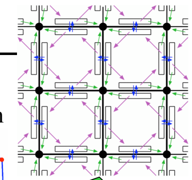
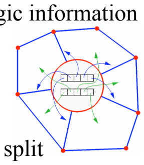
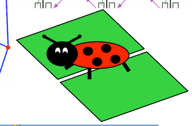
Curves & Surfaces

Last Time?

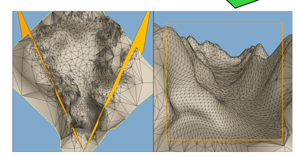


- Adjacency Data Structures
  - Geometric & topologic information
  - Dynamic allocation
  - Efficiency of access



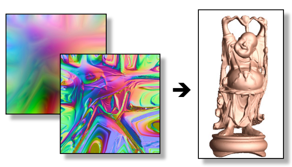


- Mesh Simplification
  - edge collapse/vertex split
  - geomorphs
  - progressive transmission
  - view-dependent refinement

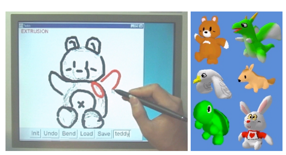


Readings for Today *(pick one)*

- "Geometry Images", Gu, Gortler, & Hoppe, SIGGRAPH 2002



- "Teddy: A Sketching Interface for 3D Freeform Design", Igarashi et al., SIGGRAPH 1999



Today

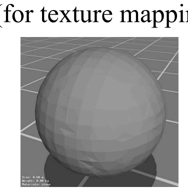
- Limitations of Polygonal Models
  - Interpolating Color & Normals in OpenGL
  - Some Modeling Tools & Definitions
- What's a Spline?
  - Interpolation Curves vs. Approximation Curves
  - Linear Interpolation
- Bézier Spline
- BSpline (NURBS)
- Extending to Surfaces – Tensor Product

Today

- Limitations of Polygonal Models
  - Interpolating Color & Normals in OpenGL
  - Some Modeling Tools & Definitions
- What's a Spline?
  - Interpolation Curves vs. Approximation Curves
  - Linear Interpolation
- Bézier Spline
- BSpline (NURBS)
- Extending to Surfaces – Tensor Product

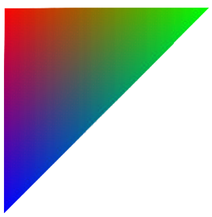
Limitations of Polygonal Meshes

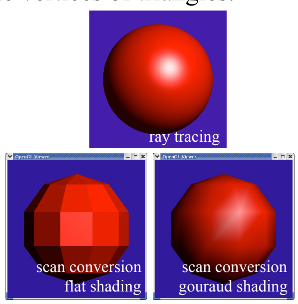
- Planar facets (& silhouettes)
- Fixed resolution
- Deformation is difficult
- No natural parameterization (for texture mapping)



Color & Normal Interpolation

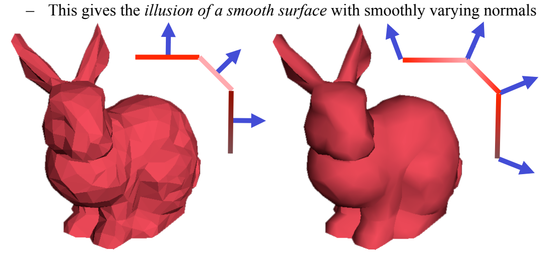
- It's easy in OpenGL to specify different colors and/or normals at the vertices of triangles:
- Why is this useful?





What is Gouraud Shading?

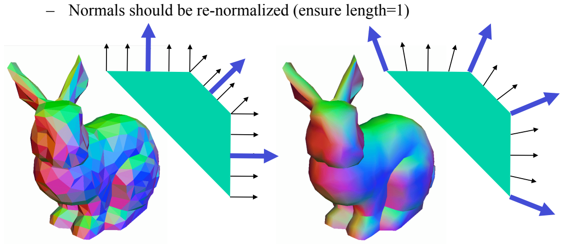
- Instead of shading with the normal of the triangle, we'll shade the vertices with the *average normal* and *interpolate the shaded color* across each face
  - This gives the *illusion of a smooth surface* with smoothly varying normals



- How do we compute Average Normals? Is it expensive??

