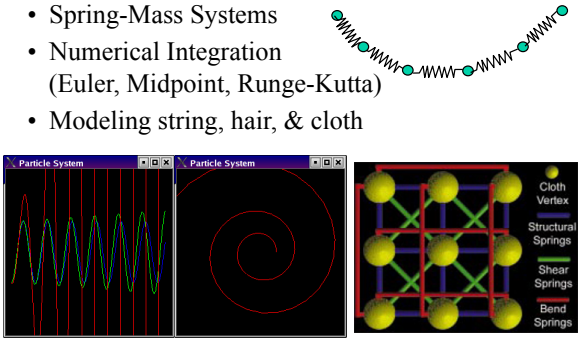
Navier-Stokes & Flow Simulation

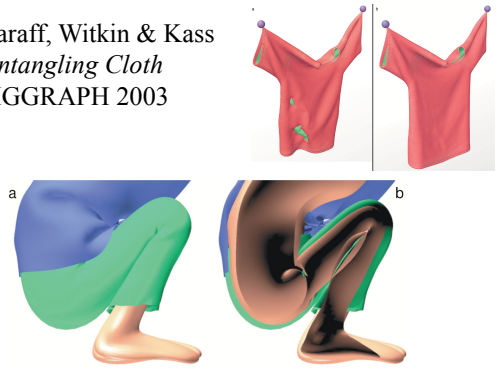
Last Time?

- Spring-Mass Systems
- Numerical Integration (Euler, Midpoint, Runge-Kutta)
- Modeling string, hair, & cloth

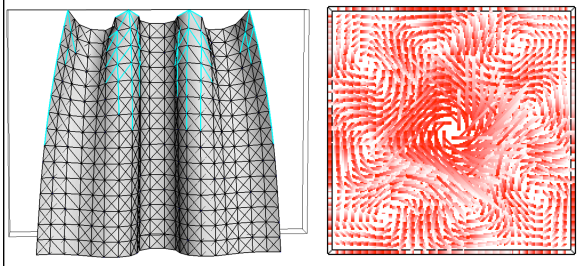


Optional Reading for Last Time:

- Baraff, Witkin & Kass *Untangling Cloth* SIGGRAPH 2003



HW2: Cloth & Fluid Simulation



Today

- Flow Simulations in Computer Graphics
  - water, smoke, viscous fluids
- Navier-Stokes Equations
  - incompressibility, conservation of mass
  - conservation of momentum & energy
- Fluid Representations
- Basic Algorithm
- Data Representation

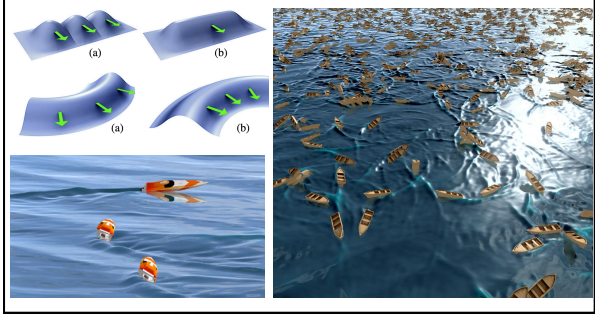
Flow Simulations in Graphics

- Random velocity fields
  - with averaging to get simple background motion
- Shallow water equations
  - height field only, can't represent crashing waves, etc.
- Full Navier-Stokes

- *note: typically we ignore surface tension and focus on macroscopic behavior*

Heightfield Wave Simulation

- Cem Yuksel, Donald H. House, and John Keyser, "Wave Particles", SIGGRAPH 2007



Flow in a Voxel Grid

- conservation of mass:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

For a single phase simulation (e.g., water only, air only)

