Declarative Computation Model

Kernel language semantics revisited (VRH 2.4.5)
From kernel to practical language (VRH 2.6)
Exceptions (VRH 2.7)

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October 2, 2006

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Sequential declarative computation model

- The kernel language semantics revisited.
  - Suspendable statements:
    - if,
    - case,
    - procedure application.
  - Procedure values
  - Procedure introduction
  - Procedure application.

Conditional

- The semantic statement is
  (if ⟨x⟩ then ⟨s$_1$⟩ else ⟨s$_2$⟩ end , *E*)
- If the activation condition (*E*(⟨x⟩) is determined) is true:
  - If *E*(⟨x⟩) is not Boolean (true, false), raise an error
  - *E*(⟨x⟩) is true, push (⟨s$_1$⟩ , *E*) on the stack
  - *E*(⟨x⟩) is false, push (⟨s$_2$⟩ , *E*) on the stack
- If the activation condition (*E*(⟨x⟩) is determined) is false:
  - Suspend

Case statement

- The semantic statement is
  (case ⟨x⟩ of ⟨*l*⟩ (⟨f_1⟩ : ⟨x_1⟩ … ⟨f_n⟩ : ⟨x_n⟩)
  then ⟨s$_1$⟩
  else ⟨s$_2$⟩ end , *E*)
- If the activation condition (*E*(⟨x⟩) is determined) is true:
  - If *E*(⟨*x*⟩) is a record, the label of *E*(⟨*x*⟩) is ⟨*l*⟩ and its arity is [⟨f_1⟩ … ⟨f_n⟩]:
    push (local ⟨x_1⟩ = ⟨*x*⟩. ⟨f_1⟩ … ⟨x_n⟩ = ⟨*x*⟩. ⟨f_n⟩ in ⟨s$_1$⟩ end, *E*)
    on the stack
  - Otherwise, push (⟨s$_2$⟩, *E*) on the stack
- If the activation condition (*E*(⟨x⟩) is determined) is false:
  - Suspend

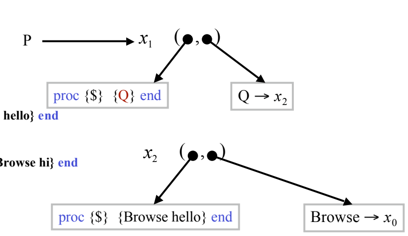
Procedure values

- Constructing a procedure value in the store is not simple because a procedure may have external references

```
local P Q in
   Q = proc {$} {Browse hello} end
   P = proc {$}  {Q} end
   local Q in
      Q = proc {$} {Browse hi} end
      {P}
   end
end
```


Procedure values (2)

```
local P Q in
   Q = proc {$} {Browse hello} end
   P = proc {$}  {Q} end
   local Q in
      Q = proc {$} {Browse hi} end
      {P}
   end
end
```



Procedure values (3)

- The semantic statement is
  ($\langle x\rangle$ = proc {\$ $\langle y_1\rangle \ldots \langle y_n\rangle$} $\langle s\rangle$ end, E)
- $\langle y_1\rangle \ldots \langle y_n\rangle$ are the (*formal*) parameters of the procedure
- Other free identifiers in $\langle s\rangle$ are called *external references* $\langle z_1\rangle \ldots \langle z_k\rangle$
- These are defined by the environment E where the procedure is declared (lexical scoping)
- The contextual environment of the procedure CE is $E\,|_{\{\langle z_1\rangle \ldots \langle z_k\rangle\}}$
- When the procedure is called CE is used to construct the environment for execution of $\langle s\rangle$

```
(proc {$ ⟨y1⟩ … ⟨yn⟩ }
    ⟨s⟩
 end ,
CE)
```