Phong Normal Interpolation (Not Phong *Shading*)

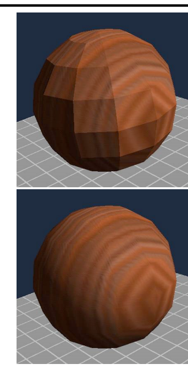
- *Interpolate the average vertex normals* across the face and compute *per-pixel shading*
  - Normals should be re-normalized (ensure length=1)



- Before shaders, per-pixel shading was not possible in hardware (Gouraud shading is actually a decent substitute!)

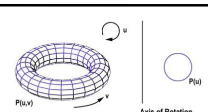
Gouraud not always good enough

- Still low, fixed resolution (missing fine details)
- Still have polygonal silhouettes
- Intersection depth is planar (e.g. ray tracing visualization)
- Collisions problems for simulation
- Solid Texturing problems
- ...

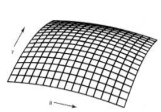


Some Non-Polygonal Modeling Tools

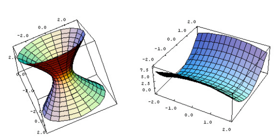
Extrusion



Surface of Revolution

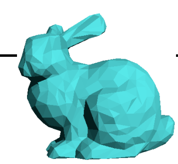


Spline Surfaces/Patches



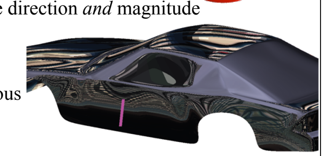
Quadrics and other implicit polynomials

Continuity definitions:



- C^0 continuous
  - curve/surface has no breaks/gaps/holes
- G^1 continuous
  - tangent at joint has same direction

- C^1 continuous
  - curve/surface derivative is continuous
  - tangent at joint has same direction *and* magnitude
- C^n continuous
  - curve/surface through n^{th} derivative is continuous
  - important for shading



"Shape Optimization Using Reflection Lines", Tosun et al., 2007

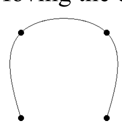
Questions?

Today

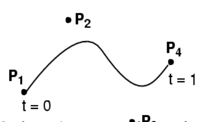
- Limitations of Polygonal Models
  - Interpolating Color & Normals in OpenGL
  - Some Modeling Tools & Definitions
- What's a Spline?
  - Interpolation Curves vs. Approximation Curves
  - Linear Interpolation
- Bézier Spline
- BSpline (NURBS)
- Extending to Surfaces – Tensor Product

Definition: What's a Spline?

- Smooth curve defined by some control points
- Moving the control points changes the curve

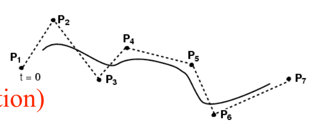


Interpolation

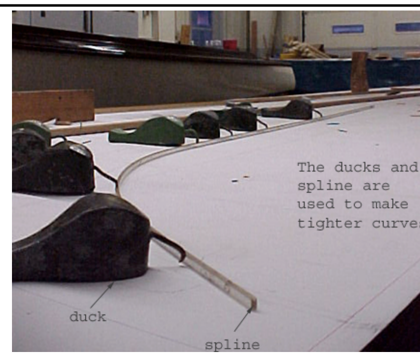


Bézier (approximation)

BSpline (approximation)

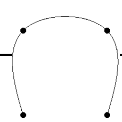


Interpolation Curves / Splines



www.abm.org

Interpolation Curves

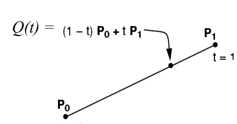


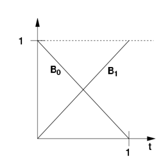
- Curve is constrained to pass through all control points
- Given points $P_0, P_1, \ldots P_n$, find lowest degree polynomial which passes through the points

$$x(t) = a_{n-1}t^{n-1} + \ldots + a_2t^2 + a_1t + a_0$$
$$y(t) = b_{n-1}t^{n-1} + \ldots + b_2t^2 + b_1t + b_0$$

