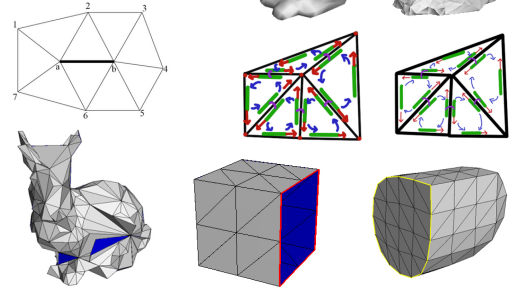# Implicit Surfaces, Collision Detection, & Volumetric Data Structures
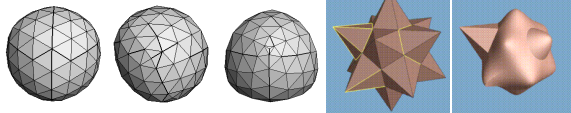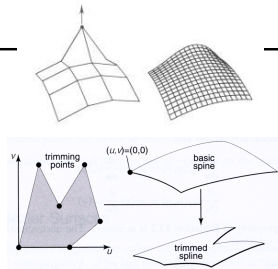

Homework 1:

- Questions/Comments?

Last Time?

- Spline Surfaces
  - complex topology is challenging, requires trimming curves
- Subdivision Zoo
  - Doo-Sabin
  - Loop
  - Catmull-Clark
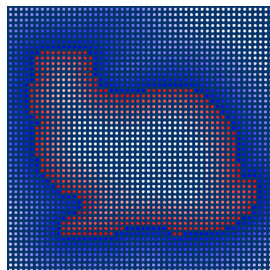- Subdivision w/ Creases

---

Today

- Implicit Surfaces, Voxels, & Marching Cubes
- Collision Detection
- Conservative Bounding Region
- Spatial Acceleration Data Structures
  - Fixed Grid
  - Nested Grid
  - Octree
  - Binary Space Partition
  - K-d tree
  - Bounding Volume Hierarchy
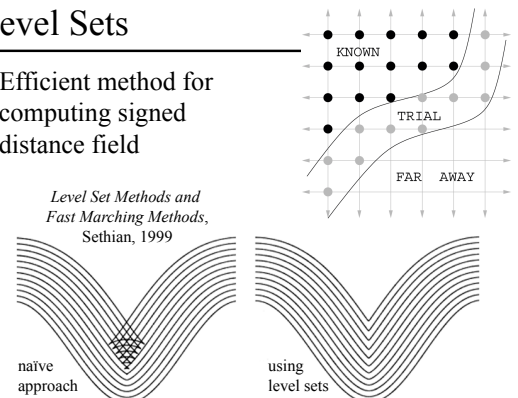- Misc Vocabulary & Advanced Papers

---

Implicit Surfaces

- For a sphere:
  $$H(x,y,z) = x^2 + y^2 + z^2 - r^2$$

- If $H(x,y,z) = 0$, on surface
- If $H(x,y,z) > 0$, outside surface
- If $H(x,y,z) < 0$, inside surface

Level Sets

- Efficient method for computing signed distance field

Level Set Methods and Fast Marching Methods, Sethian, 1999

naïve approach   using level sets

KNOWN

TRIAL

FAR AWAY

1

Marching Cubes

- Polygonization: extract triangle mesh from signed distance field

"Marching Cubes: A High Resolution 3D Surface Construction Algorithm", Lorensen and Cline, SIGGRAPH '87.

"Marching Tetrahedra"

Jules Bloomenthal
"An implicit surface polygonizer"
Graphics Gems IV

"When the Blobs Go Marching Two by Two",
Jeff Lander, Gamasutra

"Marching Tetrahedra"

Similarly, we can create volumetric models:

o < α
α ≤ o ≤ β
β < •

"Interval volume tetrahedrization"
Visualization '97
Nielson & Sung

Questions?

Today

- Implicit Surfaces, Voxels, & Marching Cubes
- Collision Detection
- Conservative Bounding Region
- Spatial Acceleration Data Structures
  - Fixed Grid
  - Nested Grid
  - Octree
  - Binary Space Partition
  - K-d tree
  - Bounding Volume Hierarchy
- Misc Vocabulary & Advanced Papers

---

Collisions

- Detection
- Response
- Overshooting problem (when we enter the solid)

Detecting Collisions

- Easy with implicit equations of surfaces
- H(x,y,z)=0 at surface
- H(x,y,z)<0 inside surface
- So just compute H and you know that you're inside if it's negative

- More complex with other surface definitions

Collision Detection for Solids

- How to detect collision between 2 polyhedra?
- Need an inside/outside test
- Test if a vertex is inside the other polyhedron
- But treat also edge-edge intersection
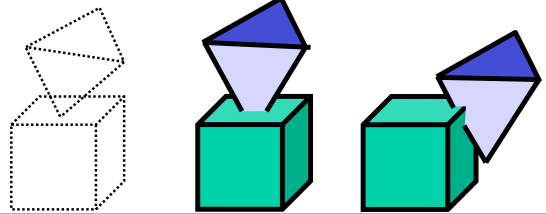
Cost of Detection?

