CSCI 4150 Introduction to Artificial Intelligence, Fall 2002
Assignment 5 (118 points): out Tuesday October 22, due Friday November 1

1. (10 points) Show that the following inference rule is sound. $\dfrac{\neg Q \quad P \Rightarrow Q}{\neg P}$

2. (12 points) Transform the following sentences into CNF.

   (a) (P $\bigvee$ (Q $\bigwedge$ R)) $\Rightarrow$ X

   (b) $\neg$(A $\Rightarrow$ (B $\bigvee$ C)) $\bigwedge$ D

   (c) ($\neg$P $\bigwedge$ R) $\bigvee$ (S $\Rightarrow \neg$T)

   (d) $\forall_x A(x) \Rightarrow (\forall_y B(x, y) \Rightarrow \neg C(y))$

   (e) $\neg\, \forall_x A(x) \Rightarrow \neg B(x)$

   (f) $\forall_x (\exists_y A(x, y) \wedge \neg B(y)) \Rightarrow D(x)$

3. (12 points) Translate the following English sentences into FOL. Use the predicates `Likes(x, y)` (i.e. `x` likes `y`) and `Hates(x, y)` (i.e. `x` hates `y`)

   (a) Alice likes everyone that hates Bob.

   (b) Bob hates everyone that Alice likes.

   (c) Not everyone hates the people that like Alice.

   (d) There is someone who likes everyone that Alice hates.

Reasoning in the kinship domain

For these problems, you will build a knowledge base in the "kinship domain" and answer a number of questions in this domain. The predicates you may use in this problem are:

Brother(x, y)	Mother(x, y)	Male(x)
Sister (x, y)	Father(x, y)	Female(x)
Sibling(x, y)	Parent(x, y)	

Remember that in FOL, *Predicate*(x, y) is always interpreted as "x is the *Predicate* of y."

4. (16 points) Translate the following English sentences into CNF in FOL. Note that some English sentences may result in more than one FOL sentence, as indicated.

   - Sentence 1: The sibling relationship is commutative, i.e. if x is the sibling of y then y is the sibling of x
   - Sentences 2–3: A brother or a sister is a sibling.
   - Sentence 4: A male sibling is a brother.
   - Sentence 5: A female sibling is a sister.
   - Sentence 6: A parent is a mother or father. (You should treat this "or" as an "inclusive or.")
   - Sentences 7–8: A mother or father is a parent.
   - Sentence 9: A mother is female.
   - Sentence 10: A father is male.
   - Sentence 11: If x and y have the same parent, then they are siblings.
   - Sentence 12: If x is y's parent and y and z are siblings, then x is also z's parent.

- Sentences 13–14: A person is either male or female but not both.

5. (18 points) Suppose Alice and Bob are siblings, and Carl is Alice's father. Using the knowledge base (sentences 1–14), do a proof by contraduction using the resolution inference rule and the set of support strategy to show that Carl is Bob's father.

   Your proof should take the following form:

sentence number	sentence	explanation
15	<your negated conclusion>	Negated conclusion
16	<your first resolvant>	from sentences 15 and X, θ={ ... }
⋮	⋮	⋮

6. (12 points) For the previous problem, although you can figure out the proof yourself and then just write it down, a computer program would have to search for ways to apply the resolution inference rule. The computer might make several "wrong turns" before it found a proof (i.e. a "path" to the "goal").

   We can treat the knowledge base as the state. At each step, the computer would find all ways to apply the resolution inference rule to a pair of sentences in the knowledge base and resolve a new sentence which would then be added to the knowledge base, creating a new state. In general, there are many possible ways to apply the resolution inference rule, resulting in many possible child states.

   Consider the first three steps of your proof for the previous problem. The first step goes from the starting node (i.e. the initial knowledge base) to another node that is on a path to the goal. In order to get a sense of the complexity of this problem, count how many unifications are attempted and how many child nodes there are for those first three steps.

   An attempted unification would be between any two predicates, and a child state would be the result of a successful unification. For example, trying to unify Brother(x, y) with Sister(Alice, Bob) counts as one attempted unification. Trying to unify ¬Brother(x, y) with ¬Brother(Bob, z) also counts as an attempted unification, but since it is successful, this will lead to one possible child state.

   Your counts should reflect all the computation necessary at each step to compute all the child states from that node. Present your results in a table:

state before step #	# attempted unifications	# child states
15		
16		
17		

7. (6 points) Does the state space (as described above) have cycles? Briefly explain your answer.

8. (16 points) Using the knowledge base above, do a proof by contradiction using the resolution inference rule and the set of support strategy to show that a sibling is a brother or a sister. (Again, assume that this "or" is an "inclusive or.")

9. (16 points) Bob is Carl's father, and Alice is Carl's mother, and Dianne is Bob's sister. Use answer extraction to answer the question: Does Carl have a parent who has a sibling? Your solution should reveal who the parent and the sibling are.

⋆10. (5 points) Consider the following riddle:

> Brothers and sisters have I none, but that man's father is my father's son. Who is "that man"?

   This cannot be solved using answer extraction in conjunction with a FOL resolution proof. Explain why.