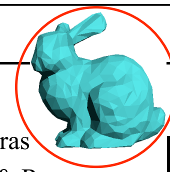
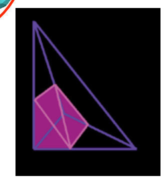
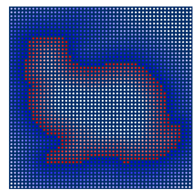
Navier-Stokes
&
Flow Simulation

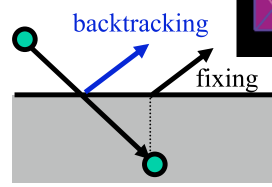
Last Time?

- Implicit Surfaces
- Marching Cubes/Tetras
- Collision Detection & Response
- Conservative Bounding Regions









Today

- Flow Simulations in Computer Graphics
  - water, smoke, viscous fluids
- Navier-Stokes Equations
  - incompressibility, conservation of mass
  - conservation of momentum & energy
- Fluid Representations
- Basic Algorithm
- Data Representation

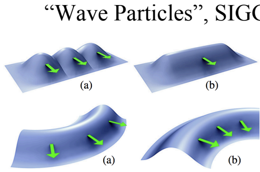
Flow Simulations in Graphics

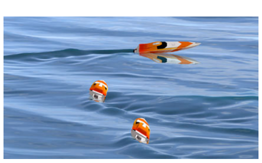
- Random velocity fields
  - with averaging to get simple background motion
- Shallow water equations
  - height field only, can't represent crashing waves, etc.
- Full Navier-Stokes

- *note: typically we ignore surface tension and focus on macroscopic behavior*

Heightfield Wave Simulation

- Cem Yuksel, Donald H. House, and John Keyser, "Wave Particles", SIGGRAPH 2007





Flow in a Voxel Grid

- conservation of mass:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

For a single phase simulation (e.g., water only, air only)

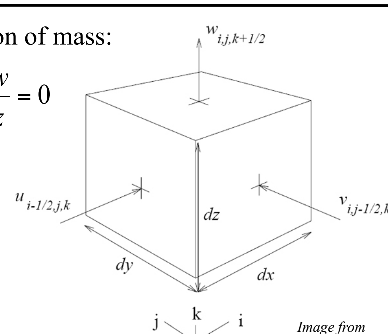


Image from Foster & Metaxas, 1996

Navier-Stokes Equations

- conservation of momentum:

gravity (& other external forces)
pressure
viscosity

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = -\frac{\partial p}{\partial x} + g_x + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\frac{\partial v}{\partial t} + \frac{\partial vu}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} = -\frac{\partial p}{\partial y} + g_y + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

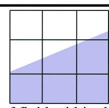
$$\frac{\partial w}{\partial t} + \frac{\partial wu}{\partial x} + \frac{\partial wv}{\partial y} + \frac{\partial w^2}{\partial z} = -\frac{\partial p}{\partial z} + g_z + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

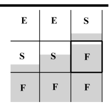
acceleration
Convection: internal movement in a fluid (e.g., caused by variation in density due to a transfer of heat)
drag

Today

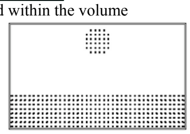
- Flow Simulations in Computer Graphics
- Navier-Stokes Equations
- Fluid Representations
- Basic Algorithm
- Data Representation

Modeling the Air/Water Surface

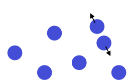




- Volume-of-fluid tracking
  - a scalar saying how "full" each cell is
- Marker and Cell (MAC)
  - *volume marker particles* identify location of fluid within the volume
  - (optional) *surface marker particles* track the detailed shape of the fluid/air boundary
  - But… marker particles don't have or represent a mass/volume of fluid
  - Harlow & Welch, "Numerical calculation of time-dependent viscous incompressible flow of fluid with free surface", *The Physics of Fluids*, 1965.



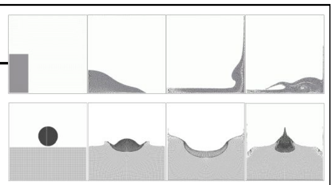
- Smoothed Particle Hydrodynamics (SPH)
  - "Meshless" (no voxel grid)
  - Each particle represents a specific mass of fluid
  - Repulsive forces between neighboring particles maintain constant volume



Comparing Representations

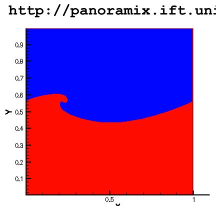
- How do we render the resulting surface?
- Are we guaranteed not to lose mass/volume? (is the simulation incompressible?)
- How is each affected by the grid resolution and timestep?
- Can we guarantee stability?

