday

Reading for Friday

- "Deformation Constraints in a Mass-Spring Model to Describe Rigid Cloth Behavior", Provot, 1995.



Simple mass-spring system Improved solution

Post a comment or question on the LMS discussion by 10am on Friday