JOIN CONTINUATIONS

Consider:

$$
\begin{aligned}
\mathrm{treeprod} = \mathrm{rec}(\lambda f.\ \lambda \mathrm{tree}.\ & \\
& \mathrm{if}(\mathrm{isnat}(\mathrm{tree}), \\
& \quad \mathrm{tree}, \\
& \quad f(\mathrm{left}(\mathrm{tree})) * f(\mathrm{right}(\mathrm{tree})) \\
& \quad))
\end{aligned}
$$

which multiplies all leaves of a tree, which are numbers.

You can do the "left" and "right" computations concurrently.

TREE PRODUCT BEHAVIOR

$B_{treeprod}$ =

```
rec(λb. λself. λm.
    seq( become(b(self)),
         if(isnat(tree(m)),
            send(cust(m), tree(m)),
            letactor{newcust := B_joincont
                                 (cust(m), ∅, nil)}
              seq(send(self,
                    pr(left(tree(m)), newcust)),
                  send(self,
                    pr(right(tree(m)), newcust)))
         )
    )
)
```

$B_{joincont}$ =

rec (λb. λcust, λnargs, λfirstnum, λnum

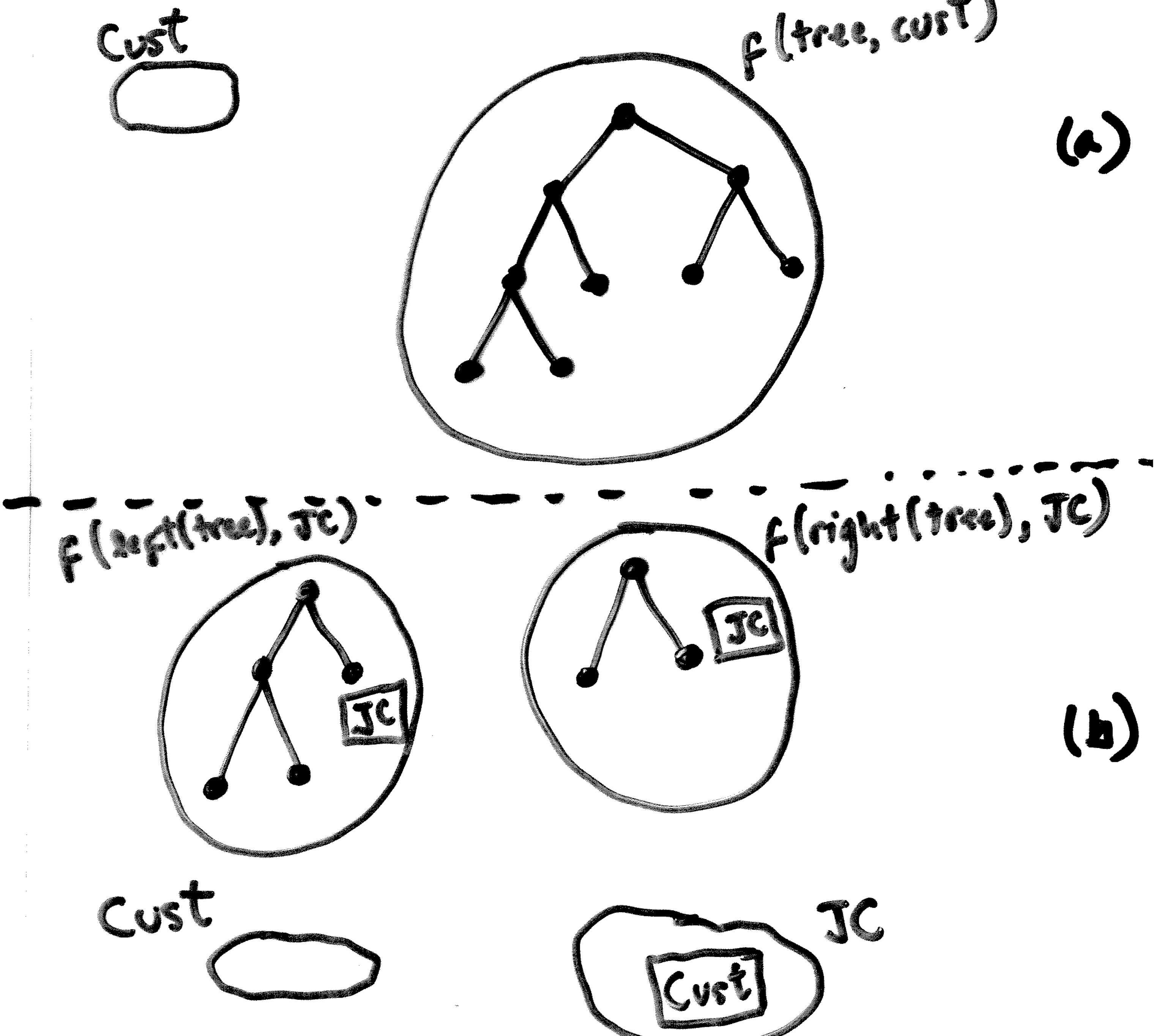
if (eq (nargs, 0),

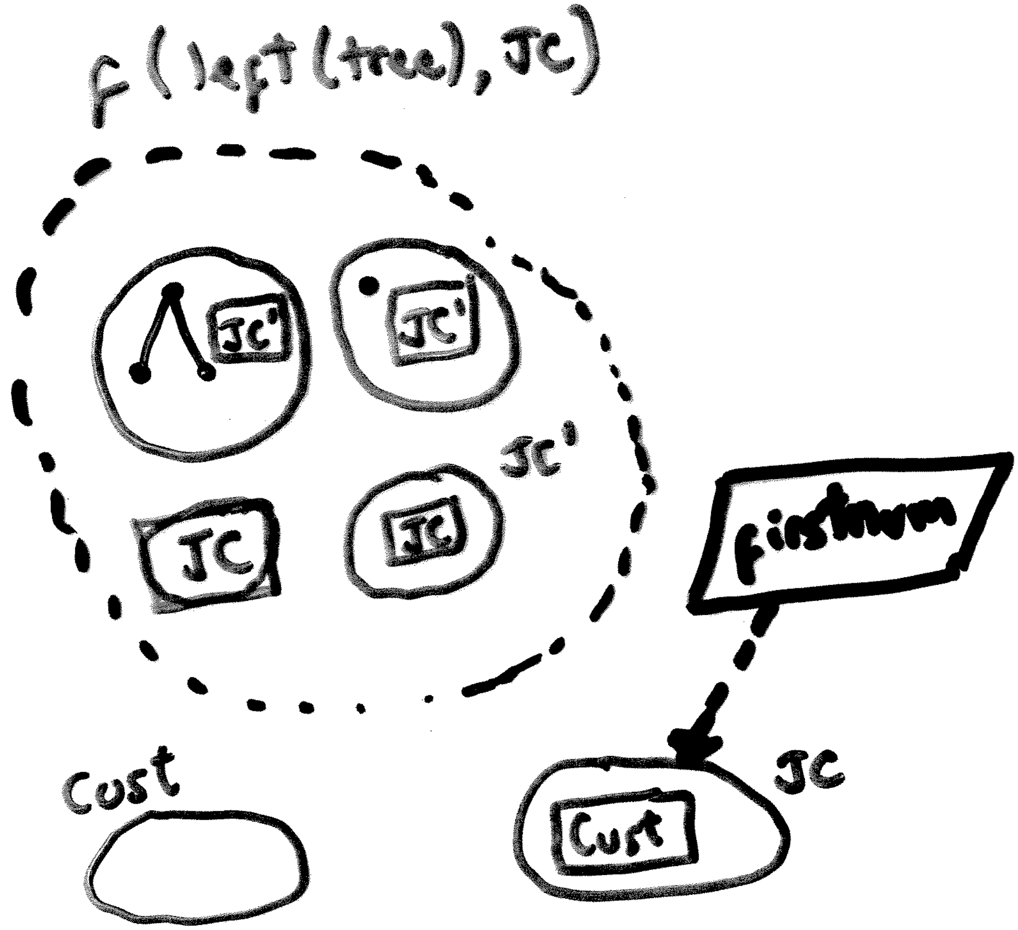
become (b (cust, 1, num)),

seq (become (sink),

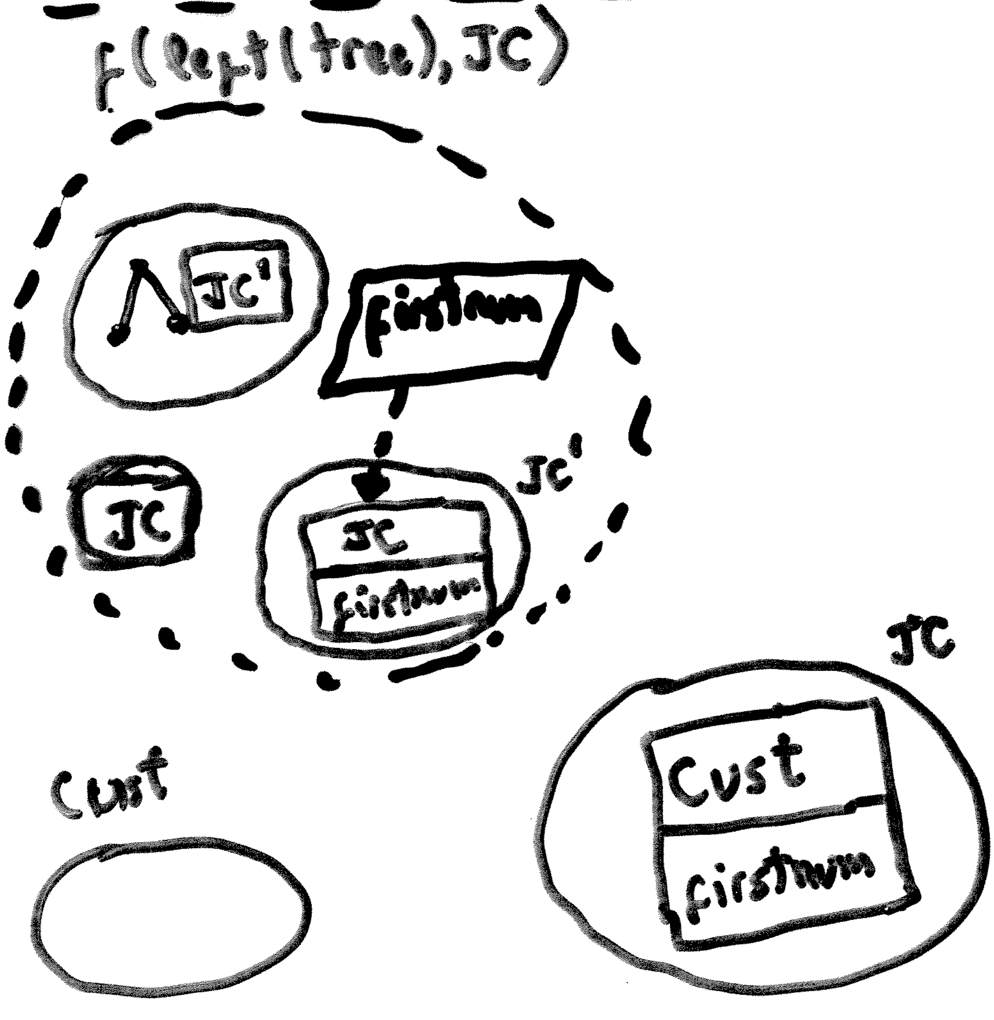
send (cust, firstnum * num))))