Procedure introduction

- The semantic statement is
  ($\langle x\rangle$ = proc {\$ $\langle y_1\rangle \ldots \langle y_n\rangle$} $\langle s\rangle$ end, E)
- Create a contextual environment:
  $CE = E\,|_{\{\langle z_1\rangle \ldots \langle z_k\rangle\}}$ where $\langle z_1\rangle \ldots \langle z_k\rangle$ are external references in $\langle s\rangle$.
- Create a new procedure value of the form:
  (proc {\$ $\langle y_1\rangle \ldots \langle y_n\rangle$} $\langle s\rangle$ end, CE) , refer to it by the variable x_P
- Bind the store variable $E(\langle x\rangle)$ to x_P
- Continue to next execution step

Procedure application

- The semantic statement is
  ({ $\langle x\rangle \langle y_1\rangle \ldots \langle y_n\rangle$ } , E)
- If the activation condition ($E(\langle x\rangle)$ is determined) is true:
  - If $E(\langle x\rangle)$ is not a procedure value, or it is a procedure with arity that is not equal to n, raise an error
  - If $E(\langle x\rangle)$ is (proc {\$ $\langle z_1\rangle \ldots \langle z_n\rangle$} $\langle s\rangle$ end, CE) , push
    ($\langle s\rangle$, $CE + \{\langle z_1\rangle \rightarrow E(\langle y_1\rangle) \ldots \langle z_n\rangle \rightarrow E(\langle y_n\rangle)\}$)
    on the stack
- If the activation condition ($E(\langle x\rangle)$ is determined) is false:
  - Suspend

Execution examples

```
⟨s⟩1 {  local Max C in
  ⟨s⟩2 {  proc {Max X Y Z}
          ⟨s⟩3  if X >= Y then Z=X else Z=Y end
        end
        {Max 3 5 C}
     end
```


Execution examples (2)

```
⟨s⟩1 {  local Max C in
  ⟨s⟩2 {  proc {Max X Y Z}
          ⟨s⟩3  if X >= Y then Z=X else Z=Y end
        end
        ⟨s⟩4 {Max 3 5 C}
     end
```

- Initial state ([($\langle s\rangle_1$, $\varnothing$)], $\varnothing$)
- After local Max C in …
  ([($\langle s\rangle_2$, {Max $\rightarrow m$, C $\rightarrow c$})], {m, c})
- After Max binding
  ([($\langle s\rangle_4$, {Max $\rightarrow m$, C $\rightarrow c$})],
  {m = (proc{\$ X Y Z} $\langle s\rangle_3$ end , $\varnothing$) , c})

Execution examples (3)

```
⟨s⟩1 {  local Max C in
  ⟨s⟩2 {  proc {Max X Y Z}
          ⟨s⟩3  if X >= Y then Z=X else Z=Y end
        end
        ⟨s⟩4 {Max 3 5 C}
     end
```

- After Max binding
  ([($\langle s\rangle_4$, {Max $\rightarrow m$, C $\rightarrow c$})],
  {m = (proc{\$ X Y Z} $\langle s\rangle_3$ end , $\varnothing$) , c})
- After procedure call
  ([($\langle s\rangle_3$, {X $\rightarrow t_1$, Y $\rightarrow t_2$, Z $\rightarrow c$})],
  {m = (proc{\$ X Y Z} $\langle s\rangle_3$ end , $\varnothing$) , t_1=3, t_2=5, c})

Execution examples (4)

$\langle s \rangle_1$ = whole statement, $\langle s \rangle_2$ = body between `in` and `end`

```
local Max C in
   proc {Max X Y Z}
      ⟨s⟩3   if X >= Y then Z=X else Z=Y end
   end
   ⟨s⟩4 {Max 3 5 C}
end
```