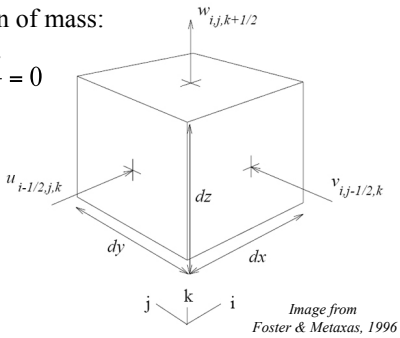


Image from Foster & Metaxas, 1996

Navier-Stokes Equations

- conservation of momentum:

gravity (& other external forces)

pressure viscosity

$$\frac{\partial u}{\partial t}+\frac{\partial u^2}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z} = -\frac{\partial p}{\partial x}+g_x+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\frac{\partial v}{\partial t}+\frac{\partial vu}{\partial x}+\frac{\partial v^2}{\partial y}+\frac{\partial vw}{\partial z} = -\frac{\partial p}{\partial y}+g_y+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$\frac{\partial w}{\partial t}+\frac{\partial wu}{\partial x}+\frac{\partial wv}{\partial y}+\frac{\partial w^2}{\partial z} = -\frac{\partial p}{\partial z}+g_z+\nu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

acceleration

Convection: internal movement in a fluid (e.g., caused by variation in density due to a transfer of heat)

drag

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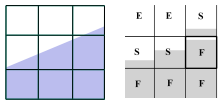
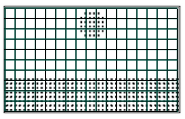
Modeling the Air/Water Surface

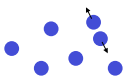
- Volume-of-fluid tracking



- Marker and Cell (MAC)



- Smoothed Particle Hydrodynamics (SPH)

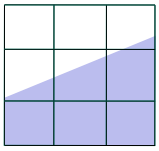


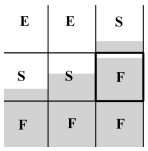
Comparing Representations

- How do we render the resulting surface?
- Are we guaranteed not to lose mass/volume? (is the simulation incompressible?)
- How is each affected by the grid resolution and timestep?
- Can we guarantee stability?

Volume-of-fluid-tracking

- Each cell stores a scalar value indicating that cell's "full"-ness

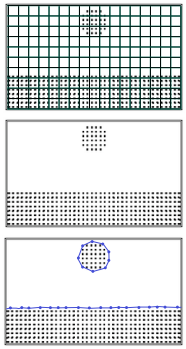




+ preserves volume
- difficult to render
- very dependent on grid resolution

Marker and Cell (MAC)

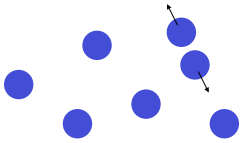
- Harlow & Welch, "Numerical calculation of time-dependent viscous incompressible flow of fluid with free surface", *The Physics of Fluids*, 1965.
- *Volume marker particles* identify location of fluid within the volume
- (Optional) *surface marker particles* track the detailed shape of the fluid/air boundary
- But… marker particles don't have or represent a mass/volume of fluid



+ rendering
- does not preserve volume
- dependent on grid resolution

Smoothed Particle Hydrodynamics (SPH)

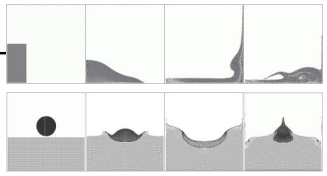
- Each particle represents a specific mass of fluid
- "Meshless" (no voxel grid)
- Repulsive forces between neighboring particles maintain constant volume



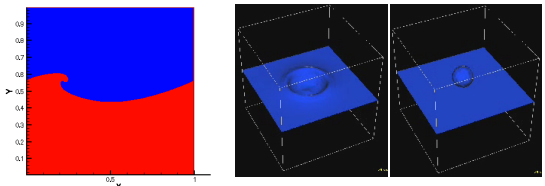
+ no grid resolution concerns (now accuracy depends on number/size of particles)
+ volume is preserved*
+ render similar to MAC
- much more expensive (particle-particle interactions)

Demos

- Nice Marker and Cell (MAC) videos at:



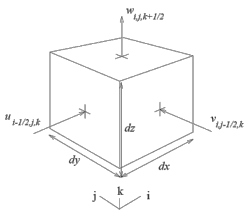
`http://panoramix.ift.uni.wroc.pl/~maq/eng/cfdthesis.php`

