Statistical and Learning Techniques in Computer Vision
Homework 2: Due Thursday September 14, 2006

1. (**12 points**)

   (a) In class we used the fact that when

   $$p(x) \propto e^{-(1/2)(ax^2-2bx)},$$

   then $p(x)$ is a normal distribution with variance $\sigma^2 = 1/a$ and mean b/a. Prove the vector version of this result:

   $$p(\mathbf{x}) \propto e^{(-1/2)[\mathbf{x}^T\mathbf{A}\mathbf{x}-2\mathbf{y}^T\mathbf{x}]}$$

   with symmetric, positive definite matrix $\mathbf{A}$, then p is multivariate normal. In doing so, derive the mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$ of $\mathbf{x}$ in terms of $\mathbf{A}$ and $\mathbf{y}$. Recall that the general form of the multivariate normal distribution is

   $$f(\mathbf{u};\boldsymbol{\mu},\boldsymbol{\Sigma}) = Ce^{-(1/2)(\mathbf{x}-\boldsymbol{\mu})^\top\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})},$$

   where C is chosen to ensure that f integrates to 1.

   (b) Use the foregoing result to derive the posterior distribution of the mean $\boldsymbol{\mu}$ of a multivariate normal distribution given samples $\mathcal{X} = \{\mathbf{x}_1, \ldots, \mathbf{x}_N\}$, known covariance $\boldsymbol{\Sigma}$, and a multivariate normal prior on $\boldsymbol{\mu}$ with mean $\boldsymbol{\mu}_0$ and covariance $\boldsymbol{\Sigma}_0$.

2. (**8 points**) Derive the mean and variance of $\hat{p}(x)$ using Parzen windows with a Gaussian kernel function.

3. (**10 points**) Prove that the k-nearest neighbor approximation to the density is not differentiable and is not a density. To do this, consider points in only 1 dimension, let $k = 2$, and assume the points are distinct.