- After procedure call
  $([(\langle s \rangle_3,\ \{X \rightarrow t_1, Y \rightarrow t_2, Z \rightarrow c\})],$
  $\{m = (\text{proc}\{\$\ X\ Y\ Z\}\ \langle s \rangle_3\ \text{end}\ , \emptyset)\ ,\ t_1{=}3,\ t_2{=}5,\ c\}\ )$
- After T = (X>=Y)
  $([(\langle s \rangle_3,\ \{X \rightarrow t_1, Y \rightarrow t_2, Z \rightarrow c, T \rightarrow t\})\ ],$
  $\{m = (\text{proc}\{\$\ X\ Y\ Z\}\ \langle s \rangle_3\ \text{end}\ , \emptyset)\ ,\ t_1{=}3,\ t_2{=}5,\ c,\ t{=}\text{false}\}\ )$
- $([(Z{=}Y\ ,\ \{X \rightarrow t_1, Y \rightarrow t_2, Z \rightarrow c, T \rightarrow t\})\ ],$
  $\{m = (\text{proc}\{\$\ X\ Y\ Z\}\ \langle s \rangle_3\ \text{end}\ , \emptyset)\ ,\ t_1{=}3,\ t_2{=}5,\ c,\ t{=}\text{false}\}\ )$

Execution examples (5)

$\langle s \rangle_1$ = whole statement, $\langle s \rangle_2$ = body between `in` and `end`

```
local Max C in
   proc {Max X Y Z}
      ⟨s⟩3   if X >= Y then Z=X else Z=Y end
   end
   ⟨s⟩4 {Max 3 5 C}
end
```

- $([(Z{=}Y\ ,\ \{X \rightarrow t_1, Y \rightarrow t_2, Z \rightarrow c, T \rightarrow t\})\ ],$
  $\{m = (\text{proc}\{\$\ X\ Y\ Z\}\ \langle s \rangle_3\ \text{end}\ , \emptyset)\ ,\ t_1{=}3,\ t_2{=}5,\ c,\ t{=}\text{false}\}\ )$
- $([\ ],$
  $\{m = (\text{proc}\{\$\ X\ Y\ Z\}\ \langle s \rangle_3\ \text{end}\ , \emptyset)\ ,\ t_1{=}3,\ t_2{=}5,\ c{=}5,\ t{=}\text{false}\}\ )$

Procedures with external references

$\langle s \rangle_1$ = whole statement, $\langle s \rangle_2$ = body between `in` and `end`

```
local LB Y C in
   Y = 5
   proc {LB X Z}
      ⟨s⟩3 if X >= Y then Z=X else Z=Y end
   end
   {LB 3 C}
end
```


Procedures with external references

$\langle s \rangle_1$ = whole statement, $\langle s \rangle_2$ = body between `in` and `end`

```
local LB Y C in
   Y = 5
   proc {LB X Z}
      ⟨s⟩3   if X >= Y then Z=X else Z=Y end
   end
   {LB 3  C}
end
```

- The procedure value of LB is
- $(\text{proc}\{\$\ X\ Z\}\ \langle s \rangle_3\ \text{end}\ ,\ \{Y \rightarrow y\})$
- The store is $\{y = 5, \ldots\}$

Procedures with external references

$\langle s \rangle_1$ = whole statement, $\langle s \rangle_2$ = body between `in` and `end`

```
local LB Y C in
   Y = 5
   proc {LB X Z}
      ⟨s⟩3   if X >= Y then Z=X else Z=Y end
   end
   {LB 3 C}
end
```

- The procedure value of LB is
- $(\text{proc}\{\$\ X\ Z\}\ \langle s \rangle_3\ \text{end}\ ,\ \{Y \rightarrow y\})$
- The store is $\{y = 5, \ldots\}$
- STACK: $[(\ \{LB\ T\ C\}\ ,\ \{Y \rightarrow y\ ,\ LB \rightarrow lb,\ C \rightarrow c,\ T \rightarrow t\})\ ]$
- STORE: $\{y = 5,\ lb = (\text{proc}\{\$\ X\ Z\}\ \langle s \rangle_3\ \text{end}\ ,\ \{Y \rightarrow y\})\ ,\ t = 3,\ c\}$

