Fall 2009, Final Exam, Data Structures and Algorithms

Name:
Section:
Email id:

21st December, 2009 (Winter Solstice)

This is an open book, open notebook exam.
Answer all twelve questions. Each Question is worth 10 points. You have 180 minutes to complete the exam.
Happy Holidays and Have a Great New Year.

1. **NP-complete**

   (a) Do Problem 8.13 c (DG)(Prove that the problem is NP-complete)
   MAX SAT Given a CNF formula and an integer g, find a truth assignment that satisfies at least g clauses.

   (b) Prove that clique problem can be solved in polynomial time for regular graphs of degree 3.

2. **Local Search**

Problem 9.6 (DG)

In the Minimum Steiner Tree Problem, the input consists of: a graph $G = (V, E)$ with distances d_{vw} between all pairs of nodes (distances satisfy metric property); and a distinguishable set of terminal nodes $V' \subseteq V$. The goal is to find an efficient ratio-2 approximation algorithm for finding a minimum Steiner tree that includes vertices V'. This tree may or may not include vertices $V - V'$. (Hint: Construct all pairs of shortest distances between vertices in V' and what can you say about these distances? Then construct a Minimum Spanning Tree and select edges from the original graph).

3. **Graphs/DFS/Path**

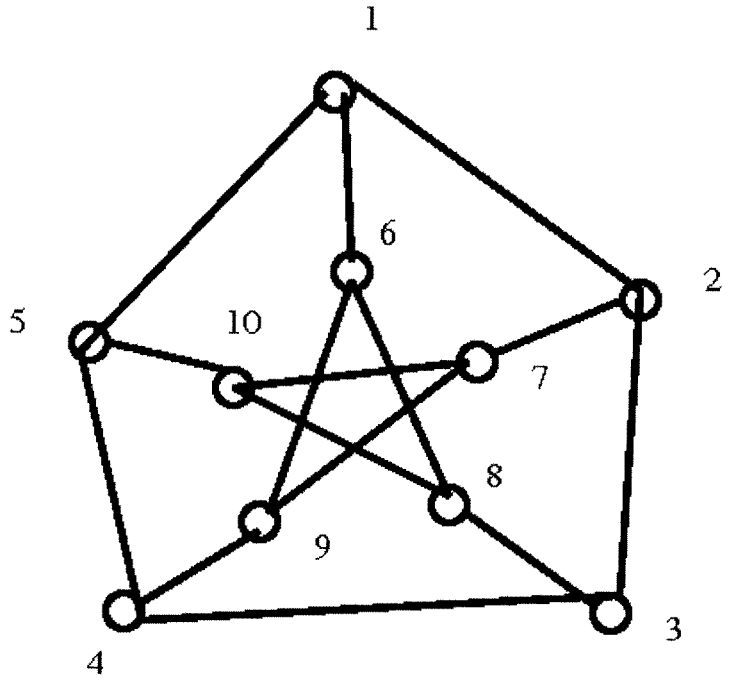
Consider the following graph with 10 vertices and 15 edges.
.[**10 points**]

(a) Draw a BFS tree starting at vertex 1.

(b) Draw a DFS tree starting at vertex 1.

(c) What is the longest path from vertex 1 and vertex 10.

(d) Draw two vertex disjoint paths from vertex 1 to vertex 2.



4. **Shortest Path Algorithms/Spanning Tree** Consider 8 of 3 bit binary numbers from 000 to 111. Treat them as vertices of an undirected graph. Two vertices are adjacent if they differ in exactly in one bit position. For example vertex labeled 001 is adjacent to 101, 011, 000.

   (a) Draw a graph of this 8 vertex undirected graph.

   (b) Draw a cycle of length 8 from vertex labeled 000 to vertex labeled 000.

5. **Linear Programming and Matching**

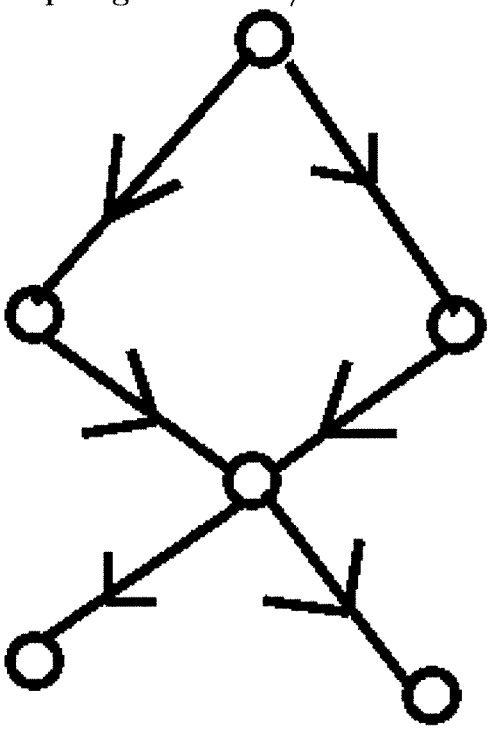
(a) Do problem 7.3 (just the LP formulation); A cargo plane can carry a maximum weight of 100 tons and maximum volume of 60 cubic meters. There are three materials to be transported, and the cargo company may choose to carry any element of each, up to the maximum available limits given below.

