Declarative Computation Model

Single assignment store (VRH 2.2)
Kernel language syntax (VRH 2.3)

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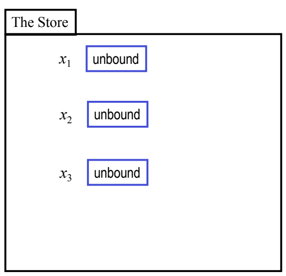
Sequential declarative computation model

- The single assignment store
  - declarative (dataflow) variables
  - partial values (variables and values are also called *entities*)
- The kernel language syntax
- The kernel language semantics
  - The environment: maps textual variable names (variable identifiers) into entities in the store
  - Interpretation (execution) of the kernel language elements (statements) by the use of an abstract machine
  - Abstract machine consists of an execution stack of statements transforming the store

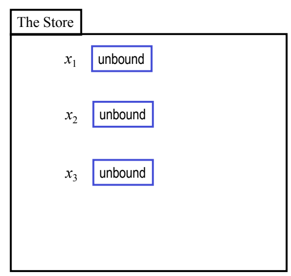
Single assignment store

- A single assignment store is a store (set) of variables
- Initially the variables are unbound, i.e. do not have a defined value
- Example: a store with three variables, x_1, x_2, and x_3



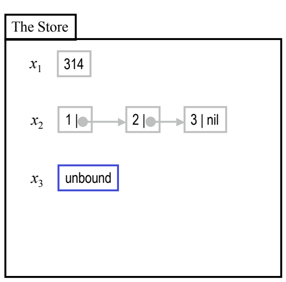
Single assignment store (2)

- Variables in the store may be bound to values
- Example: assume we allow as values, integers and lists of integers



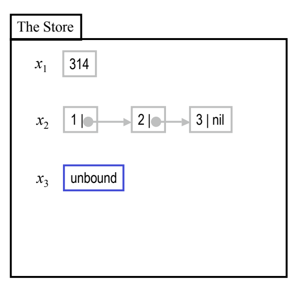
Single assignment store (3)

- Variables in the store may be bound to values
- Assume we allow as values, integers and lists of integers
- Example: x_1 is bound to the integer 314, x_2 is bound to the list [1 2 3], and x_3 is still unbound



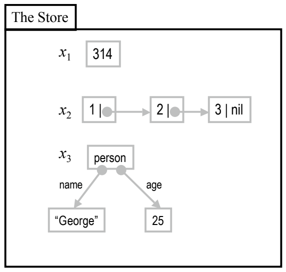
Declarative (single-assignment) variables

- A declarative variable starts out as being unbound when created
- It can be bound to exactly one value
- Once bound it stays bound through the computation, and is indistinguishable from its value



Value store

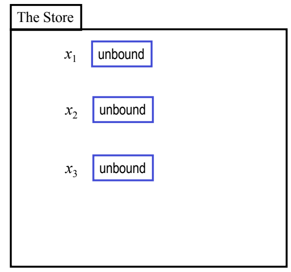
- A store where all variables are bound to values is called a value store
- Example: a value store where x_1 is bound to integer 314, x_2 to the list [1 2 3], and x_3 to the record (labeled tree) person(name: "George" age: 25)
- Functional programming computes functions on values, needs only a value store
- This notion of value store is enough for functional programming (ML, Haskell, Scheme)



Operations on the store (1) Single assignment

$\langle x \rangle = \langle v \rangle$

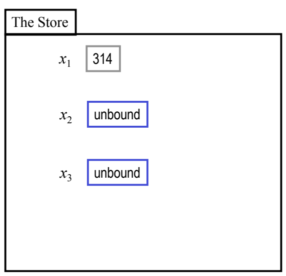
- $x_1 = 314$
- $x_2 = [1\ 2\ 3]$
- This assumes that $\langle x \rangle$ is unbound



Single-assignment

$\langle x \rangle = \langle \text{value} \rangle$

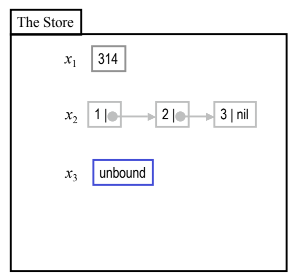
- $x_1 = 314$
- x2 = [1 2 3]



Single-assignment (2)

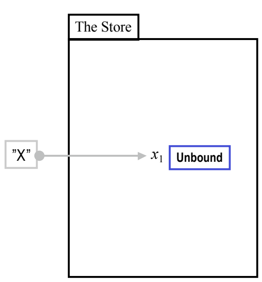
$\langle x \rangle = \langle v \rangle$

- $x_1 = 314$
- $x_2 = [1\ 2\ 3]$
- The *single assignment operation* ('=') constructs the $\langle v \rangle$ in the store and binds the variable $\langle x \rangle$ to this value
- If the variable is already bound, the operation will test the compatibility of the two values
- If the test fails an error is raised