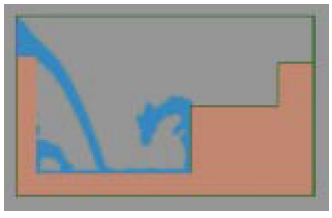


`http://mme.uwaterloo.ca/~fslien/free_surface/free_surface.htm`

Reading for Today

- "Realistic Animation of Liquids", Foster & Metaxas, 1996





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- Navier-Stokes Equations
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Each Grid Cell Stores:

- Velocity at the cell faces (offset grid)
- Pressure
- List of particles

This is a critically important detail! (and makes correct implementation rather annoying)

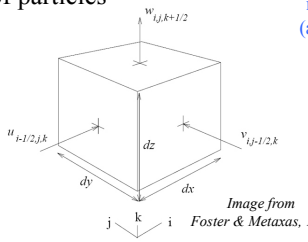


Image from Foster & Metaxas, 1996

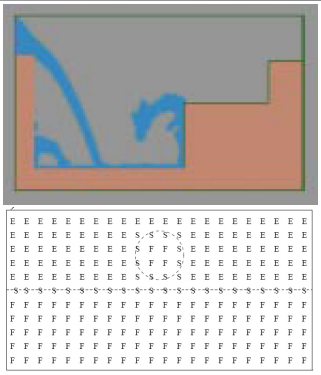
Initialization

- Choose a voxel resolution
- Choose a particle density
- Create grid & place the particles
- Initialize pressure & velocity of each cell
- Set the viscosity & gravity
- Choose a timestep & go!

At each Timestep:

- Identify which cells are Empty, Full, or on the Surface
- Compute new velocities
- Adjust the velocities to maintain an incompressible flow
- Move the particles
  - Interpolate the velocities at the faces
- Render the geometry and repeat!

Empty, Surface & Full Cells



Images from Foster & Metaxas, 1996

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Compute New Velocities

$$\tilde{u}_{i+1/2,j,k} = u_{i+1/2,j,k} + \delta t\{(1/\delta x)[(u_{i,j,k})^2 - (u_{i+1,j,k})^2]$$
$$+(1/\delta y)[(uv)_{i+1/2,j-1/2,k} - (uv)_{i+1/2,j+1/2,k}]$$
$$+(1/\delta z)[(uw)_{i+1/2,j,k-1/2} - (uw)_{i+1/2,j,k+1/2}] + g_x$$
$$+(1/\delta x)(p_{i,j,k} - p_{i+1,j,k}) + (\nu/\delta x^2)(u_{i+3/2,j,k}$$
$$-2u_{i+1/2,j,k} + u_{i-1/2,j,k}) + (\nu/\delta y^2)(u_{i+1/2,j+1,k}$$
$$-2u_{i+1/2,j,k} + u_{i+1/2,j-1,k}) + (\nu/\delta z^2)(u_{i+1/2,j,k+1}$$
$$-2u_{i+1/2,j,k} + u_{i+1/2,j,k-1})\},$$

Note: some of these values are the *average velocity* within the cell rather than the velocity at a cell face

At each Timestep:

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Adjusting the Velocities

- Calculate the *divergence* of the cell (the extra in/out flow)
- The divergence is used to update the *pressure* within the cell
- Adjust each face velocity uniformly to bring the divergence to zero
- Iterate across the entire grid until divergence is < ε

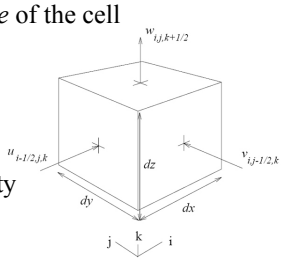
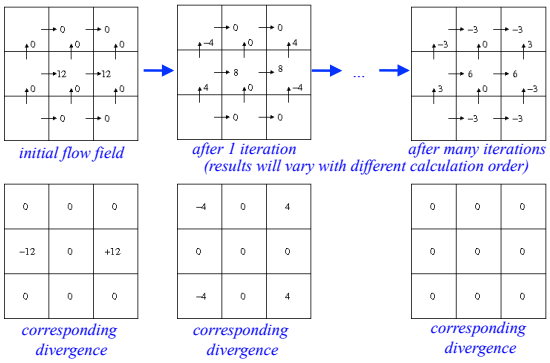


