Introduction to Programming Concepts (VRH 1.1-1.8)

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Introduction

- An introduction to programming concepts
- Declarative variables
- Functions
- Structured data (example: lists)
- Functions over lists
- Correctness and complexity
- Lazy functions
- Concurrency and dataflow
- State, objects, and classes
- Nondeterminism and atomicity

Variables

- Variables are short-cuts for values, they cannot be assigned more than once

```
declare
V = 9999*9999
{Browse V*V}
```

- Variable identifiers: is what you type
- Store variable: is part of the memory system
- The **declare** statement creates a store variable and assigns its memory address to the identifier 'V' in the environment

Functions

- Compute the factorial function: $n! = 1 \times 2 \times \cdots \times (n-1) \times n$
- Start with the mathematical definition

$$0! = 1$$
$$n! = n \times (n-1)! \text{ if } n > 0$$

```
declare
fun {Fact N}
   if N==0 then 1 else N*{Fact N-1} end
end
```

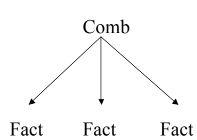
- Fact is declared in the environment
- Try large factorial {Browse {Fact 100}}

Composing functions

- Combinations of r items taken from n.
- The number of subsets of size r taken from a set of size n

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

```
declare
fun {Comb N R}
  {Fact N} div ({Fact R}*{Fact N-R})
end
```



- Example of functional abstraction

Structured data (lists)

- Calculate Pascal triangle
- Write a function that calculates the nth row as one structured value
- A list is a sequence of elements: [1 4 6 4 1]
- The empty list is written nil
- Lists are created by means of "|" (cons)

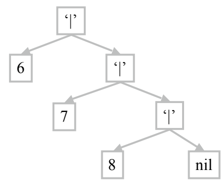
```
declare
H=1
T = [2 3 4 5]
{Browse H|T}  % This will show [1 2 3 4 5]
```

```
        1
      1   1
    1   2   1
  1   3   3   1
1   4   6   4   1
```

Lists (2)

- Taking lists apart (selecting components)
- A cons has two components: a head, and a tail

```
declare L = [5 6 7 8]
L.1 gives 5
L.2 give [6 7 8]
```



Pattern matching

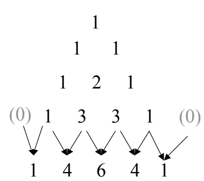
- Another way to take a list apart is by use of pattern matching with a case instruction

```
case L of H|T then {Browse H} {Browse T} end
```


Functions over lists

- Compute the function {Pascal N}
- Takes an integer N, and returns the Nth row of a Pascal triangle as a list

1. For row 1, the result is [1]
2. For row N, shift to left row N-1 and shift to the right row N-1
3. Align and add the shifted rows element-wise to get row N

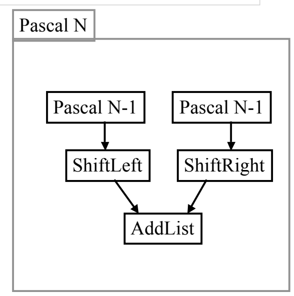


Shift right [0 1 3 3 1]

Shift left [1 3 3 1 0]

Functions over lists (2)

```
declare
fun {Pascal N}
  if N==1 then [1]
  else
    {AddList
     {ShiftLeft {Pascal N-1}}
     {ShiftRight {Pascal N-1}}}
  end
end
```



Functions over lists (3)

```
fun {ShiftLeft L}
  case L of H|T then
    H|{ShiftLeft T}
  else [0]
  end
end

fun {ShiftRight L} 0|L end
```

```
fun {AddList L1 L2}
  case L1 of H1|T1 then
    case L2 of H2|T2 then
      H1+H2|{AddList T1 T2}
    end
  else nil end
end
```


Top-down program development

- Understand how to solve the problem by hand
- Try to solve the task by decomposing it to simpler tasks
- Devise the main function (main task) in terms of suitable auxiliary functions (subtasks) that simplifies the solution (ShiftLeft, ShiftRight and AddList)
- Complete the solution by writing the auxiliary functions

Is your program correct?

- "A program is correct when it does what we would like it to do"
- In general we need to reason about the program:
- **Semantics for the language**: a precise model of the operations of the programming language
- **Program specification**: a definition of the output in terms of the input (usually a mathematical function or relation)
- Use mathematical techniques to reason about the program, using programming language semantics

Mathematical induction

- Select one or more inputs to the function
- Show the program is correct for the *simple cases* (base cases)
- Show that if the program is correct for a *given case*, it is then correct for the *next case*.
- For natural numbers, the base case is either 0 or 1, and for any number n the next case is n+1
- For lists, the base case is nil, or a list with one or a few elements, and for any list T the next case is H|T

Correctness of factorial

```
fun {Fact N}
  if N==0 then 1 else N*{Fact N-1} end
end
```

$$\underbrace{1 \times 2 \times \cdots \times (n-1)}_{Fact(n-1)} \times n$$

- Base Case N=0: {Fact 0} returns 1
- Inductive Case N>0: {Fact N} returns N*{Fact N-1} assume {Fact N-1} is correct, from the spec we see that {Fact N} is N*{Fact N-1}

Complexity

- Pascal runs very slow, try {Pascal 24}
- {Pascal 20} calls: {Pascal 19} twice, {Pascal 18} four times, {Pascal 17} eight times, ..., {Pascal 1} 2^{19} times
- Execution time of a program up to a constant factor is called the program's *time complexity*.
- Time complexity of {Pascal N} is proportional to 2^N (exponential)
- Programs with exponential time complexity are impractical

```
declare
fun {Pascal N}
  if N==1 then [1]
  else
    {AddList
     {ShiftLeft {Pascal N-1}}
     {ShiftRight {Pascal N-1}}}
  end
end
```


Faster Pascal

- Introduce a local variable L
- Compute {FastPascal N-1} only once
- Try with 30 rows.
- FastPascal is called N times, each time a list on the average of size N/2 is processed
- The time complexity is proportional to N^2 (polynomial)
- Low order polynomial programs are practical.

```
fun {FastPascal N}
  if N==1 then [1]
  else
     local L in
        L={FastPascal N-1}
        {AddList {ShiftLeft L} {ShiftRight L}}
     end
  end
end
```


Lazy evaluation

- The functions written so far are evaluated eagerly (as soon as they are called)
- Another way is lazy evaluation where a computation is done only when the results is needed

- Calculates the infinite list: 0 | 1 | 2 | 3 | ...

```
declare
fun lazy {Ints N}
  N|{Ints N+1}
end
```

Lazy evaluation (2)

- Write a function that computes as many rows of Pascal's triangle as needed
- We do not know how many beforehand
- A function is *lazy* if it is evaluated only when its result is needed
- The function PascalList is evaluated when needed

```
fun lazy {PascalList Row}
  Row | {PascalList
          {AddList
             {ShiftLeft Row}
             {ShiftRight Row}}}
end
```


Lazy evaluation (3)

- Lazy evaluation will avoid redoing work if you decide first you need the 10th row and later the 11th row
- The function continues where it left off

```
declare
L = {PascalList [1]}
{Browse L}
{Browse L.1}
{Browse L.2.1}
```

```
L<Future>
[1]
[1 1]
```


Exercises

32. Define Add in Oz using the Zero and Succ functions representing numbers in the lambda-calculus.
33. Prove that Add is correct using induction.
34. *Prove the correctness of AddList and ShiftLeft using induction
35. *VRH Exercise 1.18.5.