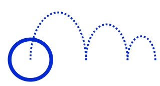
Ray Tracing

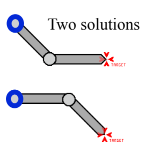
Announcements: Quiz

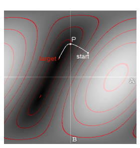
- On Friday (3/2), in class
- One 8.5x11 sheet of notes allowed
- Sample quiz (from last year) is posted online
- Focus on “reading comprehension” and material for Homeworks 0, 1, & 2

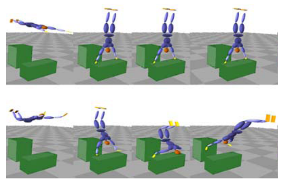
Last Time?

- Keyframing
- Procedural Animation
- Physically-Based Animation
- Forward and Inverse Kinematics
- Motion Capture







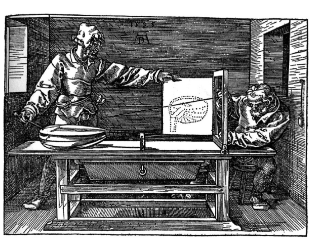


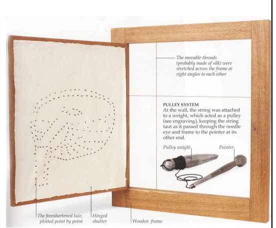
Today

- Ray Casting
  - Ray-Plane Intersection
  - Ray-Sphere Intersection
  - Point in Polygon
- Ray Tracing
- Recursive Ray Tracing
- Distribution Ray Tracing

Durer’s Ray Casting Machine

- Albrecht Durer, 16th century

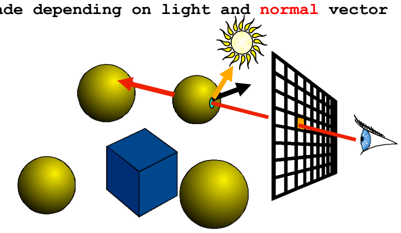




Ray Casting

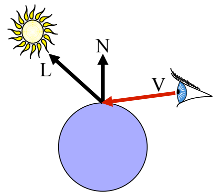
```
For every pixel
  Construct a ray from the eye
  For every object in the scene
    Find intersection with the ray
    Keep if closest
  Shade depending on light and normal vector
```

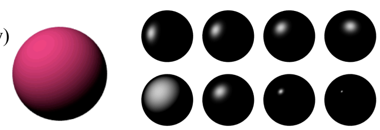
Finding the intersection and normal is the central part of ray casting



A Note on *Local* Shading

- Surface/Scene Characteristics:
  - surface normal
  - direction to light
  - viewpoint
- Material Properties
  - color/texture
  - diffuse (matte)
  - specular (shiny)
  - …
- More later!

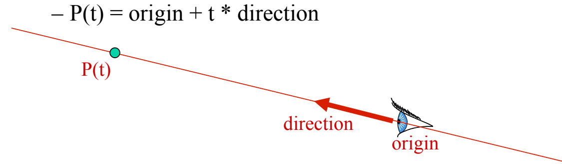




Diffuse sphere *Specular spheres*

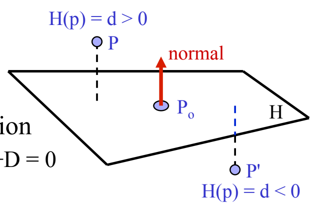
Ray Representation?

- Two vectors:
  - Origin
  - Direction (normalized is better)
- Parametric line (*explicit* representation)
  - P(t) = origin + t * direction



3D Plane Representation?

- Plane defined by
  - $P_o = (x,y,z)$
  - n = (A,B,C)
- *Implicit* plane equation
  - H(P) = Ax+By+Cz+D = 0
    = n·P + D = 0
- Point-Plane distance?
  - If n is normalized, distance to plane, d = H(P)
  - d is the *signed distance*!



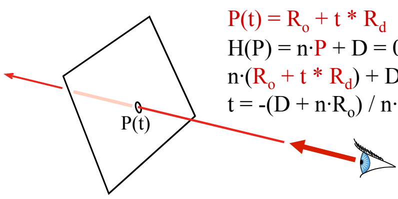
Explicit vs. Implicit?