Linear Interpolation

- Simplest "curve" between two points





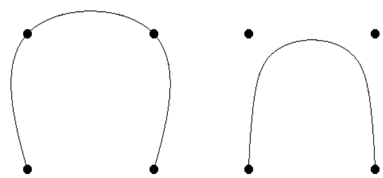
Spline Basis Functions

a.k.a. Blending Functions

$$Q(t) = \begin{pmatrix} Q_x(t) \\ Q_y(t) \\ Q_z(t) \end{pmatrix} = \begin{pmatrix} (P_0) & (P_1) \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} t \\ 1 \end{pmatrix}$$

$$Q(t) = \mathbf{GBT(t)} = \text{Geometry } \mathbf{G} \cdot \text{Spline Basis } \mathbf{B} \cdot \text{Power Basis } \mathbf{T(t)}$$

Interpolation vs. Approximation Curves



Interpolation

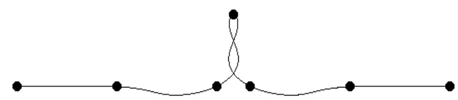
curve must pass through control points

Approximation

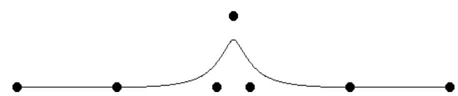
curve is influenced by control points

Interpolation vs. Approximation Curves

- Interpolation Curve – over constrained → lots of (undesirable?) oscillations



- Approximation Curve – more reasonable?



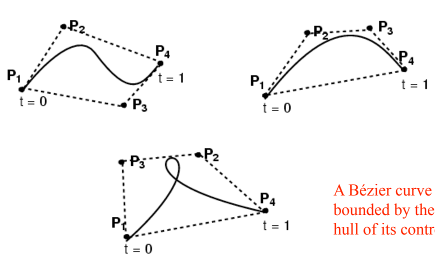
Questions?

Today

- Limitations of Polygonal Models
  - Interpolating Color & Normals in OpenGL
  - Some Modeling Tools & Definitions
- What's a Spline?
  - Interpolation Curves vs. Approximation Curves
  - Linear Interpolation
- Bézier Spline
- BSpline (NURBS)
- Extending to Surfaces – Tensor Product

Cubic Bézier Curve

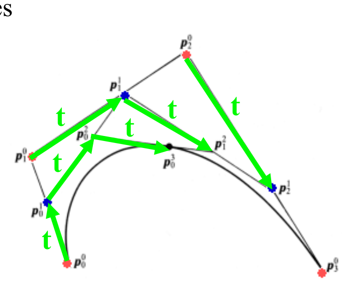
- 4 control points
- Curve passes through first & last control point
- Curve is tangent at $\mathbf{P}_1$ to $(\mathbf{P}_2\text{-}\mathbf{P}_1)$ and at $\mathbf{P}_4$ to $(\mathbf{P}_4\text{-}\mathbf{P}_3)$



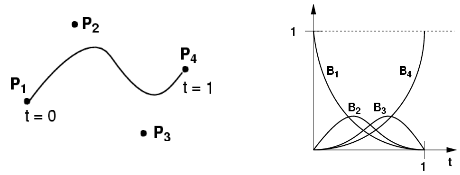
A Bézier curve is bounded by the convex hull of its control points.

