EQUIVALENCE OF EXPRESSIONS

$\left.\begin{array}{l}\text{Operational equivalence}\\ \text{Testing equivalence}\end{array}\right\}$ Observational Equivalence

Two program expressions are said to be equivalent if they <u>behave the same</u> when placed in <u>any observing context.</u>

An observing context is a complete program with a hole, such that all free variables in expressions being evaluated become captured, when placed in the hole.

EVENTS AND OBSERVING CONTEXTS

A new event primitive operator is introduced.

The $\mapsto$ reduction relation is extended:

$\langle e: a \rangle$

$$\langle\!\langle \alpha, [R[\mathrm{event}()]]_a \mid \mu \rangle\!\rangle^{\rho}_{x}$$

$$\mapsto \langle\!\langle \alpha, [R[\mathrm{nil}]]_a \mid \mu \rangle\!\rangle^{\rho}_{x}$$

An observing configuration is one of the form:

$$\langle\!\langle \alpha, [C]_a \mid \mu \rangle\!\rangle$$

where C is a hole-containing expression, or context.

OBSERVATIONS

Let K be a configuration of the extended language, and let $\pi = [K_i \xrightarrow{l_i} K_{i+1} \mid i < \bowtie]$ be a fair path, i.e. $\pi \in F(K)$. Define:

$$obs(\pi) = \begin{cases} s & \text{if } (\exists i < \bowtie, a)(l_i = \langle e : a \rangle) \\ f & \text{otherwise} \end{cases}$$

$$Obs(K) = \begin{cases} s & \text{if } (\forall \pi \in F(K))(obs(\pi) = s) \\ sf & \text{otherwise} \\ f & \text{if } (\forall \pi \in F(K))(obs(\pi) = f) \end{cases}$$

THREE EQUIVALENCES

The natural equivalence is equal observations are made in all closing configuration contexts.

Other two equivalences (weaker) arise if sf observations are considered as good as s observations; or if sf observations are considered as bad as f observations.

TESTING OR CONVEX OR PLOTKIN OR EGLI-MILNER

$$e_0 \cong_1 e_1 \text{ iff } Obs(O[e_0]) = Obs(O[e_1]).$$

MUST OR UPPER OR SMYTH

$$e_0 \cong_2 e_1 \text{ iff } Obs(O[e_0]) = s \Longleftrightarrow Obs(O[e_1]) = s$$

MAY OR LOWER OR HOARE

$$e_0 \cong_3 e_1 \text{ iff } Obs(O[e_0]) = f \Longleftrightarrow Obs(O[e_1]) = f$$

Congruence

$$e_0 \cong_j e_1 \Rightarrow C[e_0] \cong_j C[e_1] \quad \text{for } j = 1, 2, 3$$

By contrustion, all equivalences defined are congruences.

Partial Collapse

$\cong_1$

e_0 \ e_1	s	sf	f
s	✓	✗	✗
sf	✗	✓	✗
f	✗	✗	✓

$\cong_2$

e_0 \ e_1	s	sf	f
s	✓	✗	✗
sf	✗	✓	*
f	✗	*	✓

$\cong_3$

e_0 \ e_1	s	sf	f
s	✓	✓	✗
sf	✓	✓	✗
f	✗	✗	✓

$(1 = 2)$ $e_0 \cong_1 e_1$ iff $e_0 \cong_2 e_1$ (due to fairness)

$(1 \Rightarrow 3)$ $e_0 \cong_1 e_1$ implies $e_0 \cong_3 e_1$