SAMPLE EXECUTION

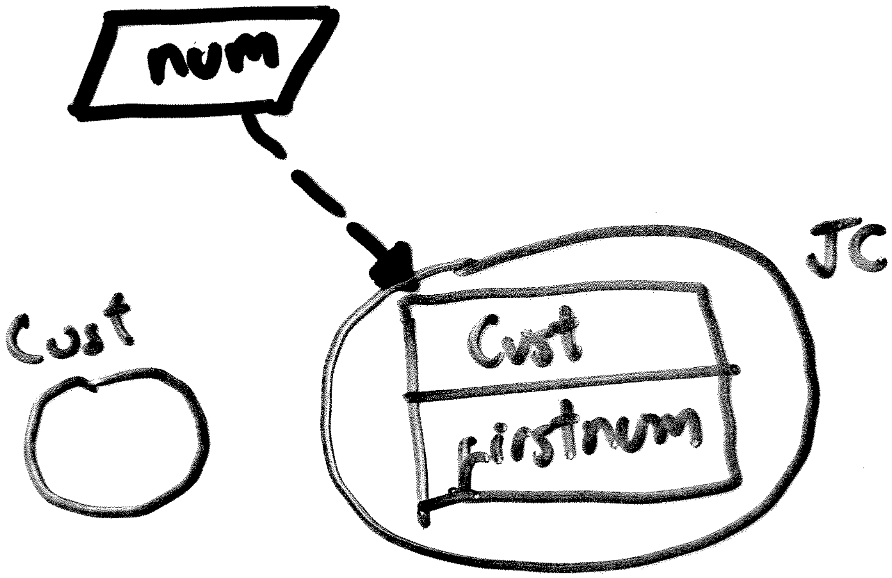



f(left(tree), JC)
JC'
JC'
JC'
JC
JC
firstnum
Cust
Cust
JC

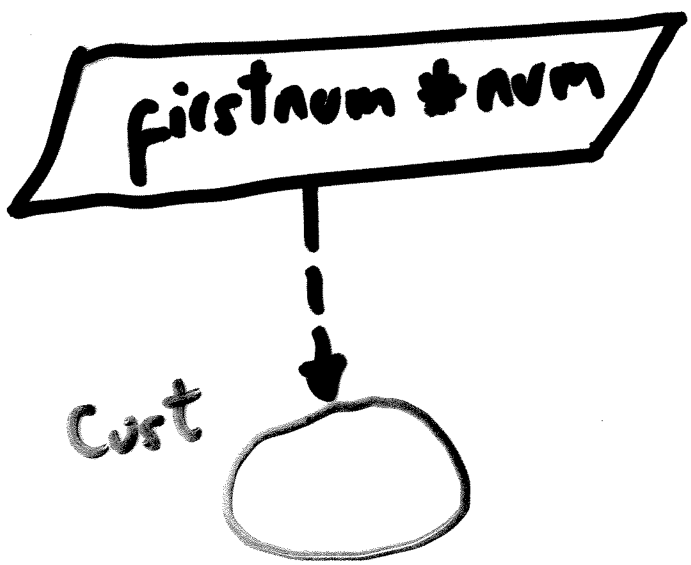
(c)


f(left(tree), JC)
JC'
firstnum
JC'
JC
JC
firstnum
JC
Cust
Cust
firstnum

(d)



(e)



(f)

OPERATIONAL SEMANTICS FOR ACTOR CONFIGURATIONS

$$\kappa = \langle\langle \alpha \mid \mu \rangle\rangle_{\chi}^{\rho}$$

α is a function mapping actor names (represented as variables) to actor states.

μ is a multi-set of messages "en-route".

ρ is a set of receptionists

χ is a set of external actors.

Given $A = Dom(\alpha)$:

$$\rho \subseteq A \quad , \quad A \cap \chi = \emptyset$$

- if $\alpha(a) = (?_{a'})$, then $a' \in A$
- if $a \in A$, then $FV(\alpha(a)) \subseteq A \cup \chi$.
- if $\langle v_0 \Leftarrow v_1 \rangle \in \mu$, $FV(v_0, v_1) \subseteq A \cup \chi$.

$\underline{\alpha} \in \mathbb{X} \xrightarrow{f} A_s$

$A_s = (?_x) \cup (\mathbb{V}) \cup [\mathbb{E}]$

$\underline{\mu} \in M_\omega[\mathbb{M}]$

$\mathbb{M} = \langle \mathbb{V} \Leftarrow \mathbb{V} \rangle$

$\langle a \Leftarrow s \rangle$

$\underline{\rho}, \underline{\chi} \in P_\omega[\mathbb{X}]$

$\mathbb{V} = \mathbb{A}t \cup \mathbb{X} \cup \lambda\mathbb{X}.\mathbb{E} \cup pr(\mathbb{V}, \mathbb{V})$

$\mathbb{E} = \mathbb{V} \cup app(\mathbb{E}, \mathbb{E}) \cup \mathbb{F}_n(\mathbb{E}^n)$

Labelled Transition Relation ($\mapsto$)

$\langle \text{fun} : a \rangle$

$$e \xmapsto{\lambda}_{Dom(\alpha) \cup \{a\}} e' \Rightarrow$$

$$\langle\!\langle \alpha, [e]_a \mid \mu \rangle\!\rangle^{\rho}_{\chi} \mapsto \langle\!\langle \alpha, [e']_a \mid \mu \rangle\!\rangle^{\rho}_{\chi}$$

$\langle \text{newactor} : a, a' \rangle$

$$\langle\!\langle \alpha, [R[\text{newactor}(e)]]_a \mid \mu \rangle\!\rangle^{\rho}_{\chi} \mapsto$$

$$\langle\!\langle \alpha, [R[a']]_a, [e]_{a'} \mid \mu \rangle\!\rangle^{\rho}_{\chi} \quad a' \text{ fresh}$$

$\langle \text{send} : a, v_0, v_1 \rangle$

$$\langle\!\langle \alpha, [R[\text{send}(v_0, v_1)]]_a \mid \mu \rangle\!\rangle^{\rho}_{\chi} \mapsto$$

$$\langle\!\langle \alpha, [R[\text{nil}]]_a \mid \mu, \langle v_0 \Leftarrow v_1 \rangle \rangle\!\rangle^{\rho}_{\chi}$$