- Test each edge with each face?
  - $O(N^2)$
- How would you detect collision between two bunnies?
 - $O(N^2)$ is too expensive!
  - Use spatial hierarchy
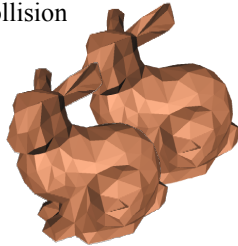
Questions?

Today

- Implicit Surfaces, Voxels, & Marching Cubes
- Collision Detection
- Conservative Bounding Region
- Spatial Acceleration Data Structures
  - Fixed Grid
  - Nested Grid
  - Octree
  - Binary Space Partition
  - K-d tree
  - Bounding Volume Hierarchy
- Misc Vocabulary & Advanced Papers
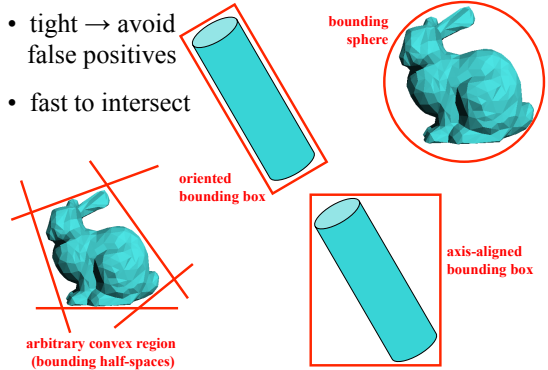
Conservative Bounding Region

- First check for an intersection with a conservative bounding region
- Early reject
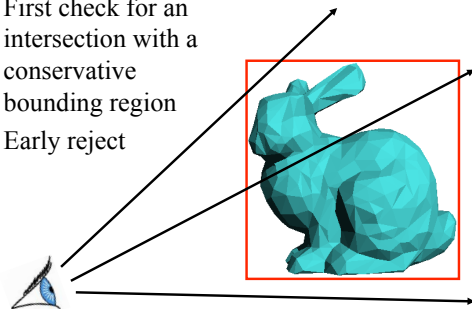
Application: Accelerate ray tracing
Intersect object & ray… *more later this semester!!*

3

Conservative Bounding Regions

- tight → avoid false positives
- fast to intersect

bounding sphere

oriented bounding box

axis-aligned bounding box

arbitrary convex region (bounding half-spaces)

Overlap test

- Overlap between two axis-aligned boxes?
  - Check if the intervals along the 3 dimensions overlap
- Overlap test between two spheres?
  - $D(center_1, center_2) < r_1 + r_2$

C_1 r_1 D r_2 C_2
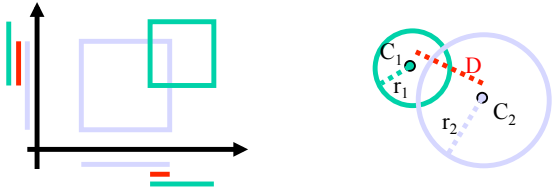
General Collision Detection

- Put a hierarchy around your objects
- Use the fast overlap test recursively
- Handle exact case at the leaves (when necessary)
- More difficult for self-collision (e.g. cloth)
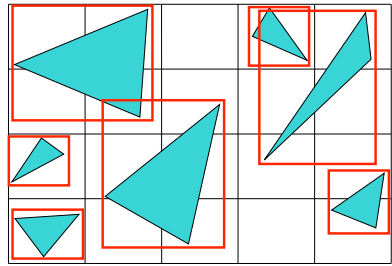 - Because there is more overlap
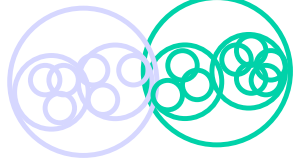
Questions?