- Ray equation is explicit $P(t) = R_o + t * R_d$
  - Parametric
  - Generates points
  - Harder to verify that a point is on the ray
- Plane equation is implicit $H(P) = n \cdot P + D = 0$
  - Solution of an equation
  - Does not generate points
  - Verifies that a point is on the plane

Ray-Plane Intersection

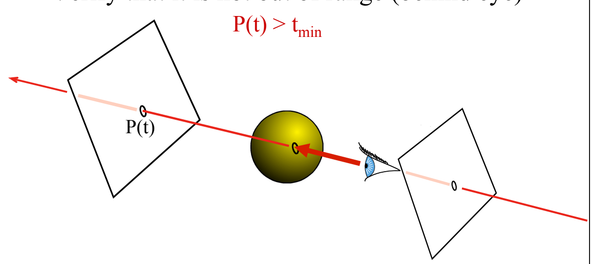
- Intersection means both are satisfied
- So, insert explicit equation of ray into implicit equation of plane & solve for t

$P(t) = R_o + t * R_d$
$H(P) = n \cdot P + D = 0$
$n \cdot (R_o + t * R_d) + D = 0$
$t = -(D + n \cdot R_o) / n \cdot R_d$



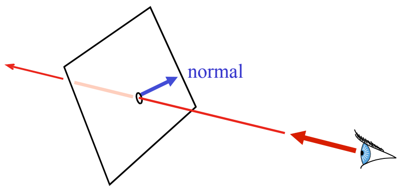
Additional Housekeeping

- Verify that intersection is closer than previous
  $P(t) < t_{current}$
- Verify that it is not out of range (behind eye)
  $P(t) > t_{min}$



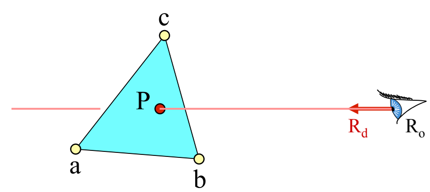
Normal

- Needed for shading
  - diffuse: dot product between light and normal
- Normal of a plane is constant!



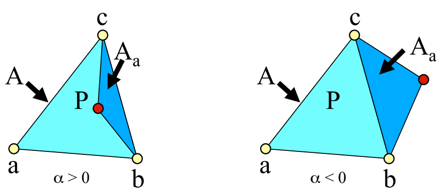
Ray-Triangle Intersection

- Use barycentric coordinates:
  - $P(\alpha, \beta, \gamma) = \alpha a + \beta b + \gamma c$
    with $\alpha + \beta + \gamma = 1$
  - If $0 < \alpha < 1$ & $0 < \beta < 1$ & $0 < \gamma < 1$
    then the point is inside the triangle!



How Do We Compute α, β, γ ?

- Ratio of opposite sub-triangle area to total area
  - $\alpha = A_a/A$ $\beta = A_b/A$ $\gamma = A_c/A$
- Use signed areas for points outside the triangle



But how do I know if the point is outside the triangle?
That's what I was trying to determine!

Using Cramer's Rule…

- Used to solve for one variable at a time in system of equations

$$\beta = \frac{\begin{vmatrix} a_x - R_{ox} & a_x - c_x & R_{dx} \\ a_y - R_{oy} & a_y - c_y & R_{dy} \\ a_z - R_{oz} & a_z - c_z & R_{dz} \end{vmatrix}}{|A|} \qquad \gamma = \frac{\begin{vmatrix} a_x - b_x & a_x - R_{ox} & R_{dx} \\ a_y - b_y & a_y - R_{oy} & R_{dy} \\ a_z - b_z & a_z - R_{oz} & R_{dz} \end{vmatrix}}{|A|}$$

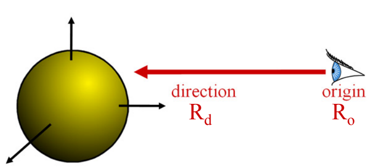
$$t = \frac{\begin{vmatrix} a_x - b_x & a_x - c_x & a_x - R_{ox} \\ a_y - b_y & a_y - c_y & a_y - R_{oy} \\ a_z - b_z & a_z - c_z & a_z - R_{oz} \end{vmatrix}}{|A|}$$

| | denotes the determinant

Can be copied mechanically into code

Sphere Representation?