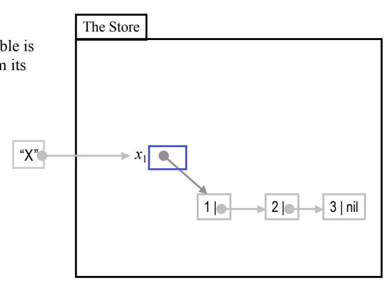
Variable identifiers

- Variable identifiers refers to store entities (variables or values)
- The environment maps variable identifiers to variables
- declare X
  :
- local X in ...
- "X" is a (variable) identifier
- This corresponds to 'environment' {"X" → x_1}



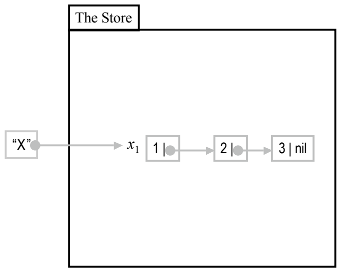
Variable-value binding revisited (1)

- X = [1 2 3]
- Once bound the variable is indistinguishable from its value



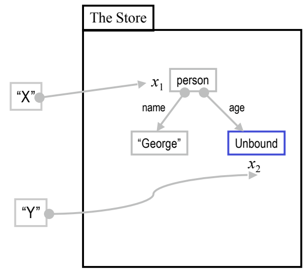
Variable-value binding revisited (2)

- X = [1 2 3]
- Once bound the variable is indistinguishable from its value
- The operation of traversing variable cells to get the value is known as *dereferencing* and is invisible to the programmer



Partial Values

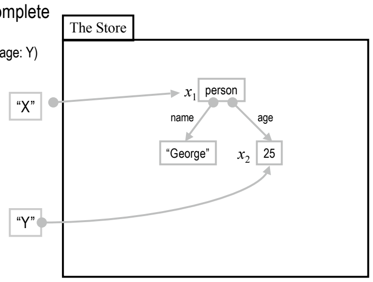
- A partial value is a data structure that may contain unbound variables
- The store contains the partial value: person(name: "George" age: x_2)
- declare Y X
  X = person(name: "George" age: Y)
- The identifier 'Y' refers to x_2



Partial Values (2)

Partial Values may be complete

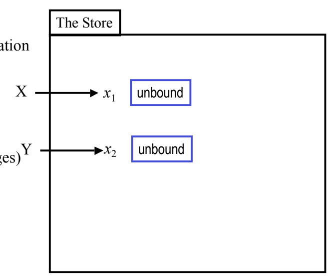
- declare Y X
  X = person(name: "George" age: Y)
- **Y = 25**



Variable to variable binding

$\langle x_1 \rangle = \langle x_2 \rangle$

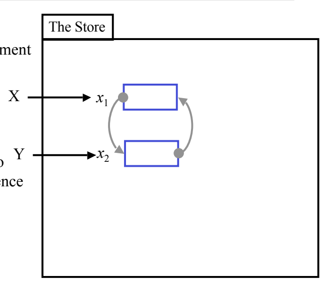
- It is to perform the bind operation between variables
- Example:
- X = Y
- X = [1 2 3]
- The operations equates (merges) the two variables



Variable to variable binding (2)

$\langle x_1 \rangle = \langle x_2 \rangle$

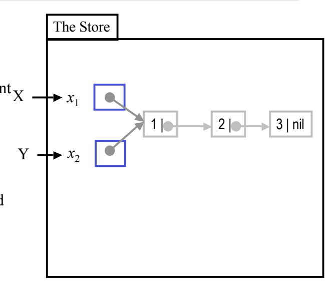
- It is to perform a single assignment between variables
- Example:
- X = Y
- X = [1 2 3]
- The operations equates the two variables (forming an equivalence class)



Variable to variable binding (3)

$\langle x_1 \rangle = \langle x_2 \rangle$

- It is to perform a single assignment between variables
- Example:
- X = Y
- **X = [1 2 3]**
- All variables (X and Y) are bound to **[1 2 3]**



Summary
Variables and partial values

- Declarative variable:
  - is an entity that resides in a single-assignment store, that is initially unbound, and can be bound to exactly one (partial) value
  - it can be bound to several (partial) values as long as they are compatible with each other
- Partial value:
  - is a data-structure that may contain unbound variables
  - when one of the variables is bound, it is replaced by the (partial) value it is bound to
  - a complete value, or *value* for short is a data structure that does not contain any unbound variables

Declaration and use of variables

- Assume that variables can be declared (introduced) and used separately
- What happens if we try to use a variable before it is bound?

