

## Practice Final Exam

1. Prove that the following language is not context-free.

$$L = \{a^i b^j c^k : 0 \leq i \leq j \leq k\}$$

2. Prove that the following language is context-free.

$$L = \{ww^R : w \neq abba, \text{ and } w \in \{a, b\}^*\}$$

3. Design a Turing Machine that accepts the following language.

$$L = \{ab(a+b)^*\}$$

4. Describe the algorithm of a three-tape Turing machine that computes the following function.

$$f(x) = x^2$$

5. Show that the class of Pushdown Automata with two stacks are equivalent with the Turing machines.

6. Describe a procedure that enumerates the following strings in proper order.

$$S = \{1^{2^n} : n \geq 0\}$$

7. Prove that the family of recursively enumerable languages is closed under union.

8. Prove that family of recursive languages are closed under intersection.

9. Prove that the problem of determining whether  $w \in L(G)$ , where  $G$  is a context-free grammar, is decidable.

10. Prove that the problem of determining whether two Turing machines accept the same language is undecidable.