Procedures with external references

$\langle s \rangle_1$ = whole statement, $\langle s \rangle_2$ = body between `in` and `end`

```
local LB Y C in
   Y = 5
   proc {LB X Z}
      ⟨s⟩3   if X >= Y then Z=X else Z=Y end
   end
   {LB 3 C}
end
```

- STACK: $[(\ \{LB\ T\ C\}\ ,\ \{Y \rightarrow y\ ,\ LB \rightarrow lb,\ C \rightarrow c,\ T \rightarrow t\})\ ]$
- STORE: $\{y = 5,\ lb = (\text{proc}\{\$\ X\ Z\}\ \langle s \rangle_3\ \text{end}\ ,\ \{Y \rightarrow y\})\ ,\ t = 3,\ c\}$
- STACK: $[(\langle s \rangle_3\ ,\ \{Y \rightarrow y\ ,\ X \rightarrow t\ ,\ Z \rightarrow c\})\ ]$
- STORE: $\{y = 5,\ lb = (\text{proc}\{\$\ X\ Z\}\ \langle s \rangle_3\ \text{end}\ ,\ \{Y \rightarrow y\})\ ,\ t = 3,\ c\}$

Procedures with external references

```
           local LB Y C in
             Y = 5
             proc {LB X Z}
⟨s⟩1  ⟨s⟩2     ⟨s⟩3   if X >= Y then Z=X else Z=Y end
             end
             {LB 3 C}
           end
```

- STACK: [(⟨s⟩$_3$, {Y → y , X → t , Z → c})]
- STORE: {y = 5, lb = (proc{$ X Z} ⟨s⟩$_3$ end , {Y → y}) , t = 3, c}
- STACK: [(Z=Y , {Y → y , X → t , Z → c})]
- STORE: {y = 5, lb = (proc{$ X Z} ⟨s⟩$_3$ end , {Y → y}) , t = 3, c}
- STACK: [ ]
- STORE: {y = 5, lb = (proc{$ X Z} ⟨s⟩$_3$ end , {Y → y}) , t = 3, c = 5}

From the kernel language to a practical language

- Interactive interface
  - the declare statement and the global environment
- Extend kernel syntax to give a full, practical syntax
  - nesting of partial values
  - implicit variable initialization
  - expressions
  - nesting the if and case statements
  - andthen and orelse operations
- Linguistic abstraction
  - Functions
- Exceptions

The interactive interface (declare)

- The interactive interface is a program that has a single global environment

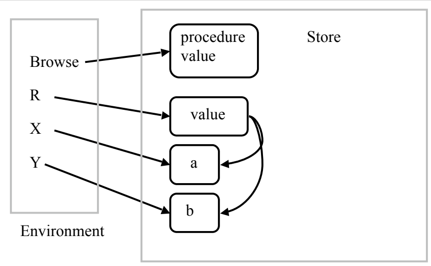
declare X Y

- Augments (and overrides) the environment with new mappings for X and Y

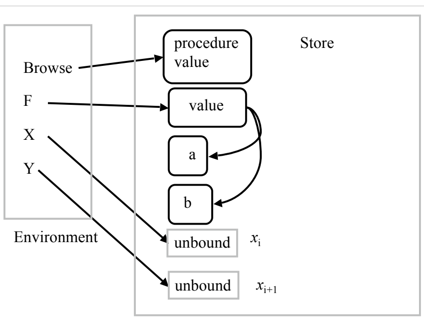
{Browse X}

- Inspects the store and shows partial values, and incremental changes

The interactive interface (declare)



declare X Y



Syntactic extensions

- **Nested partial values**
  - person(name: “George” age:25)

    local A B in A= “George” B=25 person(name:A age:B) end
- **Implicit variable initialization**
  - local ⟨pattern⟩ = ⟨expression⟩ in ⟨statement⟩ end
- **Example:**
  assume T has been defined, then

  local tree(key:A left:B right:C value:D) = T in ⟨statement⟩ end

  is the same as:

  local A B C D in
      T = tree(key:A left:B right:C value:D) <statement>
  end

Extracting fields in local statement

```
declare T
   :
   T = tree(key:seif age:48 profession:professor)
   :
local
   tree(key:A ...) = T
in
   ⟨statement⟩
end
```


