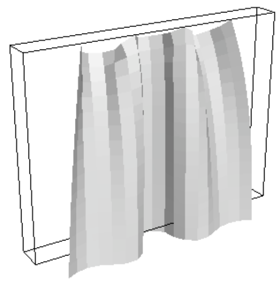
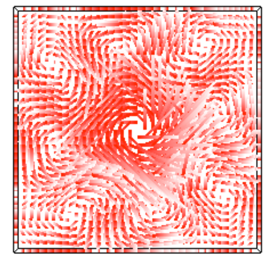
Rigid Body Dynamics, Collision Response, & Deformation

HW2: Cloth & Fluid Simulation

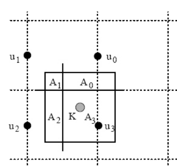


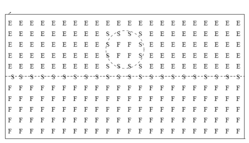


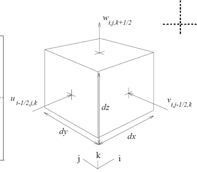
Last Time?

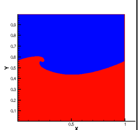
$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = -\frac{\partial p}{\partial x} + g_x + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$
$$\frac{\partial v}{\partial t} + \frac{\partial vu}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} = -\frac{\partial p}{\partial y} + g_y + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$
$$\frac{\partial w}{\partial t} + \frac{\partial wu}{\partial x} + \frac{\partial wv}{\partial y} + \frac{\partial w^2}{\partial z} = -\frac{\partial p}{\partial z} + g_z + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

- Navier-Stokes Equations
- Conservation of Momentum & Mass
- Incompressible Flow







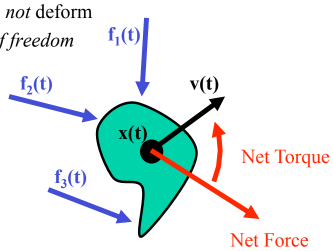


Today

- Rigid Body Dynamics
- Collision Response
- Non-Rigid Objects
- Finite Element Method
- Deformation
- Level-of-Detail

Rigid Body Dynamics

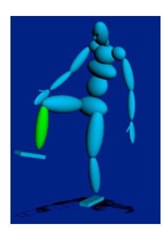
- How do we simulate this object's motion over time?
- We could discretize the object into many particles…
  - But a rigid body does *not* deform
  - Only a few *degrees of freedom*
- Instead, we use only one particle at the center of mass
- Compute net force & net torque

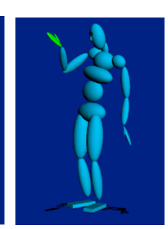


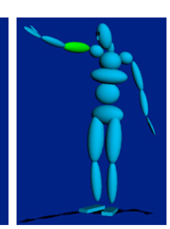
Nice Reference Material: http://www.pixar.com/companyinfo/research/pbm2001/

Degree of Freedom (DOF)

- Rotations:


1 DOF: knee

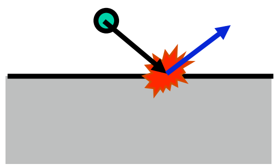

2 DOF: wrist


3 DOF: arm

- Translations count too… ➔ 6 DOF

Collisions

- Detection
- Response
- Overshooting problem
  (when we enter the solid)

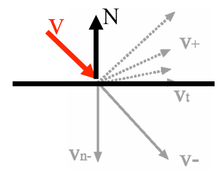


Collision Response

- tangential velocity v_t unchanged
- normal velocity v_n reflects:

$$v = v_t + v_n$$

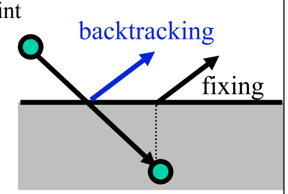
$$v \leftarrow v_t - \varepsilon v_n$$



- coefficient of restitution
  – 1 for elastic
  – 0 for plastic
- change of velocity = *-(1+ϵ)v*
- change of momentum *Impulse* = *-m(1+ϵ)v*