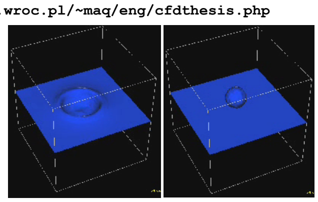
Demos



- Nice Marker and Cell (MAC) videos at:

http://panoramix.ift.uni.wroc.pl/~maq/eng/cfdthesis.php

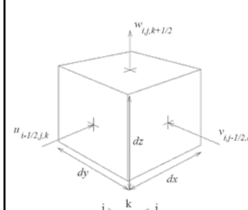


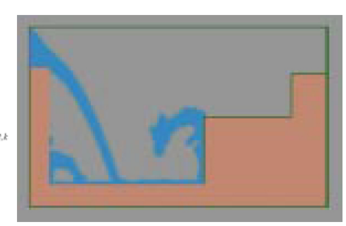


http://mme.uwaterloo.ca/~fslien/free_surface/free_surface.htm

Reading for Today

- "Realistic Animation of Liquids", Foster & Metaxas, 1996





Today

- Flow Simulations in Computer Graphics
- Navier-Stokes Equations
- Fluid Representations
- Basic Algorithm
- Data Representation

Each Grid Cell Stores:

- Velocity at the cell faces (offset grid)
- Pressure
- List of particles

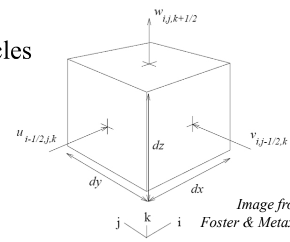


Image from Foster & Metaxas, 1996

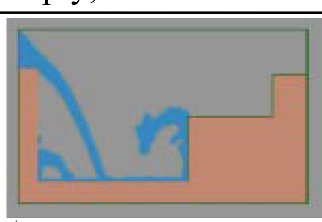
Initialization

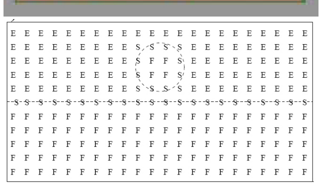
- Choose a voxel resolution
- Choose a particle density
- Create grid & place the particles
- Initialize pressure & velocity of each cell
- Set the viscosity & gravity
- Choose a timestep & go!

At each Timestep:

- Identify which cells are Empty, Full, or on the Surface
- Compute new velocities
- Adjust the velocities to maintain an incompressible flow
- Move the particles
  - Interpolate the velocities at the faces
- Render the geometry and repeat!

Empty, Surface & Full Cells





Images from Foster & Metaxas, 1996

At each Timestep:

- Identify which cells are Empty, Full, or on the Surface
- Compute new velocities
- Adjust the velocities to maintain an incompressible flow
- Move the particles
  - Interpolate the velocities at the faces
- Render the geometry and repeat!

Compute New Velocities

$$\tilde{u}_{i+1/2,j,k} = u_{i+1/2,j,k} + \delta t\{(1/\delta x)[(u_{i,j,k})^2 - (u_{i+1,j,k})^2]$$
$$+(1/\delta y)[(uv)_{i+1/2,j-1/2,k} - (uv)_{i+1/2,j+1/2,k}]$$
$$+(1/\delta z)[(uw)_{i+1/2,j,k-1/2} - (uw)_{i+1/2,j,k+1/2}] + g_x$$
$$+(1/\delta x)(p_{i,j,k} - p_{i+1,j,k}) + (\nu/\delta x^2)(u_{i+3/2,j,k}$$
$$-2u_{i+1/2,j,k} + u_{i-1/2,j,k}) + (\nu/\delta y^2)(u_{i+1/2,j+1,k}$$
$$-2u_{i+1/2,j,k} + u_{i+1/2,j-1,k}) + (\nu/\delta z^2)(u_{i+1/2,j,k+1}$$
$$-2u_{i+1/2,j,k} + u_{i+1/2,j,k-1})\},$$

Note: some of these values are the *average velocity* within the cell rather than the velocity at a cell face

At each Timestep:

- Identify which cells are Empty, Full, or on the Surface
- Compute new velocities
- Adjust the velocities to maintain an incompressible flow
- Move the particles
  - Interpolate the velocities at the faces
- Render the geometry and repeat!