Nested if and case statements

- Observe a pair notation is: 1 # 2, is the tuple '#'(1 2)

```
case Xs # Ys
of  nil # Ys then ⟨s⟩1
[]  Xs # nil then ⟨s⟩2
[]  (X|Xr) # (Y|Yr) andthen X=<Y then ⟨s⟩3
else ⟨s⟩4 end
```

- Is translated into

```
case Xs of nil then ⟨s⟩1
else
   case Ys of nil then ⟨s⟩2
   else
      case Xs of X|Xr then
         case Ys of Y|Yr then
               if X=<Y then ⟨s⟩3 else ⟨s⟩4 end
         else ⟨s⟩4 end
      else ⟨s⟩4 end
   end
end
```


Expressions

- An expression is a sequence of operations that returns a value
- A statement is a sequence of operations that does not return a value. Its effect is on the store, or outside of the system (e.g. read/write a file)
- 11*11 (expression)   X=11*11 (statement)

Functions as linguistic abstraction

- {F X1 ... Xn R}
- R = {F X1 ... Xn}

```
fun {F X1 ... Xn}
   ⟨statement⟩
   ⟨expression⟩
end
```

⟨statement⟩

```
proc {F X1 ... Xn R}
   ⟨statement⟩
   R = ⟨expression⟩
end
```

⟨statement⟩

Nesting in data structures

- Ys = {F X}|{Map Xr F}
- Is unnested to:


```
local Y Yr in
      Ys = Y|Yr
      {F X Y}
      {Map Xr F Yr}
end
```

- The unnesting of the calls occurs after the data structure

Functional nesting

- Nested notations that allows expressions as well as statements
- ```
  local R in
        {F X1 ... Xn R}
        {Q R ...}
  end
  ```
- Is written as (equivalent to):
- {Q {F X1 ... Xn} ...}

  {F X1 ... Xn}: expression

  {Q {F X1 ... Xn} ...}: statement
  ```

Conditional expressions

```
R = if ⟨expr⟩1 then
        ⟨expr⟩2
    else ⟨expr⟩3 end
```
⟨expression⟩

➡

```
if ⟨expr⟩1 then
   R = ⟨expr⟩2
else R = ⟨expr⟩3 end
```
⟨statement⟩

```
fun {Max X Y}
   if X>=Y then X
   else Y end
end
```

```
proc {Max X Y R}
   R = ( if X>=Y then X
         else Y end )
end
```


Example

```
fun {Max X Y}
   if X>=Y then X
   else Y end
end
```

➡

```
proc {Max X Y R}
   R = ( if X>=Y then X
         else Y end )
end
```

➡

```
proc {Max X Y R}
   if X>=Y then R = X
   else R = Y end
end
```


andthen and orelse

```
⟨expr⟩1 andthen ⟨expr⟩2
```

➡

```
if ⟨expr⟩1 then
   ⟨expr⟩2
else false end
```

```
⟨expr⟩1 orelse ⟨expr⟩2
```

➡

```
if ⟨expr⟩1 then
   true
else ⟨expr⟩2 end
```


Function calls

Observe

```
{F1 {F2 X} {F3 Y}}
```

➡

```
local R1 R2 in
   R1 = {F2 X}
   R2 = {F3 Y}
   {F1 R1 R2}
end
```

The arguments of a function are evaluated first from left to right

A complete example

```
fun {Map Xs F}
   case Xs
   of nil then nil
   [] X|Xr then {F X}|{Map Xr F}
   end
end
```

➡