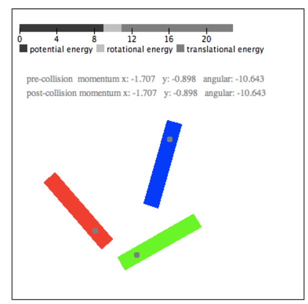
Collisions - Overshooting

- Usually, we detect collision when it's too late: we're already inside
- Solutions: back up
  – Compute intersection point
  – Compute response there
  – Advance for remaining fractional time step
- Other solution: Quick and dirty fixup
  – Just project back to object closest point



Energy & Rigid Body Collisions

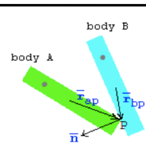
- Total Energy = Kinetic Energy + Potential Energy + Rotational Energy
- Total Energy stays constant if there is no damping and no friction
- Rotational Energy is constant between collisions



http://www.myphysicslab.com/collision.html

Collision Between Two Objects

- Suppose a vertex on body A is colliding into an edge of body B at point P. Define the following variables:



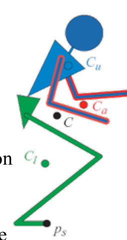
vectors involved in collision

m_a, m_b = mass of bodies A, B
$\bar{\mathbf{r}}_{ap}$ = distance vector from center of mass of body A to point P
$\bar{\mathbf{r}}_{bp}$ = distance vector from center of mass of body B to point P
ω_{a1}, ω_{b1} = initial pre-collision angular velocity of bodies A, B
ω_{a2}, ω_{b2} = final post-collision angular velocity of bodies A, B
$\bar{\mathbf{v}}_{a1}, \bar{\mathbf{v}}_{b1}$ = initial pre-collision velocities of center of mass bodies A, B
$\bar{\mathbf{v}}_{a2}, \bar{\mathbf{v}}_{b2}$ = final post-collision velocities of center of mass bodies A, B
$\bar{\mathbf{v}}_{ap1}$ = initial pre-collision velocity of impact point on body A
$\bar{\mathbf{v}}_{bp1}$ = initial pre-collision velocity of impact point on body B
$\bar{\mathbf{n}}$ = normal (perpendicular) vector to edge of body B
e = elasticity (0 = inelastic, 1 = perfectly elastic)

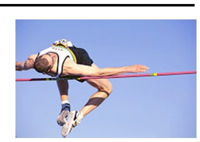
http://www.myphysicslab.com/collision.html

Center of Mass & Moment of Inertia

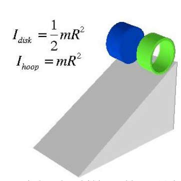
- Center of Mass: mean location of all mass in the system
- Moment of Inertia: a measure of an object's resistance to changes to its rotation
- If a solid cylinder & a hollow tube have the same radius & the same mass, which will reach the bottom of the ramp first?



Liu & Popović



http://en.wikipedia.org/wiki/Fosbury_Flop



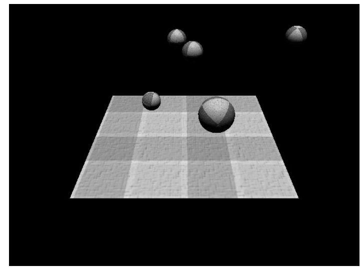
http://solomon.physics.sc.edu/~tedeschi/demo/demo12.html
http://hyperphysics.phy-astr.gsu.edu/hbase/hoocyl2.html

Advanced Collisions

- What about Friction?
- What if the contact between two objects is not a single point?
- What if more than two objects collide simultaneously?