Cubic Bézier Curve

- de Casteljau's algorithm for constructing Bézier curves



Cubic Bézier Curve



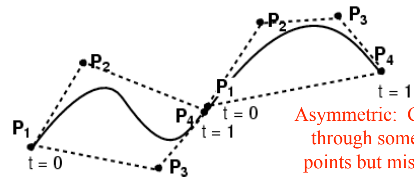
$$Q(t) = (1-t)^3 P_1 + 3t(1-t)^2 P_2 + 3t^2(1-t) P_3 + t^3 P_4$$

$$Q(t) = \mathbf{GBT(t)} \qquad B_{Bezier} = \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Bernstein Polynomials

$$B_1(t) = (1-t)^3;\ B_2(t) = 3t(1-t)^2;\ B_3(t) = 3t^2(1-t);\ B_4(t) = t^3$$

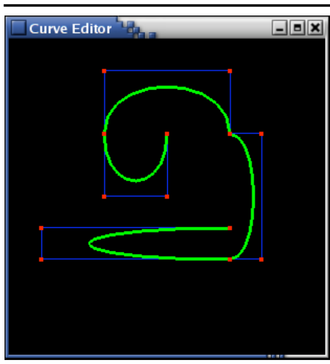
Connecting Cubic Bézier Curves



Asymmetric: Curve goes through some control points but misses others

- How can we guarantee C^0 continuity?
- How can we guarantee G^1 continuity?
- How can we guarantee C^1 continuity?
- Can't guarantee higher C^2 or higher continuity

Connecting Cubic Bézier Curves



- Where is this curve
  - C^0 continuous?
  - G^1 continuous?
  - C^1 continuous?
- What's the relationship between:
  - the # of control points, and
  - the # of cubic Bézier subcurves?

Higher-Order Bézier Curves

- > 4 control points
- Bernstein Polynomials as the basis functions

$$B_i^n(t) = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i}, \qquad 0 \le i \le n$$

- Every control point affects the entire curve
  - Not simply a local effect
  - More difficult to control for modeling

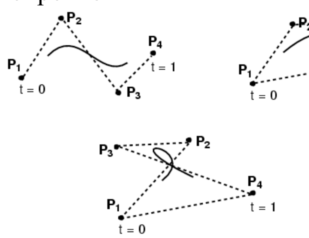
Questions?

Today

- Limitations of Polygonal Models
  - Interpolating Color & Normals in OpenGL
  - Some Modeling Tools & Definitions
- What's a Spline?
  - Interpolation Curves vs. Approximation Curves
  - Linear Interpolation
- Bézier Spline
- BSpline (NURBS)
- Extending to Surfaces – Tensor Product

Cubic BSplines

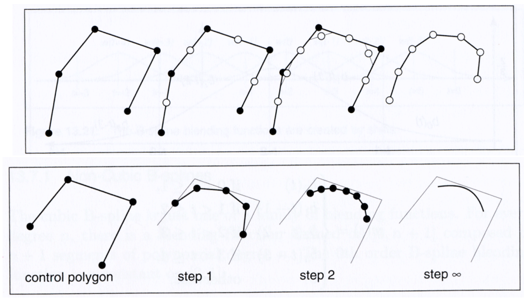
- $\geq$ 4 control points
- Locally cubic
- Curve is not constrained to pass through any control points



A BSpline curve is also bounded by the convex hull of its control points.

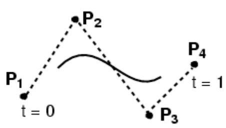
Cubic BSplines

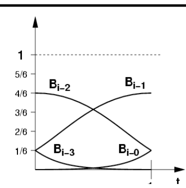
- Iterative method for constructing BSplines



Shirley, Fundamentals of Computer Graphics

Cubic BSplines



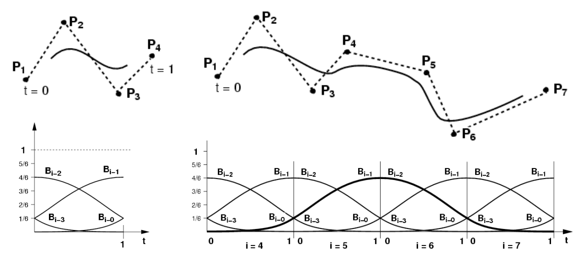


$$Q(t) = \frac{(1-t)^3}{6} P_{i-3} + \frac{3t^3 - 6t^2 + 4}{6} P_{i-2} + \frac{-3t^3 + 3t^2 + 3t + 1}{6} P_{i-1} + \frac{t^3}{6} P_i$$