- Implicit sphere equation
  - Assume centered at origin (easy to translate)
  - $H(P) = P \cdot P - r^2 = 0$



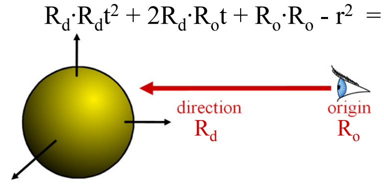
Ray-Sphere Intersection

- Insert explicit equation of ray into implicit equation of sphere & solve for t

$$P(t) = R_o + t{*}R_d \qquad H(P) = P \cdot P - r^2 = 0$$

$$(R_o + tR_d) \cdot (R_o + tR_d) - r^2 = 0$$

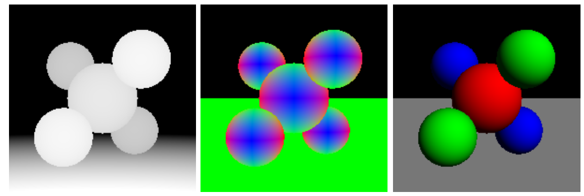
$$R_d \cdot R_d t^2 + 2R_d \cdot R_o t + R_o \cdot R_o - r^2 = 0$$



Ray-Sphere Intersection

- Quadratic: $at^2 + bt + c = 0$
  - $a = 1$ (remember, $||R_d|| = 1$)
  - $b = 2R_d \cdot R_o$
  - $c = R_o \cdot R_o - r^2$
- with discriminant $d = \sqrt{b^2 - 4ac}$
- and solutions $t_{\pm} = \frac{-b \pm d}{2a}$
- What does it mean if there are no solutions, 1 solution, or 2 solutions?

Questions?



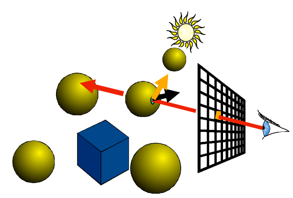
depth normals local shading

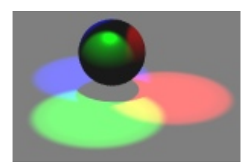
Today

- Ray Casting
- Ray Tracing
  - Shadows
  - Reflection
  - Refraction
- Recursive Ray Tracing
- Distribution Ray Tracing

How Can We Add Shadows?

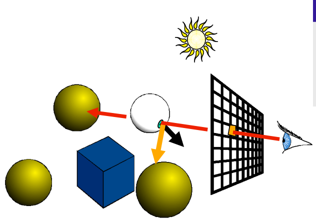
```
Find the point to be shaded
For every light,
  Construct ray from point to light
  For every object
    find intersection of ray with object
  If no objects between point and light
    Add contribution from light
```

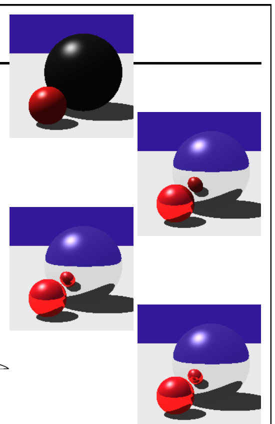




Mirror Reflection

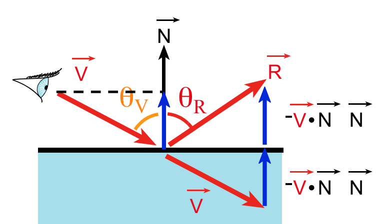
- Cast ray symmetric with respect to the normal
- Multiply by reflection coefficient (color)





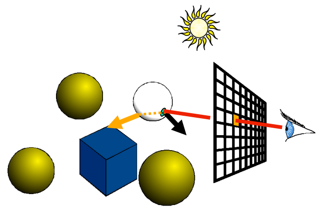
Reflection

- Reflection angle = view angle
- $\mathbf{R} = \mathbf{V} - 2\,(\mathbf{V} \cdot \mathbf{N})\,\mathbf{N}$

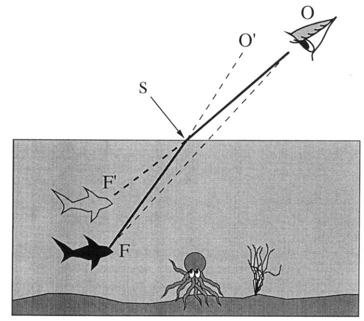


Transparency

- Cast ray in refracted direction
- Multiply by transparency coefficient (color)