Today

- Implicit Surfaces, Voxels, & Marching Cubes
- Collision Detection
- Conservative Bounding Region
- Spatial Acceleration Data Structures
  - Fixed Grid
  - Nested Grid
  - Octree
  - Binary Space Partition
  - K-d tree
  - Bounding Volume Hierarchy
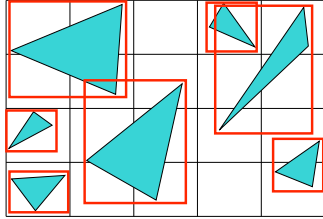- Misc Vocabulary & Advanced Papers

Collision Pruning via Uniform Grid

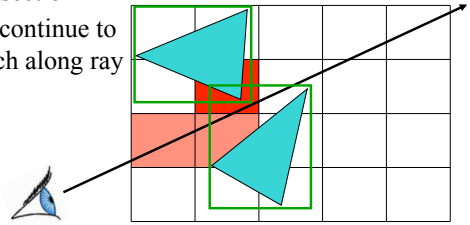- Primitives that overlap multiple cells?

Regular Grid

- Primitives that overlap multiple cells?
- Insert into multiple cells (use pointers)

For Each Cell Along a Ray

- Does the cell contain an intersection?
- Yes: return closest intersection
- No: continue to march along ray

Regular Grid Discussion

- Advantages?
  - easy to construct
  - easy to traverse

- Disadvantages?
  - may be only sparsely filled
 - geometry may still be clumped

Today

- Implicit Surfaces, Voxels, & Marching Cubes
- Collision Detection
- Conservative Bounding Region
- Spatial Acceleration Data Structures
  - Fixed Grid
  - Nested Grid
  - Octree
  - Binary Space Partition
  - K-d tree
  - Bounding Volume Hierarchy
- Misc Vocabulary & Advanced Papers

Adaptive Grids

- Subdivide until each cell contains no more than *n* elements, or maximum depth *d* is reached
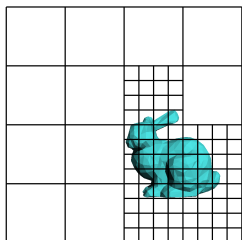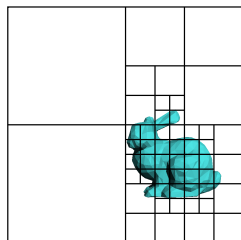
Nested Grids   Octree/(Quadtree)
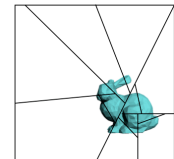
Variations of Adaptive Grids

- **When to split?** When a cell contains "lots" of geometry, but has not yet reached the max tree depth
- **Where to split?**
  - Quadtree/Octree: split *every* dimension in half, always axis aligned
  - kd-tree: choose *one* dimension (often the largest dimension) and split it axis aligned (but not necessarily at the midpoint)
  - Binary Space Partition (BSP): choose a *arbitrary* cut plane
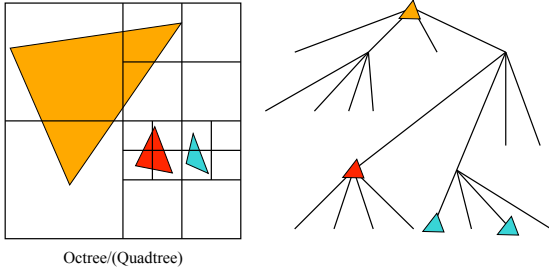- **Which one is best?** It depends…. Often they are all equally good!
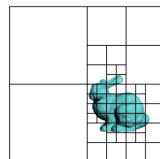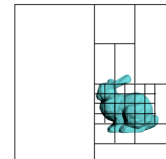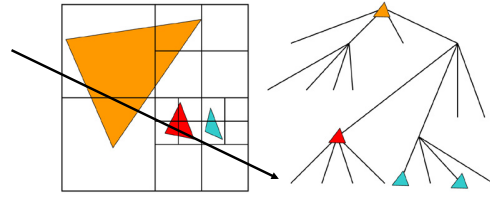
Quadtree/Octree   kd tree   BSP Tree

Primitives in an Adaptive Grid

- Can live at intermediate levels, or be pushed to lowest level of grid
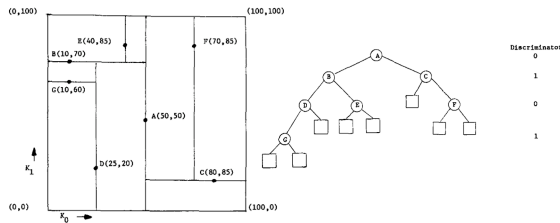

Octree/(Quadtree)

Adaptive Grid Discussion

- Advantages?
  - grid complexity matches geometric density
- Disadvantages?
  - more expensive to traverse (binary tree, lots of pointers)