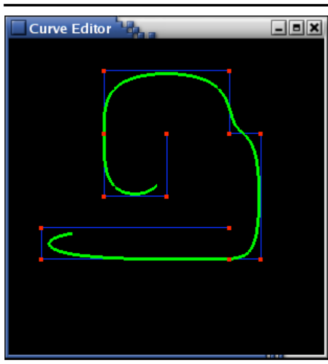
$$Q(t) = \mathbf{GBT(t)} \qquad B_{B-Spline} = \frac{1}{6}\begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 0 & 4 \\ -3 & 3 & 3 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Connecting Cubic BSpline Curves

- Can be chained together
- Better control locally (windowing)

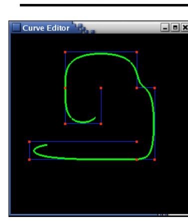


Connecting Cubic BSpline Curves

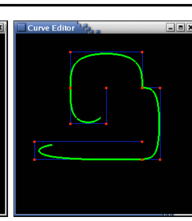


- What's the relationship between
  - the # of control points, and
  - the # of cubic BSpline subcurves?

BSpline Curve Control Points

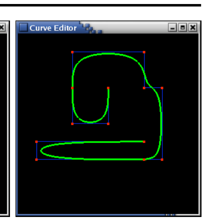


Default BSpline



BSpline with Discontinuity

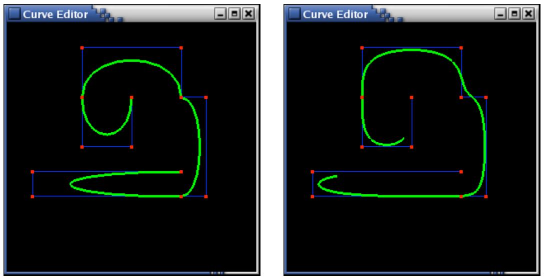
Repeat interior control point



BSpline which passes through end points

Repeat end points

Bézier is not the same as BSpline

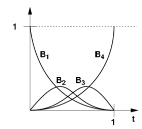


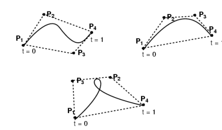
Bézier BSpline

Bézier is not the same as BSpline

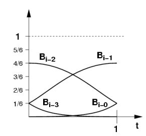
- Relationship to the control points is different

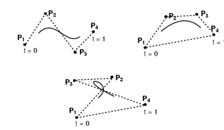
Bézier





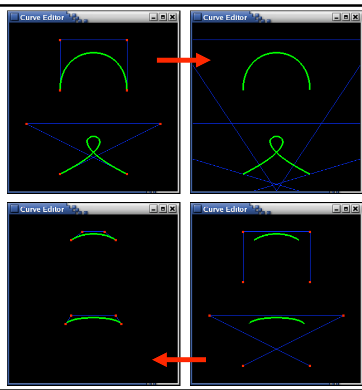
BSpline





Converting between Bézier & BSpline

original control points as Bézier



new BSpline control points to match Bézier

new Bézier control points to match BSpline

original control points as BSpline

Converting between Bézier & BSpline

- Using the basis functions:

$$B_{Bezier} = \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$B_{B-Spline} = \frac{1}{6}\begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 0 & 4 \\ -3 & 3 & 3 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

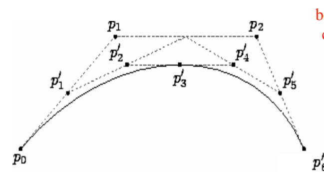
$$Q(t) = \mathbf{GBT(t)} = \text{Geometry } \mathbf{G} \cdot \text{Spline Basis } \mathbf{B} \cdot \text{Power Basis } \mathbf{T(t)}$$

NURBS (generalized BSplines)