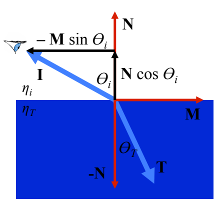


Qualitative Refraction



From “Color and Light in Nature” by Lynch and Livingston

Refraction



$\mathbf{I} = \mathbf{N} \cos \Theta_i - \mathbf{M} \sin \Theta_i$

$\mathbf{M} = (\mathbf{N} \cos \Theta_i - \mathbf{I}) / \sin \Theta_i$

$$
\begin{aligned}
\mathbf{T} &= -\mathbf{N} \cos \Theta_T + \mathbf{M} \sin \Theta_T \\
&= -\mathbf{N} \cos \Theta_T + (\mathbf{N} \cos \Theta_i - \mathbf{I}) \sin \Theta_T / \sin \Theta_i \\
&= -\mathbf{N} \cos \Theta_T + (\mathbf{N} \cos \Theta_i - \mathbf{I})\, \eta_r \\
&= [\, \eta_r \cos \Theta_i - \cos \Theta_T \,]\, \mathbf{N} - \eta_r \mathbf{I} \\
&= [\, \eta_r \cos \Theta_i - \sqrt{1 - \sin^2 \Theta_T} \,]\, \mathbf{N} - \eta_r \mathbf{I} \\
&= [\, \eta_r \cos \Theta_i - \sqrt{1 - \eta_r^2 \sin^2 \Theta_i} \,]\, \mathbf{N} - \eta_r \mathbf{I} \\
&= [\, \eta_r \cos \Theta_i - \sqrt{1 - \eta_r^2 (1 - \cos^2 \Theta_i)} \,]\, \mathbf{N} - \eta_r \mathbf{I} \\
&= [\, \eta_r (\mathbf{N} \cdot \mathbf{I}) - \sqrt{1 - \eta_r^2 (1 - (\mathbf{N} \cdot \mathbf{I})^2)} \,]\, \mathbf{N} - \eta_r \mathbf{I}
\end{aligned}
$$

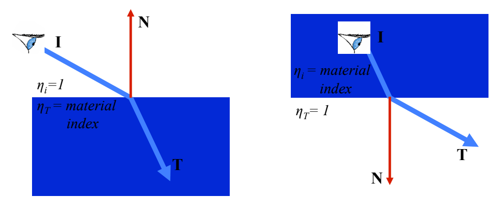
Snell-Descartes Law:
$\eta_i \sin \Theta_i = \eta_T \sin \Theta_T$

$$\frac{\sin \Theta_T}{\sin \Theta_i} = \frac{\eta_i}{\eta_T} = \eta_r$$

- **Total internal reflection when the square root is imaginary**
- **Don’t forget to normalize!**

Refraction & the Sidedness of Objects

- Make sure you know whether you’re entering or leaving the transmissive material:



- What about intersecting transparent objects?

Total Internal Reflection

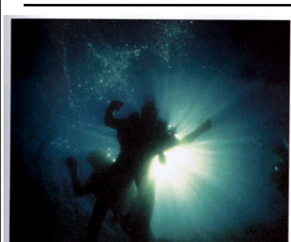


Fig. 3.7A The optical manhole. From under water, the entire celestial hemisphere is compressed into a circle only 97.2° across. The dark boundary defining the edges of the manhole is not sharp due to surface waves. The rays are analogous to the crepuscular type seen in hazy air, Section 1.9. (Photo by D. Granger)

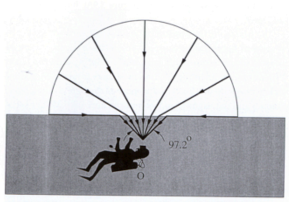


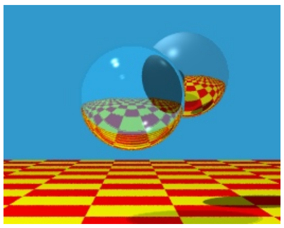
Fig. 3.7B The optical manhole. Light from the horizon (angle of incidence = 90°) is refracted downward at an angle of 48.6°. This compresses the sky into a circle with a diameter of 97.2° instead of its usual 180°.

From “Color and Light in Nature” by Lynch and Livingston

Questions?