1. Use whatever value happens to be in the memory cell occupied by the variable (C, C++)
2. The variable is initialized to a default value (Java), use the default
3. An error is signaled (Prolog). Makes sense if there is a single activity running (pure sequential programs)
4. An attempt to use the variable will wait (suspends) until another activity binds the variable (Oz/Mozart)

Declaration and use of variables (2)

- An attempt to use the variable will wait (suspends) until another activity binds the variable (Oz/Mozart)
- Declarative (single assignment) variables that have this property are called *dataflow* variables
- It allows multiple operations to proceed concurrently giving the correct result
- Example: A = 23 running concurrently with B = A+1
- Functional (concurrent) languages do not allow the separation between declaration and binding (ML, Haskell, and Erlang)

Kernel language syntax

The following defines the syntax of a statement, ⟨s⟩ denotes a statement

⟨s⟩ ::=	skip	*empty statement*
|	⟨x⟩ = ⟨y⟩	*variable-variable binding*
|	⟨x⟩ = ⟨v⟩	*variable-value binding*
|	⟨s_1⟩ ⟨s_2⟩	*sequential composition*
|	local ⟨x⟩ in ⟨s_1⟩ end	*declaration*
|	if ⟨x⟩ then ⟨s_1⟩ else ⟨s_2⟩ end	*conditional*
|	'{' ⟨x⟩ ⟨y_1⟩ ... ⟨y_n⟩ '}'	*procedural application*
|	case ⟨x⟩ of ⟨pattern⟩ then ⟨s_1⟩ else ⟨s_2⟩ end	*pattern matching*

⟨v⟩ ::= ... *value expression*

⟨pattern⟩ ::= ...

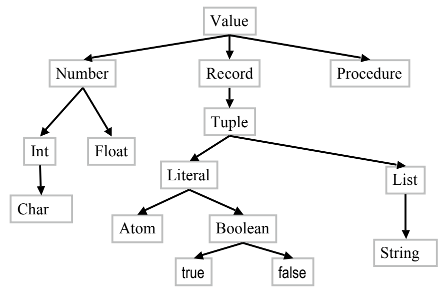
Variable identifiers

- ⟨*x*⟩ , ⟨*y*⟩, ⟨*z*⟩ stand for variables
- In the concrete kernel language variables begin with upper-case letter followed by a (possibly empty) sequence of alphanumeric characters or underscore
- Any sequence of printable characters within back-quote
- Examples:
  - X
  - Y1
  - Hello_World
  - \`hello this is a $5 bill\` (back-quote)

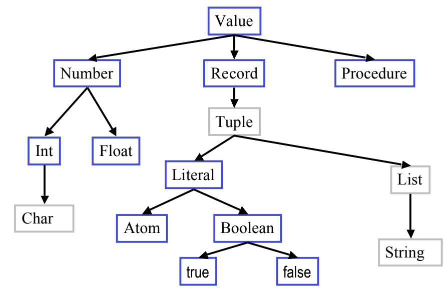
Values and types

- A *data type* is a set of values and a set of associated operations
- Example: Int is the the data type "Integer", i.e set of all integer values
- 1 is *of type* Int
- Int has a set of operations including +,-,*,div, etc
- The model comes with a set of basic types
- Programs can define other types, e.g., *abstract data types* ADT

Data types



Data types (2)



Value expressions

⟨v⟩ ::= ⟨procedure⟩ | ⟨record⟩ | ⟨number⟩

⟨procedure⟩ ::= proc '{'$ ⟨y_1⟩ … ⟨y_n⟩'}' ⟨s⟩ end

⟨record⟩, ⟨pattern⟩ ::= ⟨literal⟩
| ⟨literal⟩ ([⟨feature_1⟩ : ⟨x_1⟩ … ⟨feature_n⟩ : ⟨x_n⟩])

⟨literal⟩ ::= ⟨atom⟩ | ⟨bool⟩
⟨feature⟩ ::= ⟨int⟩ | ⟨atom⟩ | ⟨bool⟩

⟨bool⟩ ::= true | false

⟨number⟩ ::= ⟨int⟩ | ⟨float⟩

Numbers

- Integers
  - 314, 0
  - ~10 (minus 10)
- Floats
  - 1.0, 3.4, 2.0e2, 2.0E2 (2×10^2)

Atoms and booleans

- A sequence starting with a lower-case character followed by characters or digits, …
  - person, peter
  - 'Seif Haridi'
- Booleans:
  - true
  - false