Early k-d tree paper

- "Multidimensional Binary Search Trees Used for Associative Searching", Bentley, Communications of the ACM, 1975

Today

- Implicit Surfaces, Voxels, & Marching Cubes
- Collision Detection
- Conservative Bounding Region
- Spatial Acceleration Data Structures
  - Fixed Grid
  - Nested Grid
  - Octree
  - Binary Space Partition
  - K-d tree
  - Bounding Volume Hierarchy
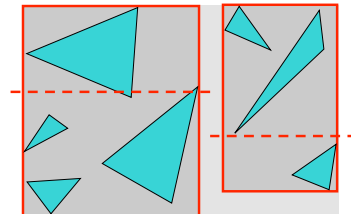- Misc Vocabulary & Advanced Papers

Bounding Volume Hierarchy

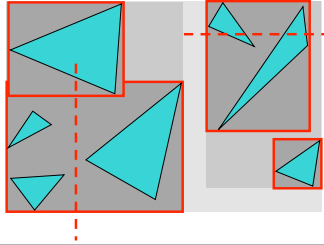- Find bounding box of objects
- Split objects into two groups
- Recurse

Bounding Volume Hierarchy

- Find bounding box of objects
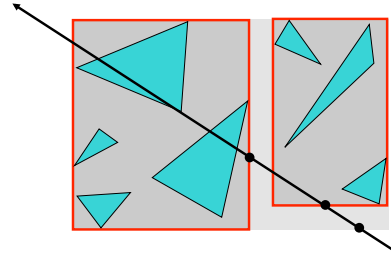- Split objects into two groups
- Recurse

6
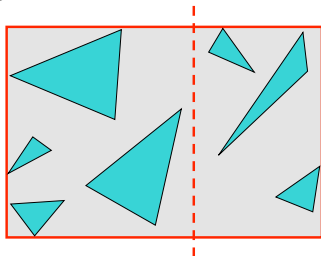
Where to split objects?

- At midpoint   OR
- Sort, and put half of the objects on each side   OR
- Use modeling hierarchy

Intersection with BVH

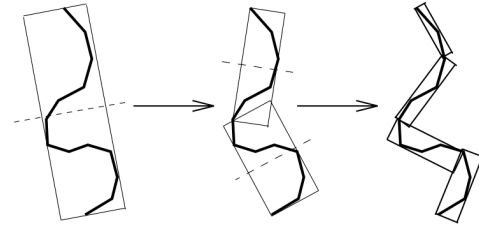- Check sub-volume with closer intersection first

Bounding Volume Hierarchy Discussion

- Advantages
  - easy to construct
  - easy to traverse
  - binary

- Disadvantages
  - may be difficult to choose a good split for a node
 - poor split may result in minimal spatial pruning

Reading for Today:

- Oriented Bounding Box (OBB):
  generalization of the (axis-aligned) BVH

OBB-Tree: A Hierarchical Structure for Rapid Interference Detection,
Gottschalk, Lin, & Manocha, SIGGRAPH 1996.

- "I-COLLIDE: An Interactive and Exact Collision Detection System for Large-scaled Environments", Cohen, Lin, Manocha, & Ponamgi, I3D 1995.
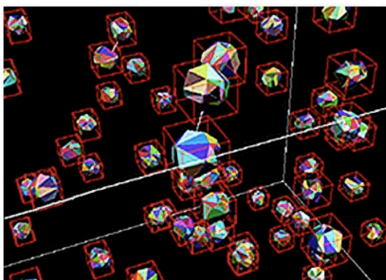
Reading for Today:

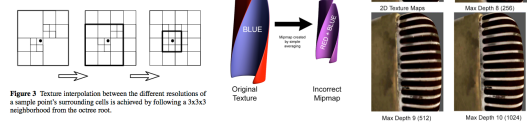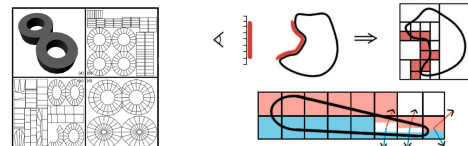- "Octree Textures", Benson & Davis, SIGGRAPH 2002

Figure 3 Texture interpolation between the different resolutions of a sample point's surrounding cells is achieved by following a 3x3x3 neighborhood from the octree root.