Readings for Today: *(read one…)*

- "An improved illumination model for shaded display" Turner Whitted, 1980.
- "Distributed Ray Tracing", Cook, Porter, & Carpenter, SIGGRAPH 1984.



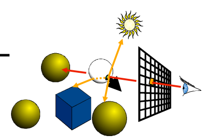
Today

- Ray Casting
- Ray Tracing
- Recursive Ray Tracing
- Distribution Ray Tracing

Ray Tracing

```
trace ray
   Intersect all objects
   color = ambient term
   For every light
      cast shadow ray
      color += local shading term
   If mirror
      color += color_refl *
               trace reflected ray
   If transparent
      color += color_trans *
               trace transmitted ray
```

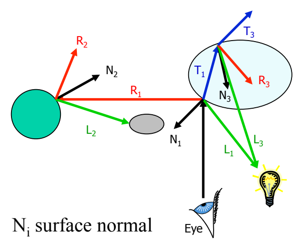
- ***Does it ever end?***



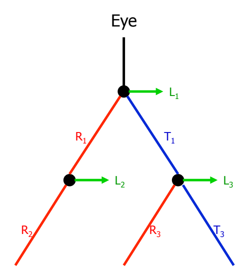
Stopping criteria:

- Recursion depth
  - Stop after a number of bounces
- Ray contribution
  - Stop if reflected / transmitted contribution becomes too small

The Ray Tree



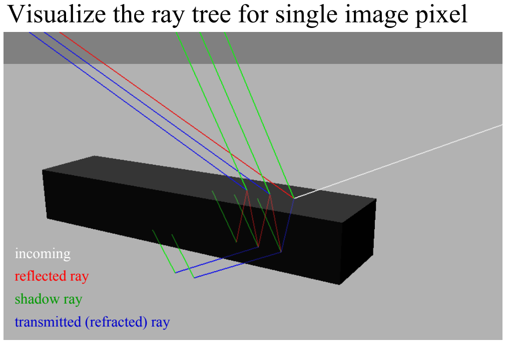
N_i surface normal
R_i reflected ray
L_i shadow ray
T_i transmitted (refracted) ray



Complexity?

Ray Debugging

- Visualize the ray tree for single image pixel

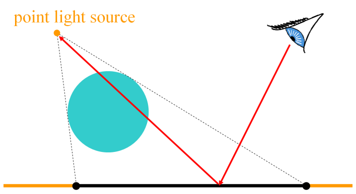


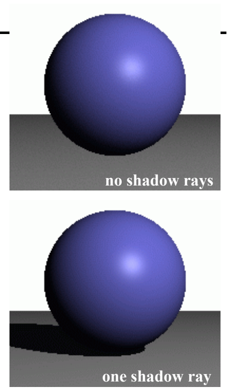
Today

- Ray Casting
- Ray Tracing
- Recursive Ray Tracing
- Distribution Ray Tracing
  - Soft shadows
  - Antialiasing (getting rid of jaggies)
  - Glossy reflection
  - Motion blur
  - Depth of field (focus)

Shadows

- one shadow ray per intersection per point light source

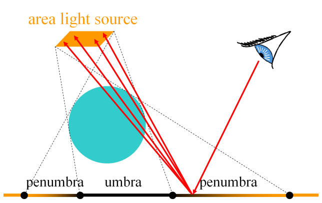


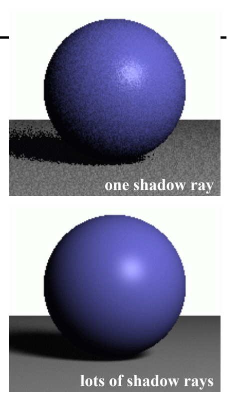


Shadows & Light Sources

Soft Shadows

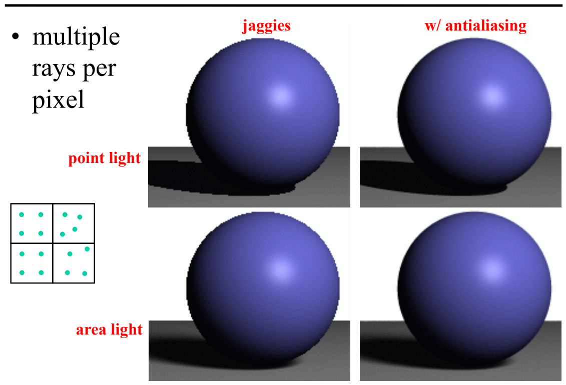
- multiple shadow rays to sample area light source