Records

- Compound representation (data-structures)
  - ⟨l⟩(⟨f_1⟩ : ⟨x_1⟩ … ⟨f_n⟩ : ⟨x_n⟩)
  - ⟨l⟩ is a literal
- Examples
  - person(age:X1 name:X2)
  - person(1:X1 2:X2)
  - '|'(1:H 2:T)
  - nil
  - person

Syntactic sugar (tuples)

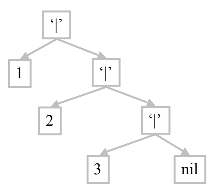
- Tuples
  $\langle l \rangle(\langle x_1 \rangle \ldots \langle x_n \rangle)$ (tuple)
- This is equivalent to the record
  $\langle l \rangle(1{:}\ \langle x_1 \rangle \ldots n{:}\ \langle x_n \rangle)$
- Example:
  person('George' 25)
- This is the record
  person(1:'George' 2:25)

Syntactic sugar (lists)

- Lists
  $\langle x_1 \rangle \mid \langle x_2 \rangle$ (a cons with the infix operator '|')
- This is equivalent to the tuple
  $\text{'|'}(\langle x_1 \rangle\ \langle x_2 \rangle)$
- Example:
  H | T
- This is the tuple
  '|'(H T)

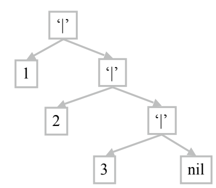
Syntactic sugar (lists)

- Lists
  $\langle x_1 \rangle \mid \langle x_2 \rangle \mid \langle x_3 \rangle$
- '|' associates to the right
  $\langle x_1 \rangle \mid (\langle x_2 \rangle \mid \langle x_3 \rangle)$
- Example:
  1 | 2 | 3 | nil
- Is
  1 | (2 | (3 | nil))



Syntactic sugar (complete lists)

- Complete lists
- Example:
  [1 2 3]
- Is
  1 | (2 | (3 | nil))



Strings

- A string is a list of character codes enclosed with double quotes
- Ex: "E=mc^2"
- Means the same as [69 61 109 99 94 50]

Procedure declarations

- According to the kernel language
  $\langle x \rangle$ = proc {$ $\langle y_1 \rangle \ldots \langle y_n \rangle$} $\langle s \rangle$ end
  is a legal statement
- It binds $\langle x \rangle$ to a procedure value
- This statement actually declares (introduces) a procedure
- Another syntactic variant which is more familiar is
  proc {$\langle x \rangle\ \langle y_1 \rangle \ldots \langle y_n \rangle$} $\langle s \rangle$ end
- This introduces (declares) the procedure $\langle x \rangle$

Operations of basic types

- Arithmetics
  - Floating point numbers: +,-,*, and /
  - Integers: +,-,*,div (integer division, i.e. truncate fractional part), mod (the remainder after a division, e.g.10 mod 3 = 1)
- Record operations
  - Arity, Label, and "."
  - X = person(name:"George" age:25)
  - {Arity X} = [age name]
  - {Label X} = person, X.age = 25
- Comparisons
  - Boolean comparisons, including ==, \= (equality)
  - Numeric comparisons, =<, <, >, >=, compares integers, floats, and atoms

Value expressions

⟨v⟩ ::= ⟨procedure⟩ | ⟨record⟩ | ⟨number⟩ | ⟨basicExpr⟩

⟨basicExpr⟩ ::= ... | ⟨numberExpr⟩ | ...

⟨numberExpr⟩ ::= $\langle x \rangle_1$ + $\langle x \rangle_2$ | ...

.....

Syntactic sugar (multiple variables)

- Multiple variable introduction

```
local X Y in ⟨statement⟩ end
```

- is transformed to

```
local X in
    local Y in ⟨statement⟩ end
end
```


Syntactic sugar (basic expressions)

- Basic expression nesting

```
if ⟨basicExpr⟩ then ⟨statement⟩1 else ⟨statement⟩2 end
```

- is transformed to

```
local T in
  T = ⟨basicExpr⟩
  if T then ⟨statement⟩1 else ⟨statement⟩2 end
end
```

- where T is a fresh ('new') variable identifier

Syntactic sugar (variables)

- Variable initialization

```
local X = ⟨value⟩ in ⟨statement⟩ end
```

- Is transformed to

```
local X in
    X = ⟨value⟩
    ⟨statement⟩
end
```


Exercises

23. Using Oz, perform a few basic operations on numbers, records, and booleans (see Appendix B1-B3)
24. Explain the behavior of the `declare` statement in the interactive environment. Give an example of an interactive Oz session where "`declare`" and "`declare … in`" produce different results. Explain why.
25. VRH Exercise 2.9.1
26. *Describe what an anonymous procedure is, and write one in Oz. When are anonymous procedures useful?