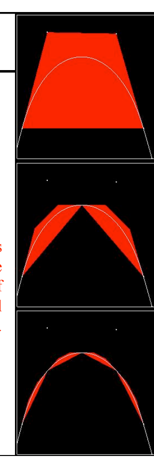
- BSpline: uniform cubic BSpline
- NURBS: Non-Uniform Rational BSpline
  - non-uniform = different spacing between the blending functions, a.k.a. knots
  - rational = ratio of polynomials (instead of cubic)

Neat Bezier Spline Trick

- A Bezier curve with 4 control points:
  - P_0 P_1 P_2 P_3
- Can be split into 2 new Bezier curves:
  - P_0 P'_1 P'_2 P'_3
  - P'_3 P'_4 P'_5 P_3



A Bézier curve is bounded by the convex hull of its control points.



Today

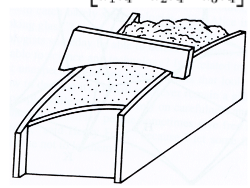
- Limitations of Polygonal Models
  - Interpolating Color & Normals in OpenGL
  - Some Modeling Tools & Definitions
- What's a Spline?
  - Interpolation Curves vs. Approximation Curves
  - Linear Interpolation
- Bézier Spline
- BSpline (NURBS)
- Extending to Surfaces – Tensor Product

Spline Surface via Tensor Product

- Of two vectors:

$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \otimes \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \end{bmatrix} = \begin{bmatrix} a_1b_1 & a_2b_1 & a_3b_1 \\ a_1b_2 & a_2b_2 & a_3b_2 \\ a_1b_3 & a_2b_3 & a_3b_3 \\ a_1b_4 & a_2b_4 & a_3b_4 \end{bmatrix}$$

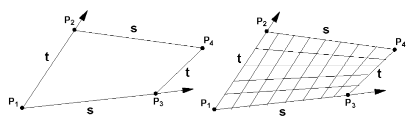
- Similarly, we can define a surface as the tensor product of two curves....



Farin, Curves and Surfaces for Computer Aided Geometric Design

Bilinear Patch

Bi-lerp a (typically non-planar) quadrilateral

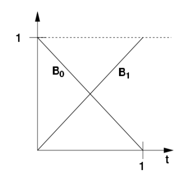


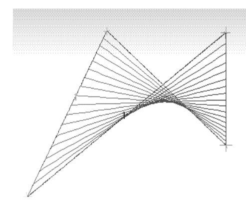
Notation: $\mathbf{L}(P_1, P_2, \alpha) \equiv (1-\alpha)P_1 + \alpha P_2$

$$Q(s,t) = \mathbf{L}(\mathbf{L}(P_1, P_2, t), L(P_3, P_4, t), s)$$

Bilinear Patch

- Smooth version of quadrilateral with non-planar vertices...



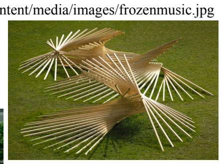


- But will this help us model smooth surfaces?
- Do we have control of the derivative at the edges?

Ruled Surfaces in Art & Architecture

http://www.bergenwood.no/wp-content/media/images/frozenmusic.jpg

Chiras Iulia
Astri Isabella
Matiss Shteinerts

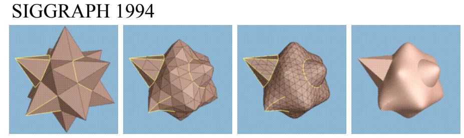


Antoni Gaudi
Children's School
Barcelona

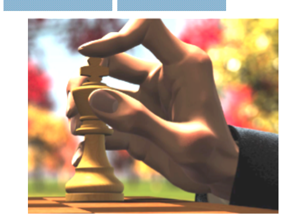
http://www.lonelyplanetimages.com/images/399954

Readings for Friday *(pick one)*

- Hoppe et al., "Piecewise Smooth Surface Reconstruction" SIGGRAPH 1994



- DeRose, Kass, & Truong, "Subdivision Surfaces in Character Animation", SIGGRAPH 1998



Post a comment or question on the LMS discussion by 10am on Tuesday

Homework 1:

- Questions/Comments?