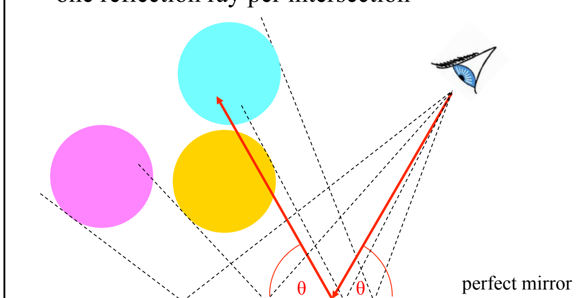
Antialiasing – Supersampling

- multiple rays per pixel



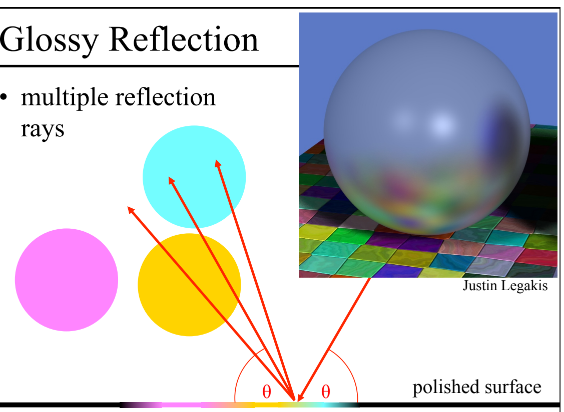
Reflection

- one reflection ray per intersection



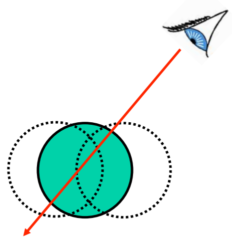
Glossy Reflection

- multiple reflection rays



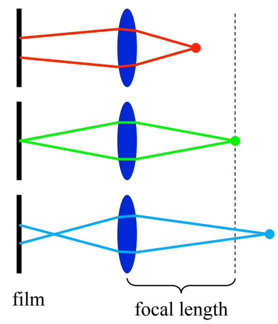
Motion Blur

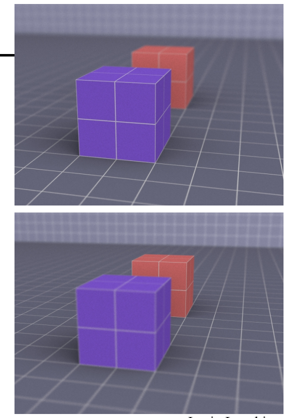
- Sample objects temporally



Depth of Field

- multiple rays per pixel





Ray Tracing Algorithm Analysis

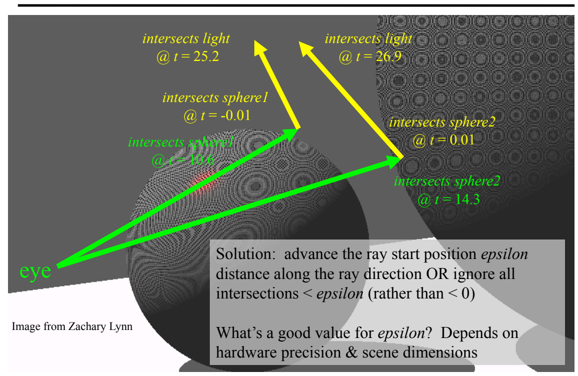
- Ray casting
- Lots of primitives
- Recursive
- Distributed Ray Tracing Effects
  - Soft shadows
  - Anti-aliasing
  - Glossy reflection
  - Motion blur
  - Depth of field

cost ≈ height * width *
num primitives *
intersection cost *
size of recursive ray tree *
num shadow rays *
num supersamples *
num glossy rays *
num temporal samples *
num focal samples *
. . .

can we reduce this?

these can serve double duty

Raytracing & Epsilon



Solution: advance the ray start position *epsilon* distance along the ray direction OR ignore all intersections < *epsilon* (rather than < 0)

What's a good value for *epsilon*? Depends on hardware precision & scene dimensions

Reading for Tuesday 3/6:

- Goral, Torrance, Greenberg & Battaile "Modeling the Interaction of Light Between Diffuse Surfaces", SIGGRAPH '84