- Material 1 has density 2 tons/cubic meter, maximum available amount is 40 cubic meter and revenue is \$1000 per cubic meter.
- Material 2 has density 1 ton/cubic meter, maximum available amount is 30 cubic meter and revenue is \$1200 per cubic meter.
- Material 3 has density 3 ton/cubic meter, maximum available amount is 20 cubic meter and revenue is \$12000 per cubic meter.

Write a linear program that optimizes revenue within the constraints. (A linear Programming Formulation will suffice You do not have to solve it).

(b) Draw a tree with 6 vertices and has a max matching with three edges.

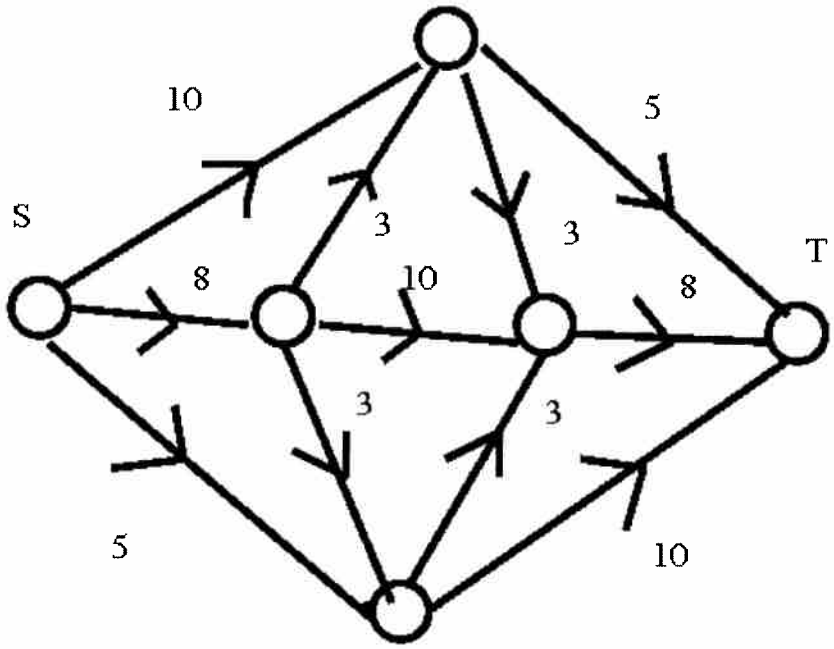
6. **Topological Order/Minimum Spanning Tree**



(a) Consider the directed acyclic graph G in the above figure. How many topological orderings does it have? (What are all the possible labeling of the vertices that preserve topological order.)

(b) Describe an algorithm to find the largest spanning tree in a given undirected weighted graph.

7. **Network Flows** Consider the following network (the numbers are edge capacities). What is the maximum flow from S to T and what is the minimum cut



8. **Dynamic Programming** Given two strings $x = x_1 x_2 \cdots x_n$ and $y = y_1 y_2 \cdots y_m$, we wish to find the length of their largest common substring, that is, the largest k for which there are indices i and j with $x_i x_{i+1} \cdots x_{i+k-1} \equiv y_j y_{j+1} \cdots y_{j+k-1}$ Show how to do this in time $O(mn)$

9. **Huffman** Suppose that the symbols a, b, c, d, e occur with frequencies of 1/2, 1/4, 1/8, 1/16, 1/16 respectively.

   (a) What is the Huffman encoding of this alphabet.

   (b) If this encoding is applied to a file consisting of 1,000,000 characters with the given frequencies, what is the length of the encoded file in bits?

10. **Extended Euclidean Algorithm, Greedy**

(a) Give a linear time algorithm that takes input as a tree T and determines a two coloring of the vertices, such that each vertex is colored with one of two colors and two adjacent vertices are not colored with same color.

(b) Solve the equation for integers x and y such that $13 \times x + 8 \times y = 1$ gcd(13,8)=1.

11. **Algorithm Design**

An array $A(1, \cdots, n)$ (random access) is said to have a majority element if more than half of its elements are the same. Given an array, design an efficient algorithm to tell whether there is a majority element, and, if so, find that element. The only question/operation that you are allowed to do is $A(i) = A(j)$? and this operation takes constant time. A pseudo code description will suffice. Analyze your algorithm.

12. **Multiplication and Divide and Conquer**

(a) Describe an efficient algorithm to multiply two n digit complex integers (Hint: Use divide and conquer strategy). (A+IB) *(C+ID) = (AC-BD)+I(BC+AD)

(b) Find the value of T(8) and then Solve the following recurrence equation for general $n = 2^k$

$$T(n) = 3T(n/2)$$

and

$$T(1) = 1$$

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