Rigid Body Dynamics

- Physics
  - Velocity
  - Acceleration
  - Angular Momentum
- Collisions
- Friction

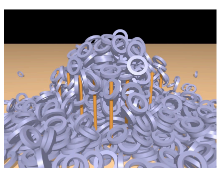


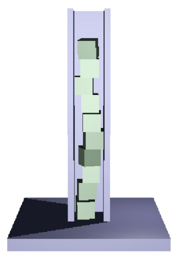
from: Darren Lewis
http://www-cs-students.stanford.edu/~dalewis/cs448a/rigidbody.html

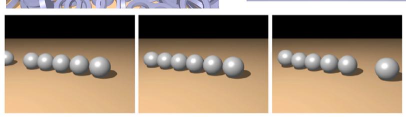
Collisions

Victor J. Milenkovic & Harald Schmidl
Optimization-Based Animation
SIGGRAPH 2001

- We know how to simulate bouncing really well
- But resting collisions are harder to manage







Guendelman, Bridson & Fedkiw
Nonconvex Rigid Bodies with Stacking
SIGGRAPH 2003

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- Finite Element Method
- Deformation
- Level-of-Detail

Simulation of Non-Rigid Objects

- We modeled string & cloth using mass-spring systems. Can we do the same?
- Yes…
- But a more physically accurate model uses *volumetric elements:*

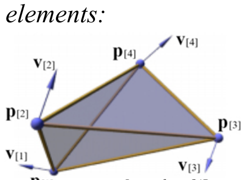
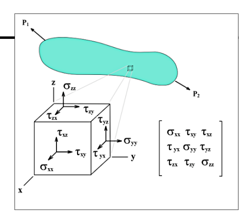


Image from O'Brien et al. 1999

Strain & Stress

- Stress
  - the internal distribution of forces within a body that balance and react to the loads applied to it
  - *normal stress & shear stress*



http://en.wikipedia.org/wiki/Image:Stress_tensor.png

- Strain
  - material deformation caused by stress.
  - measured by the change in length of a line or by the change in angle between two lines

$$\varepsilon = \frac{\Delta_l}{l_0}$$

Finite Element Method

- To solve the continuous problem (deformation of all points of the object)
  - Discretize the problem
  - Express the interrelationship
  - Solve a big linear system
- More principled than Mass-Spring

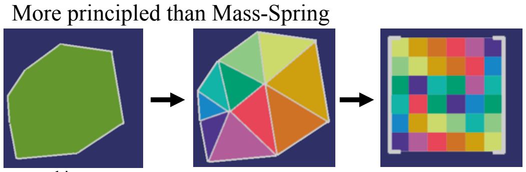


Diagram from Debunne et al. 2001

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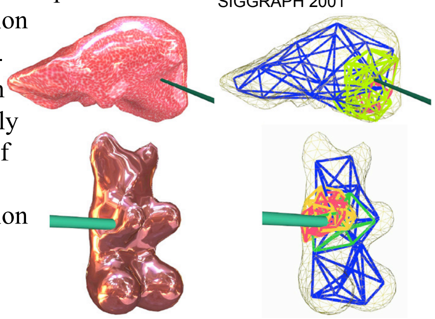
Level of Detail

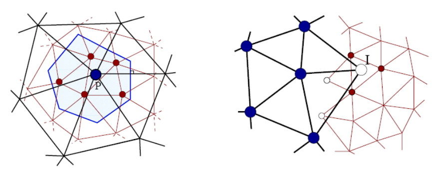
Gilles Debunne , Mathieu Desbrun, Marie-Paule Cani, & Alan H. Barr
Dynamic Real-Time Deformations using Space & Time Adaptive Sampling
SIGGRAPH 2001

- Interactive shape deformation
- Use high-resolution model only in areas of extreme deformation



Multi-Resolution Deformation

- Use Voronoi diagrams to match parent & child vertices.
- Interpolate values for inactive interface vertices from active parent/child vertices