- "Painting and Rendering Textures on Unparameterized Models", DeBry, Gibbs, Deleon, and Robins, SIGGRAPH 2002
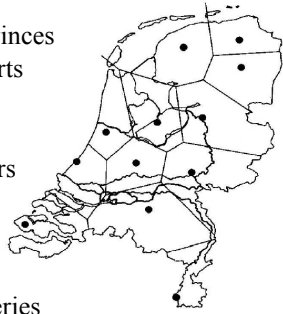
Questions?

Today

- Implicit Surfaces, Voxels, & Marching Cubes
- Collision Detection
- Conservative Bounding Region
- Spatial Acceleration Data Structures
  - Fixed Grid
  - Nested Grid
  - Octree
  - Binary Space Partition
  - K-d tree
  - Bounding Volume Hierarchy
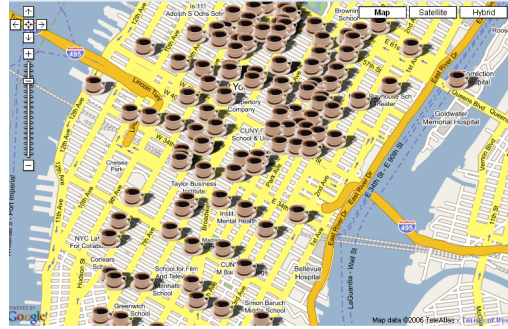- Misc Vocabulary & Advanced Papers

Voronoi Diagram/Cells/Regions

- How to re-district the Netherlands into provinces so that everyone reports to the closest capital
- Cell edges are the perpendicular bisectors of nearby points
- 2D or 3D
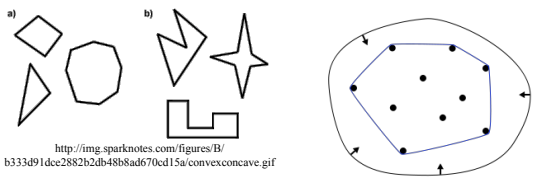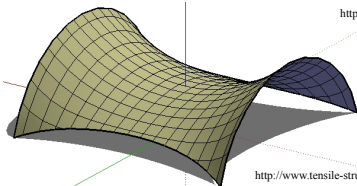- Supports efficient *Nearest Neighbor* queries

http://ccc.inaoep.mx/~rodrigo/robotica/Trigui.pdf

"Optimally" site the next Starbucks

http://findbyclick.com/coffee_s.html

Convex vs. Non-Convex

http://img.sparknotes.com/figures/B/
b333d91dce2882b2db48b8ad670cd15a/convexconcave.gif

http://en.wikipedia.org/wiki/File:ConvexHull.svg

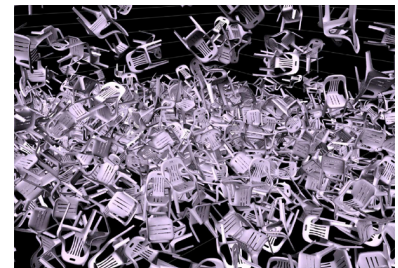http://www.tensile-structures.de/Bilder/SaddleSurface.jpg

Reduced Deformation

Doug L. James & Dinesh K. Pai
BD-Tree: Output-Sensitive Collision Detection for Reduced Deformable Models
SIGGRAPH 2004

- Collisions are expensive
- Deformation is expensive
- This is a lot of geometry!
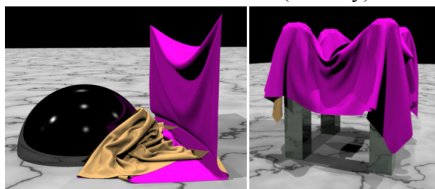- Simplify the simulation model

Cloth Collision

Robert Bridson, Ronald Fedkiw & John Anderson
Robust Treatment of Collisions, Contact and Friction for Cloth Animation
SIGGRAPH 2002

- A cloth has many points of contact
- Stays in contact
- Requires
  - Efficient collision detection
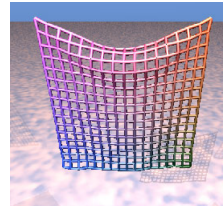 - Efficient numerical treatment (stability)

Reading for Friday:

- "Deformation Constraints in a Mass-Spring Model to Describe Rigid Cloth Behavior", Provot, 1995.
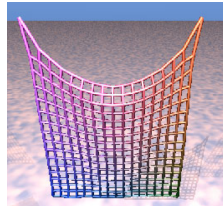
Simple mass-spring system   Improved solution

Post a comment/question on the LMS discussion by 10am

Optional Reading for Friday:

- Baraff, Witkin & Kass
  Untangling Cloth
  SIGGRAPH 2003

a   b