Adjusting the Velocities

- Calculate the *divergence* of the cell (the extra in/out flow)
- The divergence is used to update the *pressure* within the cell
- Adjust each face velocity uniformly to bring the divergence to zero
- Iterate across the entire grid until divergence is < ε

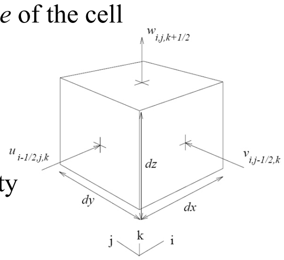
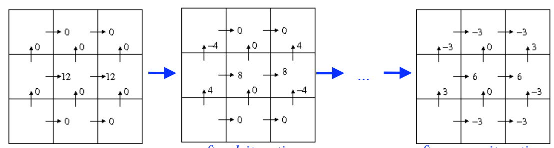


Image from Foster & Metaxas, 1996

Calculating/Eliminating Divergence



initial flow field — *after 1 iteration* — *after many iterations*
(results will vary with different calculation order)

0	0	0
−12	0	+12
0	0	0

corresponding divergence

−4	0	4
0	0	0
−4	0	4

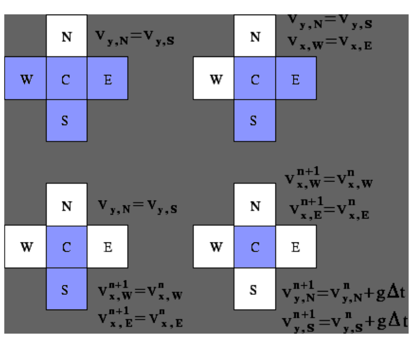
corresponding divergence

0	0	0
0	0	0
0	0	0

corresponding divergence

Handing Free Surface with MAC

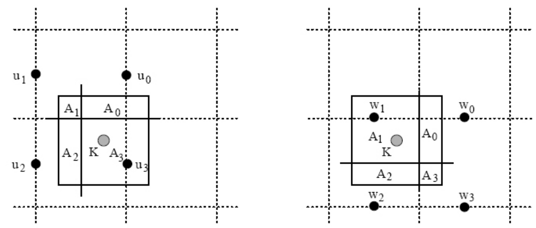
- Divergence in surface cells:
  - Is divided equally amongst neighboring empty cells
  - Or other similar strategies?
- Zero out the divergence & pressure in empty cells



At each Timestep:

- Identify which cells are Empty, Full, or on the Surface
- Compute new velocities
- Adjust the velocities to maintain an incompressible flow
- Move the particles
  - Interpolate the velocities at the faces
- Render the geometry and repeat!

Velocity Interpolation

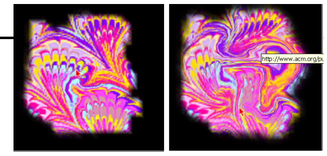


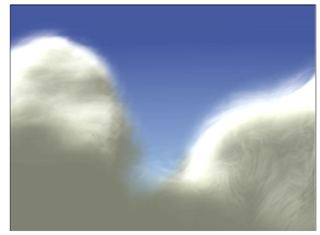
$u_k = A_0 u_0 + A_1 u_1 + A_2 u_2 + A_3 u_3$ $w_k = A_0 w_0 + A_1 w_1 + A_2 w_2 + A_3 u_3$

Image from Foster & Metaxas, 1996

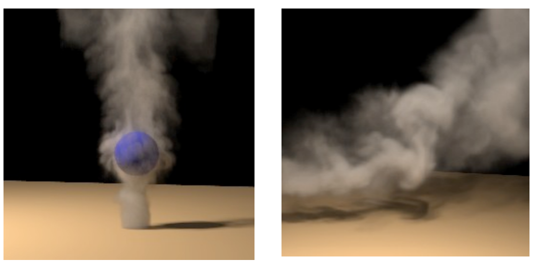
Stable Fluids

- "Stable Fluids", Jos Stam, SIGGRAPH 1999.





Efficient Smoke Simulation



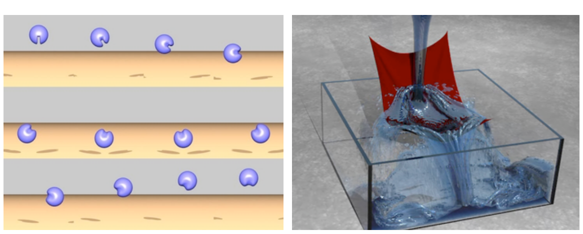
"Visual Simulation of Smoke"
Fedkiw, Stam & Jensen
SIGGRAPH 2001

Solid/Liquid: Time-varying viscosity

"Melting and Flowing"
Carlson, Mucha, Van Horn III & Turk
Symposium on Computer Animation 2002

Ron Fedkiw http://physbam.stanford.edu/~fedkiw/

- Enright, Marschner, & Fedkiw, "Animation and Rendering of Complex Water Surfaces", SIGGRAPH 2002.
- Guendelman, Selle, Losasso, & Fedkiw, "Coupling Water and Smoke to Thin Deformable and Rigid Shells", SIGGRAPH 2005.



Reading for Tuesday 2/22:

- "Real-Time Hand-Tracking with a Color Glove" SIGGRAPH 2009, Wang & Popović.