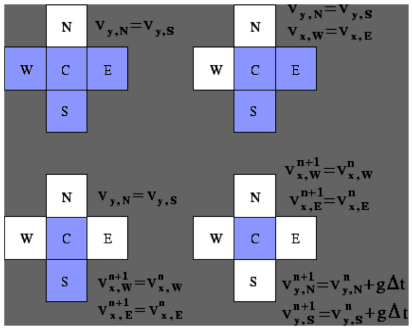
Image from Foster & Metaxas, 1996

Calculating/Eliminating Divergence



Handing Free Surface with MAC

- Divergence in surface cells:
  - Is divided equally amongst neighboring empty cells
  - Or other similar strategies?
- Zero out the divergence & pressure in empty cells



At each Timestep:

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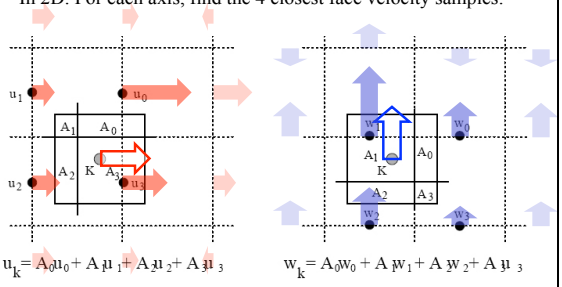
Velocity Interpolation

Original image from Foster & Metaxas, 1996

- In 2D: For each axis, find the 4 closest face velocity samples:



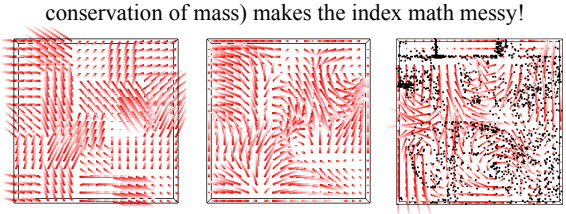
$u_k = A_0u_0 + A_1u_1 + A_2u_2 + A_3u_3$

$w_k = A_0w_0 + A_1w_1 + A_2w_2 + A_3u_3$

- (In 3D… Find 8 closest face velocities in each dimension)

Correct Velocity Interpolation

- NOTE: The complete implementation isn't particularly elegant… Storing velocities at face midpoints (req'd for conservation of mass) makes the index math messy!



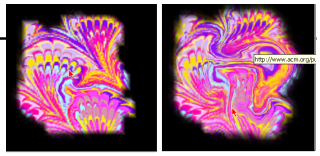
No interpolation (just use the left/bottom face velocity)
Note the discontinuities in velocity at cell boundaries

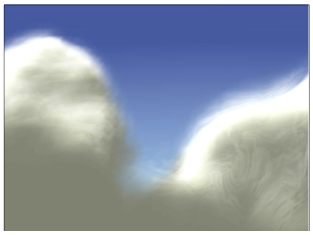
Correct Interpolation
Note that the velocity perpendicular to the outer box is zero

Buggy Interpolation
Note the clumping particles, and the discontinuities at some of the cell borders

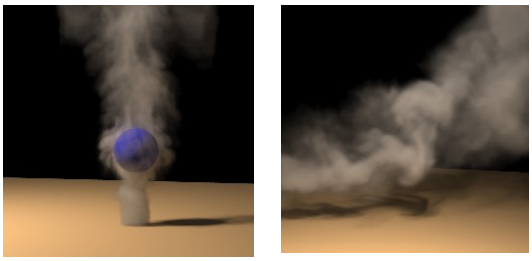
Stable Fluids

- "Stable Fluids", Jos Stam, SIGGRAPH 1999.





Smoke Simulation & Rendering



"Visual Simulation of Smoke"
Fedkiw, Stam & Jensen
SIGGRAPH 2001

Readings for Friday: *(pick one)*

"Deformable Objects Alive!" Coros, Martin, Thomaszewski, Schumacher, & Sumner, SIGGRAPH 2012

"Coupling Water and Smoke to Thin Deformable and Rigid Shells", Guendelman, Selle, Losasso, & Fedkiw, SIGGRAPH 2005.