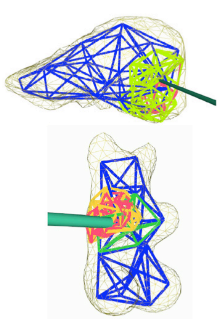


- *Need to avoid interference of vibrations between simulations at different resolutions*

Debunne et al. “Dynamic Real-Time Deformations using Space & Time Adaptive Sampling”, 2001

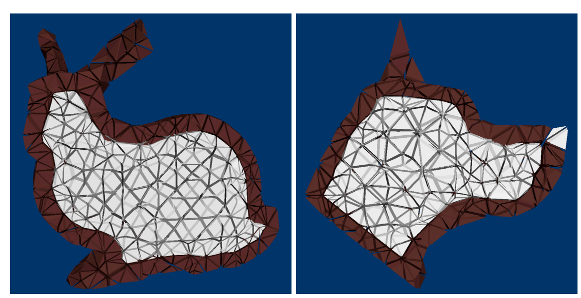
Pre-computation & Simulation

- FEM matrix pre-computed
- Level of detail coupling pre-computed for rest topology
- What to do if connectivity of elements changes?
  - Cloth is cut or torn
  - Surgery simulation



Multiple Materials

Mueller, Dorsey, McMillan, Jagnow, & Cutler
Stable Real-Time Deformations
Symposium on Computer Animation 2002



Tree Stump

Images from Cutler et al. 2002

Image from Cutler et al. 2002

Image from Cutler et al. 2002

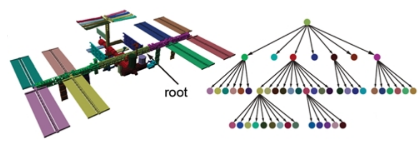
Reading for Today:

"Melting and Flowing"
Carlson, Mucha, Van Horn III & Turk
Symposium on Computer Animation 2002

- Object is always a fluid, with time-varying viscosity
- Creating a stable solver, allowing large steps in simulation, for high viscosity fluids
- Modeling temperature-varying viscosity
- 2 resolution (x64) simulation

Reading for Friday:

"Real-time Large-deformation Substructuring", Barbic & Zhao, SIGGRAPH 2011



- Mesh for simulation ≠ mesh for rendering
- Mesh hierarchy decomposition
  – not automatic
  – instanced vs duplicated/unique geometry
- Limitations
  – cycles disallowed (“close the loop” problem is common challenge)
  – boundary interfaces assumed to be (nearly) rigid
  – Vibration, lumped mass, & inertia

How to read a research paper?

(especially an advanced paper in a new area)

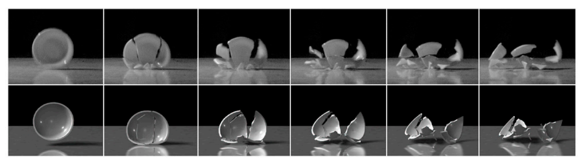
- Multiple readings are often necessary
- Don't necessarily read from front to back
- Lookup important terms
- Target application & claimed contributions
- Experimental procedure
- How well results & examples support the claims
- Scalability of the technique (order notation)
- Limitations of technique, places for future research
- Possibilities for hybrid systems with other work

Components of a well-written research paper?

- Motivation/context/related work
- Contributions of this work
- Clear description of algorithm
  – Sufficiently-detailed to allow work to be reproduced
  – Work is theoretically sound (hacks/arbitrary constants discouraged)
- Results
  – well chosen examples
  – clear tables/illustrations/visualizations
- Conclusions
  – limitations of the method are clearly stated

Reading for Tuesday: *(pick one)*

- James O’Brien & Jessica Hodgins *“Graphical Modeling and Animation of Brittle Fracture”* SIGGRAPH 1999.



- Fracture threshhold
- Remeshing
  – need connectivity info!
- Material properties
- Parameter tuning

Reading for Tuesday: *(pick one)*

Guendelman, Bridson, & Fedkiw *“Nonconvex Rigid Bodies with Stacking”* SIGGRAPH 2003