```
proc {Map Xs F Ys}
   case Xs
   of nil then Ys = nil
   [] X|Xr then Y Yr in
         Ys = Y|Yr
         {F X Y}
         {Map Xr F Yr}
   end
end
```


Exceptions

- How to handle exceptional situations in the program?
- Examples:
  - divide by 0
  - opening a nonexistent file
- Some errors are programming errors
- Some errors are imposed by the external environment
- Exception handling statements allow programs to handle and recover from errors

Exceptions

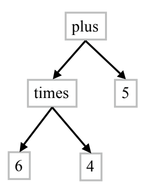
- The error confinement principle:
  - Define your program as a structured layers of components
  - Errors are visible only internally and a recovery procedure corrects the errors: either errors are not visible at the component boundary or are reported (nicely) to a higher level
- In one operation, exit from arbitrary depth of nested contexts
  - Essential for program structuring; else programs get complicated (use boolean variables everywhere, etc.)

Basic concepts

- A program that encounters an error (*exception*) should transfer execution to another part, the *exception handler* and give it a (partial) value that describes the error
- try $\langle s \rangle_1$ catch $\langle x \rangle$ then $\langle s \rangle_2$ end
- raise $\langle x \rangle$ end
- Introduce an exception marker on the semantic stack
- The execution is equivalent to $\langle s \rangle_1$ if it executes without raising an error
- Otherwise, $\langle s \rangle_1$ is aborted and the stack is popped up to the marker, the error value is transferred through $\langle x \rangle$, and $\langle s \rangle_2$ is executed

Exceptions (Example)

```
fun {Eval E}
   if {IsNumber E} then E
   else
      case E
      of plus(X Y) then {Eval X}+{Eval Y}
      [] times(X Y) then {Eval X}*{Eval Y}
      else raise illFormedExpression(E) end
      end
   end
end
```



Exceptions (Example)

```
try
   {Browse {Eval plus(5 6) }}
   {Browse {Eval plus(times(5 5) 6) }}
   {Browse {Eval plus(minus(5 5) 6) }}
catch illFormedExpression(E) then
   {System.showInfo "**** illegal expresion ****" # E}
end
```


Try semantics

- The semantic statement is
  (try $\langle s \rangle_1$ catch $\langle y \rangle$ then $\langle s \rangle_2$ end, E)
- Push the semantic statement (catch $\langle y \rangle$ then $\langle s \rangle_2$ end, E) on ST
- Push $(\langle s \rangle_1, E)$ on ST
- Continue to next execution step

Raise semantics

- The semantic statement is
  (raise $\langle x \rangle$ end, E)
- Pop elements off ST looking for a catch statement:
  - If a catch statement is found, pop it from the stack
  - If the stack is emptied and no catch is found, then stop execution with the error message "Uncaught exception"
- Let (catch $\langle y \rangle$ then $\langle s \rangle$ end, E_c) be the catch statement that is found
- Push $(\langle s \rangle, E_c + \{\langle y \rangle \rightarrow E(\langle x \rangle)\})$ on ST
- Continue to next execution step

Catch semantics

- The semantic statement is
  (catch ⟨x⟩ then ⟨s⟩ end, E)
- Continue to next execution step (like skip)

Full exception syntax

- Exception statements (expressions) with multiple patterns and finally clause
- Example:

```
:
FH = {OpenFile "xxxxx"}
:
try
   {ProcessFile FH}
catch X then
   {System.showInfo "***** Exception when processing *****" # X}
finally {CloseFile FH} end
```


Exercises

34. VRH Exercise 2.9.3 (page 107).
35. VRH Exercise 2.9.7 (page 108) –translate example to kernel language and execute using operational semantics.
36. Write an example of a program that suspends. Now, write an example of a program that never terminates. Use the operational semantics to prove suspension or non-termination.
37. *VRH Exercise 2.9.12 (page 110).
38. *Change the semantics of the case statement, so that patterns can contain variable labels and variable feature names.

Exercises

39. *Restrict the kernel language to make it strictly functional (i.e., without dataflow variables)
    - Language similar to Scheme (dynamically typed functional language)

    This is done by disallowing variable declaration (without initialization) and disallowing procedural syntax
    - Only use implicit variable initialization
    - Only use functions