LABELLED TRANSITION RELATION ($\mapsto$) CONTINUED

$\langle \mathit{receive} : v_0, v_1 \rangle$

$$\langle\!\langle \alpha, [\mathit{ready}(v)]_{v_0} \mid \langle v_0 \Leftarrow v_1 \rangle, \mu \rangle\!\rangle^{\rho}_{\chi} \mapsto$$

$$\langle\!\langle \alpha, [\mathit{app}(v, v_1)]_{v_0} \mid \mu \rangle\!\rangle^{\rho}_{\chi}$$

$\langle \mathit{out} : v_0, v_1 \rangle$

$$\langle\!\langle \alpha \mid \mu, \langle v_0 \Leftarrow v_1 \rangle \rangle\!\rangle^{\rho}_{\chi} \mapsto \langle\!\langle \alpha \mid \mu \rangle\!\rangle^{\rho'}_{\chi}$$

if $v_0 \in \chi$ and $\rho' = \rho \cup (FV(v_1) \cap Dom(\alpha))$

$\langle \mathit{in} : v_0, v_1 \rangle$

$$\langle\!\langle \alpha \mid \mu \rangle\!\rangle^{\rho}_{\chi} \mapsto \langle\!\langle \alpha \mid \mu, \langle v_0 \Leftarrow v_1 \rangle \rangle\!\rangle^{\rho}_{\chi'}$$

if $v_0 \in \rho$ and $FV(v_1) \cap Dom(\alpha) \subseteq \rho$

and $\chi' = \chi \cup (FV(v_1) - Dom(\alpha))$

COMPUTATION SEQUENCES AND PATHS

If K is a configuration, then the computation tree $\mathcal{T}(K)$ is the set of all finite sequences of labelled transitions $[K_i \xrightarrow{l_i} K_{i+1} \mid i < n]$ for some $n \in \mathbb{N}$, with $K = K_0$. Such sequences are called computation sequences.

A computation path from K is a maximal linearly ordered set of computation sequences in the computation tree, $\mathcal{T}(K)$.

$\mathcal{T}^\infty(K)$ denotes the set of all paths from K.

FAIRNESS

A path $\pi = [k_i \xrightarrow{l_i} k_{i+1} \mid i < \alpha]$ in the computation tree $\tau^\infty(k)$ is *fair* if each enabled transition eventually happens or becomes permanently disabled.

For a configuration k we define $\mathcal{F}(k)$ to be the subset of $\tau^\infty(k)$ that are fair.

COMPOSITION OF ACTOR CONFIGURATIONS

$$\kappa_0 = \langle\!\langle \alpha_0 \mid \mu_0 \rangle\!\rangle^{\rho_0}_{\chi_0}$$

$$\kappa_1 = \langle\!\langle \alpha_1 \mid \mu_1 \rangle\!\rangle^{\rho_1}_{\chi_1}$$

$$\kappa_0 \parallel \kappa_1 = \langle\!\langle \alpha_0 \cup \alpha_1 \mid \mu_0 \uplus \mu_1 \rangle\!\rangle^{\rho_0 \cup \rho_1}_{(\chi_0 \cup \chi_1) - (\rho_0 \cup \rho_1)}$$

Configurations are "composable" if

$$Dom(\alpha_0) \cap Dom(\alpha_1) = \emptyset$$

$$\chi_0 \cap Dom(\alpha_1) \subseteq \rho_1$$

$$\chi_1 \cap Dom(\alpha_0) \subseteq \rho_0$$

$$\kappa_\emptyset = \langle\!\langle \emptyset \mid \emptyset \rangle\!\rangle^{\emptyset}_{\emptyset}$$

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$$K_0 \| K_1 = K_1 \| K_0$$

$$K_0 \| K_\emptyset = K_0$$

$$(K_0 \| K_1) \| K_2 = K_0 \| (K_1 \| K_2)$$

CLOSED CONFIGURATIONS

A configuration in which both the receptionist and external actors sets are empty is said to be closed.

COMPOSITION PRESERVES COMPUTATION PATHS

$$\mathcal{T}(K_0 \| K_1) = \mathcal{T}(K_0) \| \mathcal{T}(K_1)$$

$$\mathcal{F}(K_0 \| K_1) = \mathcal{F}(K_0) \| \mathcal{F}(K_1)$$